THEORIES OF ECONOMIC GROWTH

EE 462 Development Macroeconomics

Semester 1/2022

Topics

- The Basic Growth Model
- The Harrod-Domar Model ✓

  Solow. Sawan
- The Solow (Neoclassical) Growth Model
  - Basic Equations and Solow Diagram
  - Steady State
  - Comparative Statics
  - Technological Change
- Diminishing Returns and the Production Function
- Beyond Solow: New Approaches to Growth

Introduction

- In previous lectures, we examined the basic process and patterns that characterize economic growth based on empirical approach.
- This topic explores key contributions to the theory of economic growth based on theoretical models.
- Growth models provide consistent frameworks for understanding the growth process (capital accumulation and productivity gain) and theoretical foundation for empirical studies.
  - Need to identify specific mathematical relationship between the quantity of capital and labor, their productivity, and the resulting aggregate output.

TFP

The Basic Growth Model

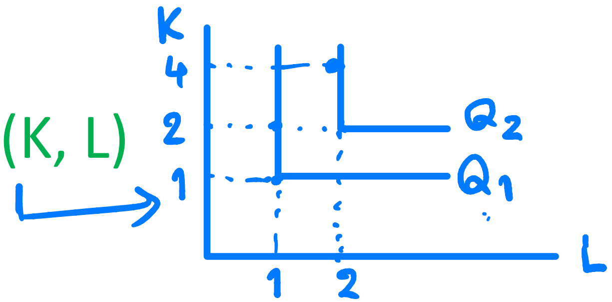
GDP

- The aggregate production function is based on five equations:
  - Aggregate production function: $Y = F(K, L)$ -- (1)
  - Saving is a fixed share of income: $S = sY$, s = mps -- (2) $s = 1 - mpc$, $0 < s < 1$
  - Assume a closed economy: $S = I$ (saving = investment)
  - Change in capital stock: $\Delta K = I - (d \times K)$, -- (3)
    where d=depreciation rate $S = sY$
  - Assume the labor force grows exactly as fast as the total population. Let n =population growth rate and L=Labor force, then: $\Delta L = n \times L$. -- (4)
- Based on equations (2) and (3), we have:

