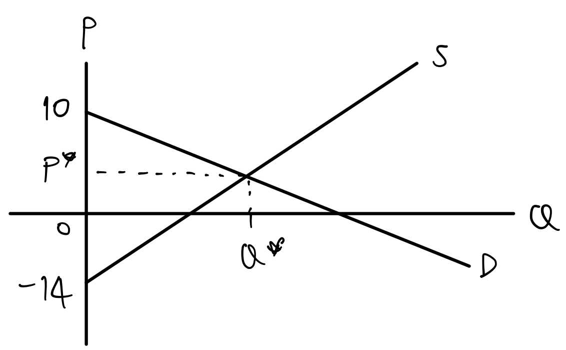
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



$D: P = 10 - Q^2$

$S: Q = a + P$

$P = a - Q$

$= -14 - Q$

1.2) $a = -14$, Find Q^* and P^* that satisfies market demand = market supply

$$Q^d = Q^s$$
$$\sqrt{10-P} = P - 14$$
$$10 - P = P^2 + 28P + 196$$
$$0 = P^2 + 29P + 186$$

1.3) The market equilibrium of quantity is expected to decrease.
The market equilibrium of price is expected to increase.

$$\text{let } u = Q^2+1$$

$$\frac{d}{du} \cdot \ln(u) = \frac{1}{u} \cdot \frac{d}{du}(u)$$

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R'(Q) = \frac{1}{Q^2+1} \cdot 2Q^{2-1} + 0 + 3 \cdot \underbrace{\frac{d}{dQ}\left(\frac{Q}{Q+1}\right)}$$

$$\text{use } \left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$\Rightarrow \frac{(Q+1)\left(\frac{d}{dQ}\cdot Q\right) - (Q)\left(\frac{d}{dQ}\cdot(Q+1)\right)}{(Q+1)^2}$$

$$\Rightarrow \frac{(Q+1)(1) - (Q)(1)}{(Q+1)^2}$$

$$\Rightarrow \frac{\cancel{Q}+1-\cancel{Q}}{(Q+1)^2} = \frac{1}{(Q+1)^2}$$

$$R'(Q) = \frac{2Q}{Q^2+1} + 3 \cdot \frac{1}{(Q+1)^2}$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

Marginal revenue function: $\frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$

suppose $Q = 0$; $\frac{2(0)}{0^2+1} + \frac{3}{(0+1)^2} = \cancel{\frac{0}{1}}^{0} + \frac{3}{1^2}$

$$= \underline{\underline{3}} > 0$$

Hence, the revenue function is an increasing function.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

Profit-maximizing output:

$$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$$

$$\frac{d\,\pi(Q)}{dQ} = -\frac{1}{3}\cdot 3\,Q^{3-1} - 2Q^{2-1} + 8\,Q^{1-1} - 0$$

$$\pi'(Q) = -Q^2 - 2Q + 8$$

$$\pi'(Q) = 0;\quad 0 = -Q^2 - 2Q + 8$$

2nd derivative:

$$\frac{d\,\pi(Q)}{dQ} = -\frac{1}{3}\cdot 3\,Q^{3-1} - 2Q^{2-1} + 8\,Q^{1-1} - 0$$

$$= -Q^2 - 2Q + 8$$

$$\frac{d^2\pi(Q)}{d^2Q} = -2Q^{2-1} - 2Q^{1-1} + 0$$

$$= -2Q - 2$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}_{2\times2}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$, calculate the following object. Show your work.

4.1 A + B

Matrices A and B are different in size ; so they cannot be added.

4.2 A*B

$$A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}_{2\times\cancel{2}} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{\cancel{2}\times3} \qquad \therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}_{\#}$$

$a_{11} = (8\cdot1)+(9\cdot4) = 44$
$a_{12} = (8\cdot2)+(9\cdot5) = 61$
$a_{13} = (8\cdot3)+(9\cdot6) = 78$
$a_{21} = (10\cdot1)+(11\cdot4) = 54$
$a_{22} = (10\cdot2)+(11\cdot5) = 75$
$a_{23} = (10\cdot3)+(11\cdot6) = 96$

4.3 det(A)

$9 \times 10 = 90$

$$A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \qquad \therefore \det A = 88 - 90 = -2_{\#}$$

$8 \times 11 = 88$

4.4 det(B)

Indeterminant since determinant is only defined for squared matrices.

4.5 det(C)

Uni

$-(3\cdot5\cdot7)$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad = 45 - 105 = -60$$

$(1\cdot5\cdot9)$

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$$

$$\frac{\partial U}{\partial x} = y^b\left[\frac{\partial}{\partial x}(x^a)\right] + \frac{\partial}{\partial x}\left[\ln\left(\frac{x}{x+y}\right)\right]$$

$$= y^b a x^{a-1} + \frac{\partial}{\partial x}\left[\ln\left(\frac{x}{x+y}\right)\right] \rightsquigarrow \frac{\partial}{\partial x}\left[\ln\left(\frac{x}{x+y}\right)\right]$$

use $\log\left(\frac{M}{N}\right) = \log(M) - \log(N)$

$$\Rightarrow \frac{\partial}{\partial x}\left[\ln(x) - \ln(x+y)\right]$$

Chain rule

$$= \left(\frac{\partial}{\partial x}\ln(x)\right) - \left(\frac{\partial}{\partial x}\ln(x+y)\right) \quad ; \ \ln(x) = \frac{1}{x}$$

$$= \frac{1}{x} - \underbrace{\left(\frac{\partial}{\partial x}\ln(x+y)\right)}$$

Chain rule: $u = x+y$

$$\frac{\partial}{\partial u}\ln(u) = \frac{1}{u}\cdot\frac{\partial}{\partial u}(u)$$

$$= \frac{1}{x} - \left(\frac{1}{u}\cdot\underbrace{\frac{\partial}{\partial x}(x+y)}_{\frac{\partial}{\partial x}(x)+\frac{\partial}{\partial x}(y)\ =\ 1+0}\right)$$

$$= \frac{1}{x} - \left(\frac{1}{x+y}\cdot 1+0\right)$$

$$= \frac{1}{x} - \frac{1}{x+y}$$

$$= y^b a x^{a-1} + \frac{1}{x} - \frac{1}{x+y}$$

$$U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$$

$$\frac{\partial U}{\partial y} = x^a \left[\frac{\partial}{\partial y}(y^b) + \frac{\partial}{\partial y}\left(\ln\left(\frac{x}{x+y}\right)\right)\right]$$

$$= x^a b y^{b-1} + \frac{\partial}{\partial y}\left(\ln\left(\frac{x}{x+y}\right)\right) \rightarrow \frac{\partial}{\partial y} \ln\left(\frac{x}{x+y}\right)$$

use $\log\left(\frac{M}{N}\right) = \log(M) - \log(N)$

$$\Rightarrow \cancel{\frac{\partial}{\partial y} \ln(x)}^{\,0} - \frac{\partial}{\partial y} \ln(x+y)$$

$$\Rightarrow -\frac{\partial}{\partial y} \underbrace{\ln(x+y)}$$

Chain rule: $u = x+y$

$$\frac{\partial}{\partial u} \ln(u) = \frac{1}{u} \cdot \frac{\partial}{\partial u}(u)$$

$$= -\frac{1}{x+y} \cdot \left[\underbrace{\frac{\partial}{\partial y}(x+y)} \right]$$

$$\cancel{\frac{\partial}{\partial y} x}^{\,0} + \underbrace{\frac{\partial}{\partial y} y}_{1}$$

$$= -\frac{1}{x+y} \cdot 1$$

$$= x^a b y^{b-1} - \frac{1}{x+y}$$