Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models

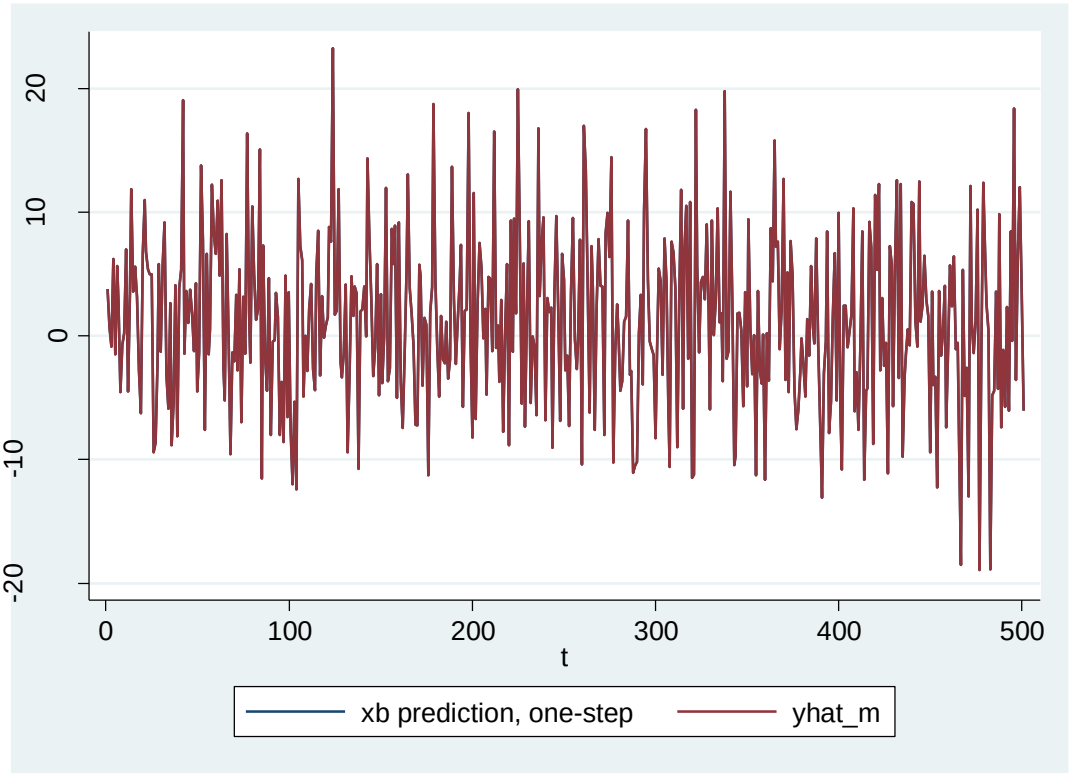
Example Simulated Data

```
set obs 501
g t=_n
tsset t
g s2=1 in 1
g u=rnormal(0,s2) in 1
g u2=u^2 in 1
forvalue i=2(1)501 {
  replace s2=0.3+0.3*l.u2+0.3*l.s2 in `i'
  replace u=rnormal(0,s2) in `i'
  replace u2=u^2 in `i'
}
g x=rnormal(1,10)
g y=0.5+0.7*x+u
reg y x if t>1
estat archlm
arch y x  if t>1, arch(1) garch(1) nolog
predict s2hat, v
twoway (line s2hat t) (scatter s2 t)
```