$$\boxed{\Delta K = sY - dK} \quad \text{---- (6)}$$

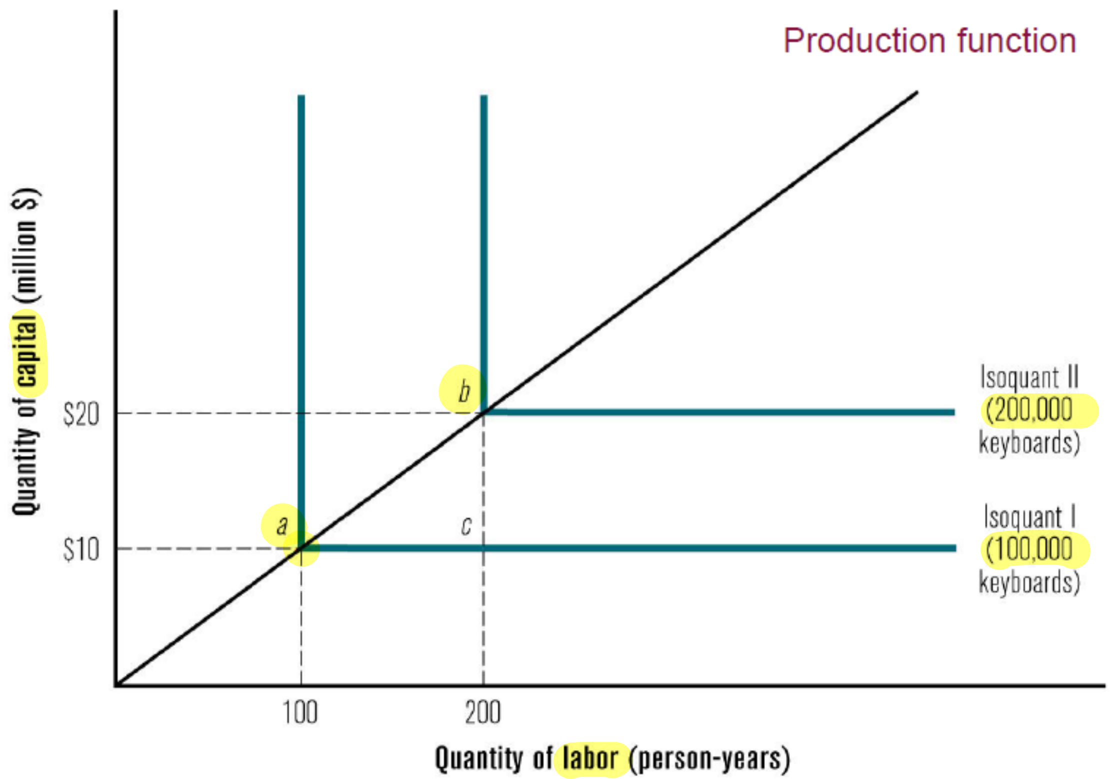
The Harrod-Domar Growth Model

- Harrod-Domar Growth model is a particular model with basic feature of fixed coefficient production function or "Leontief" production function.
  - Example: Y= F(K, L) = min(K, L)
    - ➢ Capital-labor ratio = 1:1



- It assumes *no substitution* between labor and capital.
- The production isoquant is L-shaped.
- It also shows constant returns to scale (CRS).
  - i.e. doubling inputs will double output

Fixed-Coefficient Production Function



Constant return to scale:

$Y_2 = F(K_2, L_2)$

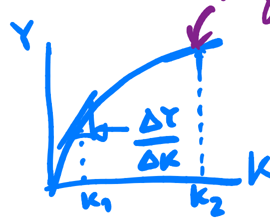
$2Y_1 = F(2K_1, 2L_1)$

$= 2F(K_1, L_1)$

- At point *a*, capital-labor ratio = 10m : 100 = 100,000: 1
- At point *b*, capital-labor ratio = 20m: 200 = 100,000: 1
  - ➢ Constant capital-labor ratio

Harrod-Domar Production Function

high v



$Y = \left(\frac{1}{v}\right)K = c \cdot K$ $\quad$ $Y = f(K)$

$\frac{\Delta Y}{\Delta K} = \frac{dY}{dK} = MP_K$ = marginal product of K = "c"

- The Harrod-Domar production function:

  $Y= (1/v) \times K$ or $Y=K/v$ $\quad$ $v = \frac{K}{Y}$

  MP_K

  ➢ $v = K/Y$: "capital output ratio" or measure of the productivity of capital or investment (indication of capital intensity)

- Incremental capital-output ratio is fixed: ICOR = v → $\Delta Y=\Delta K/v$

  $\Rightarrow \frac{\Delta Y}{\Delta K} = \frac{1}{v} = c$

  $\Delta Y = \frac{1}{v}\Delta K = c \cdot \Delta K$

- Growth rate of output: $g= \Delta Y/Y = \frac{Y_1 - Y_0}{Y_0} = \frac{\Delta Y}{Y} \approx \frac{dY}{Y}$

- Since $\Delta K = sY - dK$, growth rate of output is equal to:

$$g = \frac{\Delta Y}{Y} = \frac{\Delta K / v}{Y} = \frac{sY - dK}{vY} = \frac{s\cancel{Y}}{v\cancel{Y}} - \frac{dK}{\cancel{vY}} \quad \frac{K}{Y} = v$$

$$g = \frac{s}{v} - d \quad \Leftrightarrow \quad g = s \cdot c - d$$

➢ *K created by I is the main determinant of growth in output.*

→ higher growth rate – high s, low v, low d

↑ depreciation.

Examples: Harrod-Domar Model

$v = \frac{K}{Y}$

- If v=4, then how much investment (or capital) will be needed to produce 5 million of output?
  - ➔ Based on v = K/Y, so $\rightarrow$ $K = v \times Y$

    K = … $4 \times 5\text{ m} = 20$ million

What if $v = 2$? $K = 2 \times 5\text{ m} = 10$ million.

