# Time Series

## EE426 Semester 2/2013

Part 2: Unit Root Test and Spurious Regression

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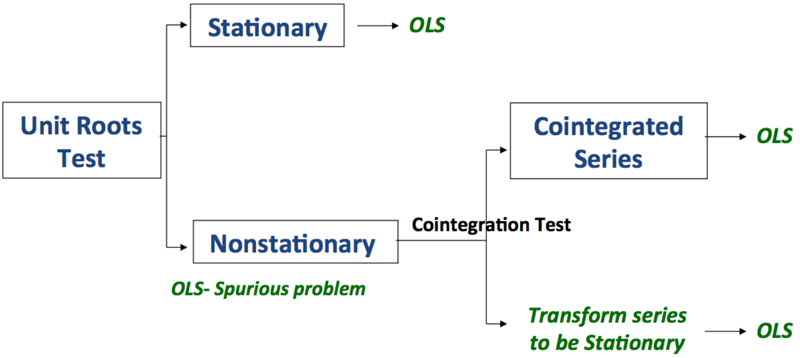
# Basic Concept of Time Series



# Unit root process

- A random walk is a special case of a unit root process
- AR(1): $y_t = \alpha + \rho y_{t-1} + e_t$ (1)
  - $E(e_t | y_{t-1}, y_{t-2}, ..., y_0) = 0$
  - $\{y_t\}$ has a unit root if, and only if, $\rho = 1$.
  - If $\alpha = 0$ and $\rho = 1$, $\{y_t\}$ follows a random walk without drift.
  - If $\alpha \neq 0$ and $\rho = 1$, $\{y_t\}$ follows a random walk with drift.

# Integrated series

- Nonstationary series can be intergrated series if the series is differentiated one or more times, the resulting series will be stationary.
- Stationary process (weakly dependent process) is integrated of order zero, I(0).
- A random walk is integrated of order one, I(1), meaning that a first difference is weakly dependent, hence I(0).
  - A time series that is I(1) is a difference-stationary process.
- If $y_t$ is nonstationary, but its $d^{th}$ difference is stationary, the series is integrated of order d, or I(d).

# Testing for Unit roots

## Model 1

- Null hypothesis: $\rho = 1$; Alternative hypothesis: $\rho < 1$ (one-sided test)
  - A test of whether $\{y_t\}$ is I(1) against alternative that $\{y_t\}$ is I(0)
- A convenient equation for carrying out the unit root test is $\triangle y_t = \alpha + \theta y_{t-1} + e_t$, where $\theta = \rho - 1$ (2)
- Then, $H_0 : \theta = 0$, againt $H_1 : \theta < 0$
- Under $H_0$, $y_{t-1}$ is I(1), so the asymptotic standard normal distribution for t stat does not apply. We need new critical values.
- We use the Dickey-Fuller (DF) test for a unit root.

# Testing for Unit roots

- Table of critical values for DF test:

**Asymptotic Critical Values for Unit Root $t$ Test: No Time Trend**

| Significance level | 1% | 2.5% | 5% | 10% |
|---|---|---|---|---|
| Critical value | −3.43 | −3.12 | −2.86 | −2.57 |

- Still use the usual t statistic ($t_{\hat{\theta}}$), but compare to this table for asymptotic critical values.
- Reject $H_0$ if $t_{\hat{\theta}} < c$
- When we fail to reject a unit root, we should only conclude that the data do not provide strong evidence against $H_0$

# Testing for Unit roots
## Model 2

- We might need to test for unit roots in models with more complicated dynamics
- If $\{y_t\}$ follows AR(1) with $\rho = 1$, then $\triangle y_t$ is serially uncorrelated.
- We can allow $\{\triangle y_t\}$ to follow an AR model by augmenting eq. (2) with additional lags
  $\triangle y_t = \alpha + \theta y_{t-1} + \gamma_1 \triangle y_{t-1} + e_t$, where $|\gamma_1| < 1$ (3)
- That is, we can add $p$ lags of $\triangle y_t$ to the equation to account for the dynamics in the process.
  - How many tags depend on the frequency of the data.
- This extended version is called the augmented Dicky-Fuller test. Still use the same critical values as in Model 1.

# Testing for Unit roots

## Model 3

- Trend-stationary vs. unit root with trend
- $\triangle y_t = \alpha + \delta t + \theta y_{t-1} + e_t$ (4)
- $H_0 : \theta = 0$, $H_1 : \theta < 0 \rightarrow \{y_t\}$ is a trend-stationary process under alternative.
- If $y_t$ has a unit root, then $\triangle y_t = \alpha + \delta t + e_t$
  - A change in $y_t$ has a mean linear in t unless $\delta = 0$
- With time trend, the critical values of the unit root test change.

**Asymptotic Critical Values for Unit Root *t* Test: Linear Time Trend**

| Significance level | 1% | 2.5% | 5% | 10% |
|---|---|---|---|---|
| Critical value | −3.96 | −3.66 | −3.41 | −3.12 |

# Spurious regression

- A situation of finding a relationship between two or more trending variables simply because each is growing over time.
  - Find high $R^2$ or statistically significant t-stat, but no meaning of this significant relationship
  - Check: $R^2 > Durbin - Watson \rightarrow$ likely to have a spurious regression
- If the series are weakly dependent about their time trends, we can solve by including time trend in the regression model.
- We are likely to find a significant t-stat on a simple regression involving two independent I(1) series even if these two series are not trending.
- If the series are I(1) or higher order of integration, check for cointegration.