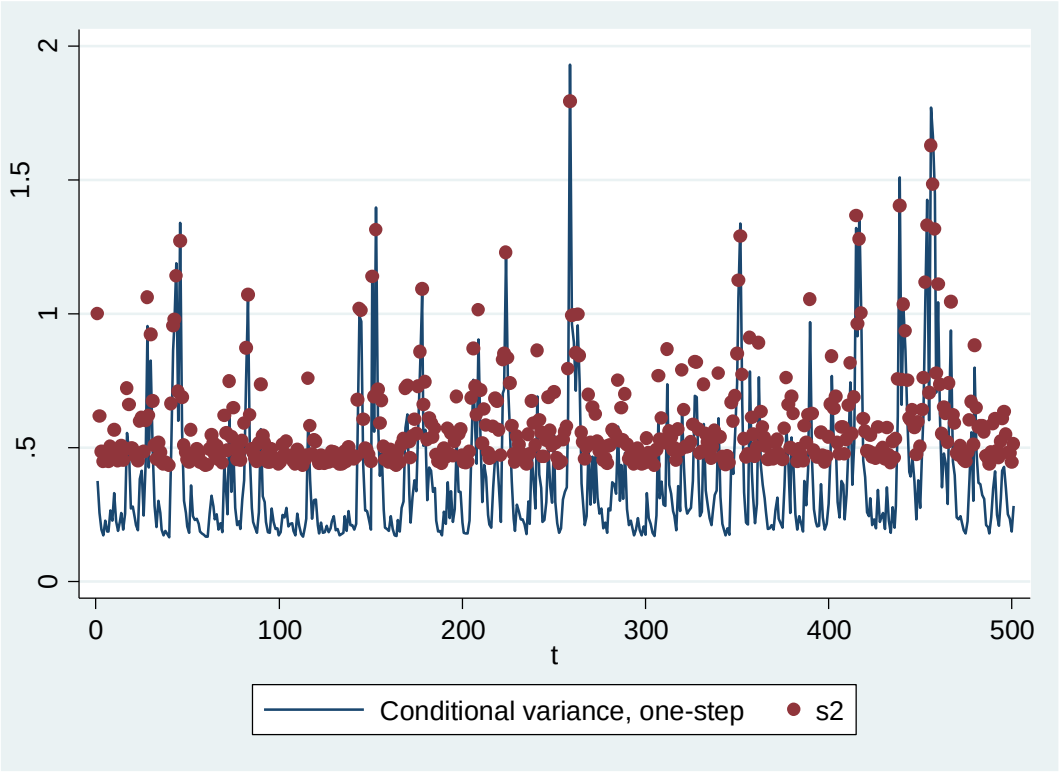
```
. set obs 501
number of observations (_N) was 0, now 501

. g t=_n

. tsset t
        time variable:  t, 1 to 501
                delta:  1 unit

. g s2=1 in 1
(500 missing values generated)

. g u=rnormal(0,s2) in 1
(500 missing values generated)

. g u2=u^2 in 1
(500 missing values generated)

. forvalue i=2(1)501 {
  2.   replace s2=0.3+0.3*l.u2+0.3*l.s2 in `i'
  3.   replace u=rnormal(0,s2) in `i'
  4.   replace u2=u^2 in `i'
  5. }

. g x=rnormal(1,10)

. g y=0.5+0.7*x+u

. reg y x if t>1
      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =  63539.73
       Model |  23398.9523         1  23398.9523   Prob > F        =    0.0000
    Residual |  183.391992       498  .368257012   R-squared       =    0.9922
-------------+----------------------------------   Adj R-squared   =    0.9922
       Total |  23582.3443       499   47.259207   Root MSE        =    .60684

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .7013338   .0027823   252.07   0.000     .6958673    .7068002
       _cons |   .4743123   .0273053    17.37   0.000     .4206645    .5279601
------------------------------------------------------------------------------
```

```
. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-----------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+---------------------------------------------------------------
       1     |         35.821               1                   0.0000
-----------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance

. arch y x  if t>1, arch(1) garch(1) nolog

ARCH family regression

Sample: 2 - 501                                   Number of obs   =        500
Distribution: Gaussian                            Wald chi2(1)    =   84227.65
Log likelihood =  -426.751                        Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
           x |   .7028459   .0024218   290.22   0.000     .6980993    .7075925
       _cons |   .4758044    .023265    20.45   0.000     .4302058    .5214029
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .3970325    .093721     4.24   0.000     .2133428    .5807223
             |
       garch |
         L1. |   .3355659    .112419     2.98   0.003     .1152287    .5559032
             |
       _cons |    .105968   .0289994     3.65   0.000     .0491302    .1628059
------------------------------------------------------------------------------

. predict s2hat, v

. predict yhat, xb

. mat beta=e(b)

. mat list beta

beta[1,5]
            y:          y:      ARCH:      ARCH:      ARCH:
                                   L.         L.
            x       _cons       arch      garch       _cons
y1  .70284588  .47580435  .39703251  .33556595  .10596802

. sca b0=el(beta,1,2)

. sca b1=el(beta,1,1)

. sca a0=el(beta,1,5)

. sca d1=el(beta,1,4)

. sca a1=el(beta,1,3)

. g yhat_m=b0+b1*x

. g ehat_m=y-yhat_m

. predict uhat, r

. g uhat2=uhat^2

. g s2hat_m=a0+d1*l.uhat2+a1*l.uhat2 in 2
```

```
(500 missing values generated)

. replace s2hat_m=a0+d1*l.s2hat_m+a1*l.uhat2 if t>2
(499 real changes made)

. line yhat yhat_m t
```




```
. twoway (line s2hat t) (scatter s2 t)
```



Example

$$SET_t = \beta_0 + \beta_1 IBR + \beta_2 GOLDB + \beta_3 USDS + \varepsilon_t$$

where: SET_t = Return on Stock Exchange of Thailand (SET).