- Suppose s = 0.24, v = 3, and d = 0.05. What is the growth rate of this economy?
  - ➔ g = … $\frac{s}{v} - d = \frac{\cancel{0.24}^{\,0.08}}{\cancel{3}} - 0.05 = 0.03 = 3\%$

    Thus, the economy will grow at …

Strengths and Weaknesses of HD Model

- *Strengths*
  - The model works well in the absence of severe economic shocks, or only in a short period of time.
  - Focus on the key role of saving – important driver of growth
- *Weaknesses*
  - It assumes saving is necessary and *sufficient* for growth.
  - The assumptions of fixed K-L, L-Y, and L-Y ratios are too rigid.
    - ➢ This is true only when K, L, and Y grow at the same rate (Ex. n = g = s/v -d). → Not always true!
    - ➢ ICOR is not constant over time (see the case of Thailand).
  - Productivity growth plays no role.
    - ➢ An increase in factor productivity implies a smaller ICOR → contradiction!

Case Study: Economic Growth in Thailand

- Thailand in 1960 was an agrarian economy with 75% of population in agriculture, GDP was about $1000.
- Beginning 1970 Thailand began to save averaging 20%, reaching 35% in 1990s, and falling to 32% in 2000s. This combined with good governance and prudent policies led to rapid economic growth.
- After the financial crisis, GDP fell by 10% in 1998. Then, between 1999-2007, the growth rebounded to 3.9%.
- During 1960-2007, average per capita income grew at 4.5%.
- *Consistent with HD*: High saving rate and ↑K ➔ increase in income
- *Contradict to HD*: ICOR rose from 2.6 in 1970s to ~5 by early 2000s (due to shifts toward more capital-intensive production).

The Solow Model

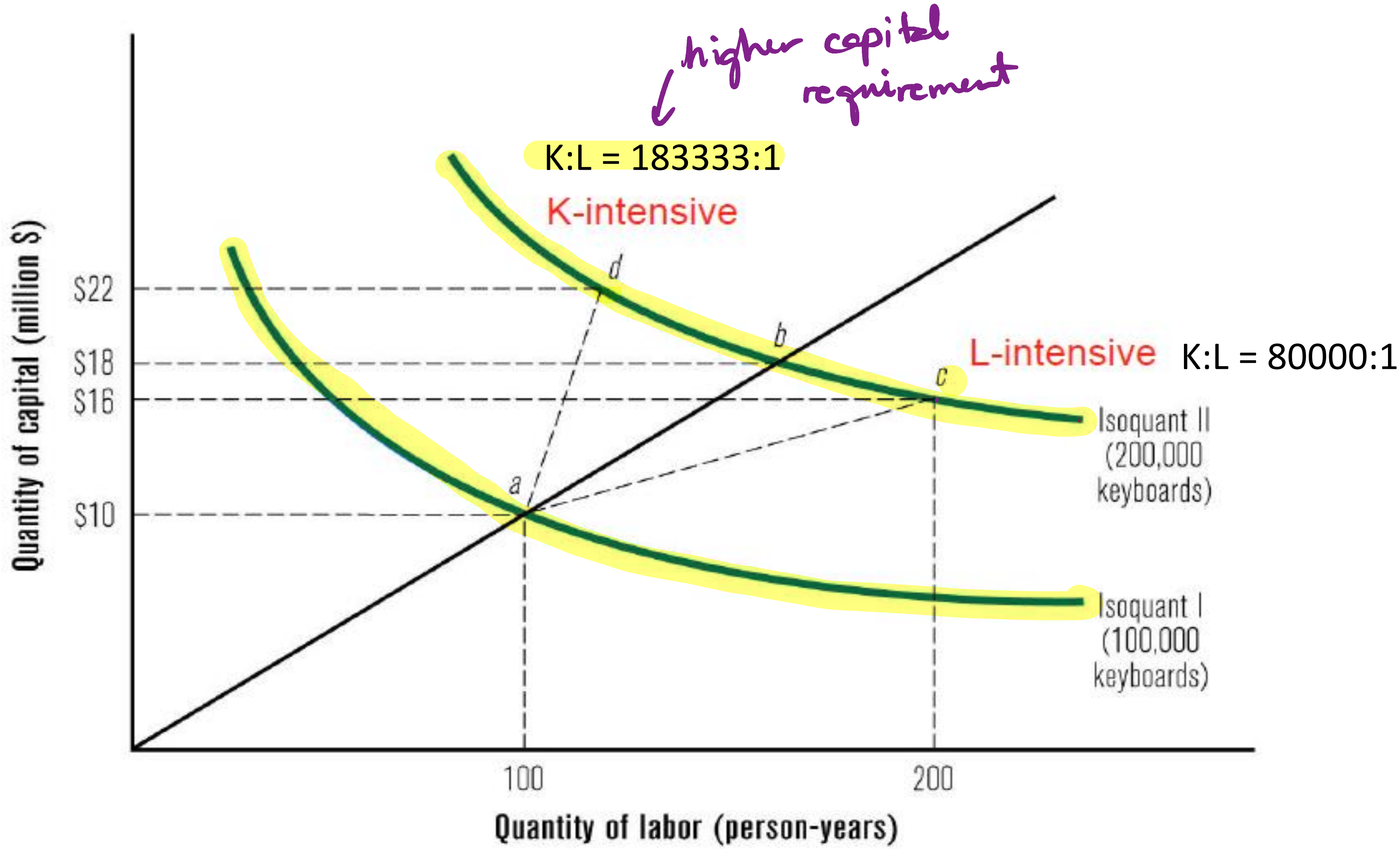
- Introduced in 1956 by Robert Solow, this model drops fixed-coefficient production function and replaces with a neoclassical production function:

