Quiz#3 EE320 Semester 1/2019: Answer the question in the area provided.

Given the short-run total cost (STC) function of a monopolist $STC = Q^3 - 8Q^2 + 20Q + 21$ where Q is the level of output. Calculate the ***level of cost-minimizing output***, and ***level of minimized cost***. Confirm your result with the **second-order derivative** test.

The first-order condition suggests that first-order derivative of STC must be equal to zero. That is, we solve for Q that puts $\frac{dSTC}{dQ} = 0$.

$$\frac{dSTC}{dQ} = 3Q^2 - 16Q + 20 = 0$$

Using the factorization method, we know that $3Q^2 - 16Q + 20 = (3Q - 10)(Q - 2)$. So, the two possible solutions are $Q = \frac{10}{3}$ and 2.

Next, we look at the second-order condition by checking at the convexity/concavity of the function. To warrant the stationary point as the minimizer, the function must be convex, i.e. $\frac{d^2STC}{dQ^2} > 0$. Note that $\frac{d^2STC}{dQ^2} = 6Q - 16$. Given the second-order derivative, we know that the second-derivative is greater than zero when $Q = \frac{10}{3}$.

The level of minimized cost is equal to $\left(\frac{10}{3}\right)^3 - 8\left(\frac{10}{3}\right)^2 + 20\left(\frac{10}{3}\right) + 21$ = 35.81

Note:

In fact, if one calculates the value of cost when Q = 0, the total cost is 21, which is lower than 35.81. The reason is that our function is not globally convex. Therefore, it might be the case that local minimizer is not the global minimizer. So, the correct answer is that Q = 0. However, if anyone reaches the final step shown above, you will get full score for what you attempted.