IBR_t = Inter-bank rate.

$GOLDB_t$ = Gold price.

$USDS_t$ = Exchange rate ($US/฿Baht).

ε_t = Residual which has GARCH(p,q) process:

$$\sigma_t^2 = \alpha_0 + \delta_1\sigma_{t-1}^2 + \delta_2\sigma_{t-2}^2 + \cdots + \delta_p\sigma_{t-p}^2 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2$$

Testing GARCH Effect

Firstly, estimate the model using OLS without ARCH(p) process.

```
. regress set ibr usds goldb

      Source |       SS       df       MS              Number of obs =     118
-------------+------------------------------           F(  3,   114) =  103.43
       Model |  215609.598     3  71869.8659           Prob > F      =  0.0000
    Residual |  79215.8193   114  694.875608           R-squared     =  0.7313
-------------+------------------------------           Adj R-squared =  0.7242
       Total |  294825.417   117  2519.87536           Root MSE      =   26.36

------------------------------------------------------------------------------
         set |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ibr |  -142.0644   68.73617    -2.07   0.041    -278.2302   -5.898551
        usds |  -78.06484    5.30484   -14.72   0.000    -88.57369     -67.556
       goldb |    .029042   .0192431     1.51   0.134    -.0090784    .0671624
       _cons |   3698.659   332.8835    11.11   0.000     3039.219    4358.099
------------------------------------------------------------------------------

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-----------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+---------------------------------------------------------------
       1     |         63.052               1                   0.0000
-----------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

In this case, since p-value of the ARCH effect test (F-statistic or Chi-Square (Obs*R-squared = (117*0.538907) = 63.05211)) is less than level of significance 0.05, thus, null hypothesis that there is no ARCH effect is rejected, thus, there exists significant ARCH effect in this model with 0.05 significant level.

Identify Order (p,q) and Estimation

The next step is to identify order of GARCH(p,q) by estimating GARCH models in several orders and choose the model with the lowest AIC or SIC.

Estimate GARCH(p,q) using MLE

```
. arch set ibr usds goldb, arch(1) nolog
BFGS stepping has contracted, resetting BFGS Hessian (0)
      ...
BFGS stepping has contracted, resetting BFGS Hessian (4)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1122.15
Log likelihood = -517.3025                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -64.13751   41.80188    -1.53   0.125    -146.0677    17.79268
        usds |  -72.48378    2.63951   -27.46   0.000    -77.65712   -67.31043
       goldb |   .0590191   .0138355     4.27   0.000      .031902    .0861362
       _cons |   3178.165    197.789    16.07   0.000     2790.506    3565.825
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9245129   .3233346     2.86   0.004     .2907888    1.558237
       _cons |   93.59632   38.20728     2.45   0.014     18.71142    168.4812
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC          BIC
-------------+---------------------------------------------------------------
           . |    118           .   -517.3025      6     1046.605    1063.229
-----------------------------------------------------------------------------

. arch set ibr usds goldb, arch(1/2) nolog
BFGS stepping has contracted, resetting BFGS Hessian (17)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1158.49
Log likelihood = -516.9879                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -61.07753   33.81901    -1.81   0.071    -127.3616    5.206507
        usds |  -72.49558   2.831238   -25.61   0.000     -78.0447   -66.94645
       goldb |   .0599955   .0140527     4.27   0.000     .0324528    .0875382
       _cons |    3167.73   207.2899    15.28   0.000      2761.45    3574.011
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9952712   .3383216     2.94   0.003      .332173    1.658369
         L2. |  -.1255426   .1038731    -1.21   0.227    -.3291301     .078045
       _cons |    117.519   40.07261     2.93   0.003     38.97815    196.0599
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC          BIC
-------------+---------------------------------------------------------------
           . |    118           .   -516.9879      7     1047.976    1067.371
-----------------------------------------------------------------------------
```

Estimate GARCH(1,1)

```
. arch set usds, arch(1) garch(1) nolog

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(1)       =    882.55
Log likelihood = -518.5556                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
        usds |  -74.04275   2.492366   -29.71   0.000    -78.92769    -69.1578
       _cons |   3607.771   97.79026    36.89   0.000     3416.106    3799.437
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   1.063916    .297246     3.58   0.000     .4813242    1.646507
       garch |
         L1. |  -.1115611   .1054421    -1.06   0.290    -.3182239    .0951017
       _cons |   112.9776   46.69068     2.42   0.016     21.46551    204.4896
------------------------------------------------------------------------------
```