$$Y = F(K, L)$$

Where labor and capital are substitutable.

- The isoquants are curved (convex and smooth) rather than L-shaped.
  - Output can be achieved with different combinations of K and L.
- This model still assumes constant returns to scale ** of the production function. → Cobb-Douglas f^{n} : $Y = AK^{\alpha}L^{1-\alpha}$, $0 < \alpha < 1$

Isoquants for a Neoclassical Production Technology



Basic Equations of the Solow Model (1)

$Y = F(K, L) \rightarrow \frac{Y}{L} = F(\frac{K}{L}, 1)$

- All key variables are expressed in terms of per-worker:

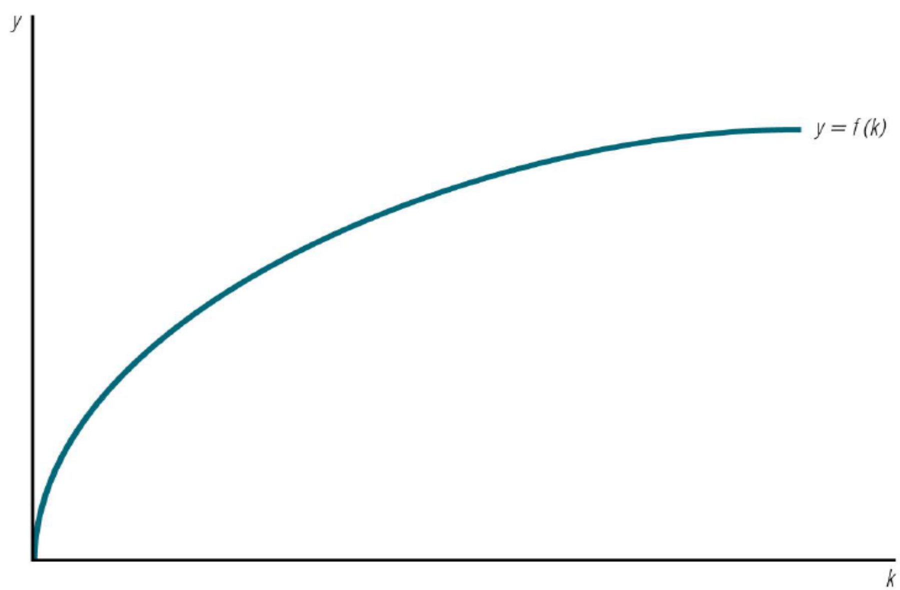
$$Y/L = F(K/L, 1)$$

output per worker

➔ $y = f(k)$ -- (1) , where $y = Y/L$ and $k = K/L$.

capital per worker

- Assume that y = f(k) exhibits diminishing returns to capital.



derivative

$\rightarrow \frac{dy}{dk} > 0$ & $\frac{d^2y}{dk^2} < 0$

Basic Equations of the Solow Model (2)

- From $\Delta K = sY - dK$, capital accumulation can be written as:

$$\Delta k = sy - (n+d)k \qquad \text{-- (2)}$$

- Derivation of equation (2):

From $\Delta K = sY - dK$, we have: $\Delta K/K = sY/K - d$

depreciation rate

Then, $\Delta k/k = \ldots \frac{\Delta K}{K} - \frac{\Delta L}{L} = \frac{sY - dK}{K} - n$

$= \Delta k$; n

$$\frac{\Delta k}{k} = s\cdot\frac{Y}{K} - d - n = s\cdot\frac{Y/L}{K/L} - (n+d)$$

$$k = \frac{K}{L} \Rightarrow \ln(k) = \ln\left(\frac{K}{L}\right)$$

$$\ln(k) = \ln(K) - \ln(L)^{*}$$

Totally differentiate:

derivative

$$\frac{1}{k}\cdot dk = \frac{dK}{K} - \frac{dL}{L}$$

$$\Rightarrow \frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta L}{L}$$

- Interpretations of equation (2):
  - *Δk is positively related to saving per worker.*
  - *Δk is negatively related to population growth.*
  - *Depreciation erodes the capital stock.*

Thus, saving (and investment) adds to capital per worker, whereas labor force growth and depreciation reduce capital per worker.

$$\frac{\Delta k}{k} = \frac{\Delta K}{K} - \overbrace{\cancel{\frac{\Delta L}{L}}}^{n} = \frac{\underbrace{sY - dK}_{=\Delta K}}{K} - n$$

$$\frac{\Delta k}{k} = s \cdot \frac{Y}{K} - d - n = s \cdot \frac{Y/L}{K/L} - (n+d)$$

$$\frac{\Delta k}{k} = s \cdot \frac{y}{k} - (n+d)$$

$$\Delta k = s \cdot \frac{y}{\cancel{k}} \cdot \cancel{k} - (n+d) \cdot k$$

$$\therefore \boxed{\Delta k = sy - (n+d)k} \quad \text{—} \; (*)$$

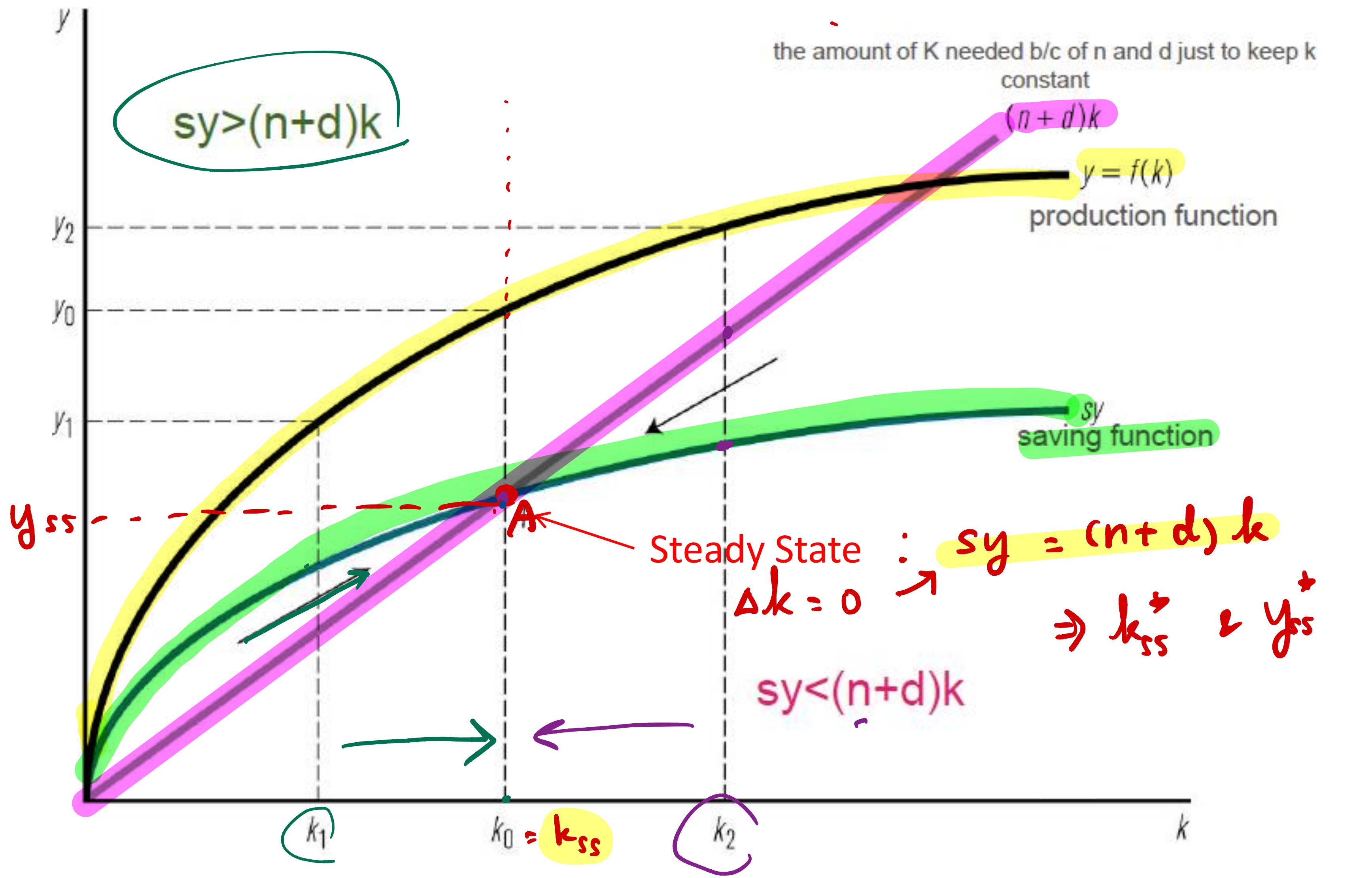
Per worker capital accumulation

$\Rightarrow$ Steady state where $\frac{\Delta y}{y} = 0$

Basic Equations of the Solow Model (3)

- *Capital deepening*: the process through which the economy increases the amount of capital per worker (i.e. $\Delta k) > 0$
- *Capital widening*: a widening of both the total amount of capital and the size of the workforce
  - This occurs when $sy = (n+d)k$ (i.e. $\Delta k = 0$).
- Thus, $\Delta k = sy - (n+d)k$ means "capital deepening (Δk) is equal to saving per worker (sy) minus the amount needed for capital widening [$(n+d)k$]."
  - If $sy > (n+d)k$, then $\Delta k > 0$ (i.e. $k\uparrow$).
  - If $sy < (n+d)k$, then $\Delta k < 0$ (i.e. $k\downarrow$).
  - If $sy = (n+d)k$, then $\Delta k = 0$. : S.S.

The Solow Diagram



Steady State of the Solow Model

- Point A is where new savings (sy) equals the amount of new capital needed for growth in the labor force and depreciation [(n+d)k]. ➔ the steady state of the Solow model.
- The output level at the steady state (y_0) is referred to as the steady state (or long run or potential) level of output per worker.

$\frac{\Delta Y}{Y} = n$; $\frac{\Delta y}{y} = 0$

- Note: at the steady state, *total output* continues to grow at the same rate the population and labor force grows (n), but output per worker (y) is constant (i.e. average income remains unchanged).
  - Why?
  - What about the rate of change in total capital and total saving.

Steady State (Continued)

Note: $y = \frac{Y}{L}$

$\ln(y) = \ln(Y) - \ln(L)$

$\Rightarrow \frac{dy}{y} = \frac{dY}{Y} - \frac{dL}{L}$

- Suppose two key equations in Solow Model are given by: $y = k^{\alpha}$ and $\Delta k = sy - (n + d)k$.

$k_{ss} = ?$

- Find steady-state quantity of capital per worker and steady-state quantity of output per worker.

$y_{ss} = ?$

$\Delta k = sy - (n+d)k$

At s.s., $\Delta k = 0 \Rightarrow sy - (n+d)k = 0$

$sy = (n+d)k$

$\because y = k^{\alpha}$

$s \cdot k^{\alpha} = (n+d)k$

$\frac{s}{(n+d)} = k^{1-\alpha}$

$$\boxed{k_{ss} = \left(\frac{s}{(n+d)}\right)^{\frac{1}{(1-\alpha)}}}$$

At s.s.

$\Delta k = 0$

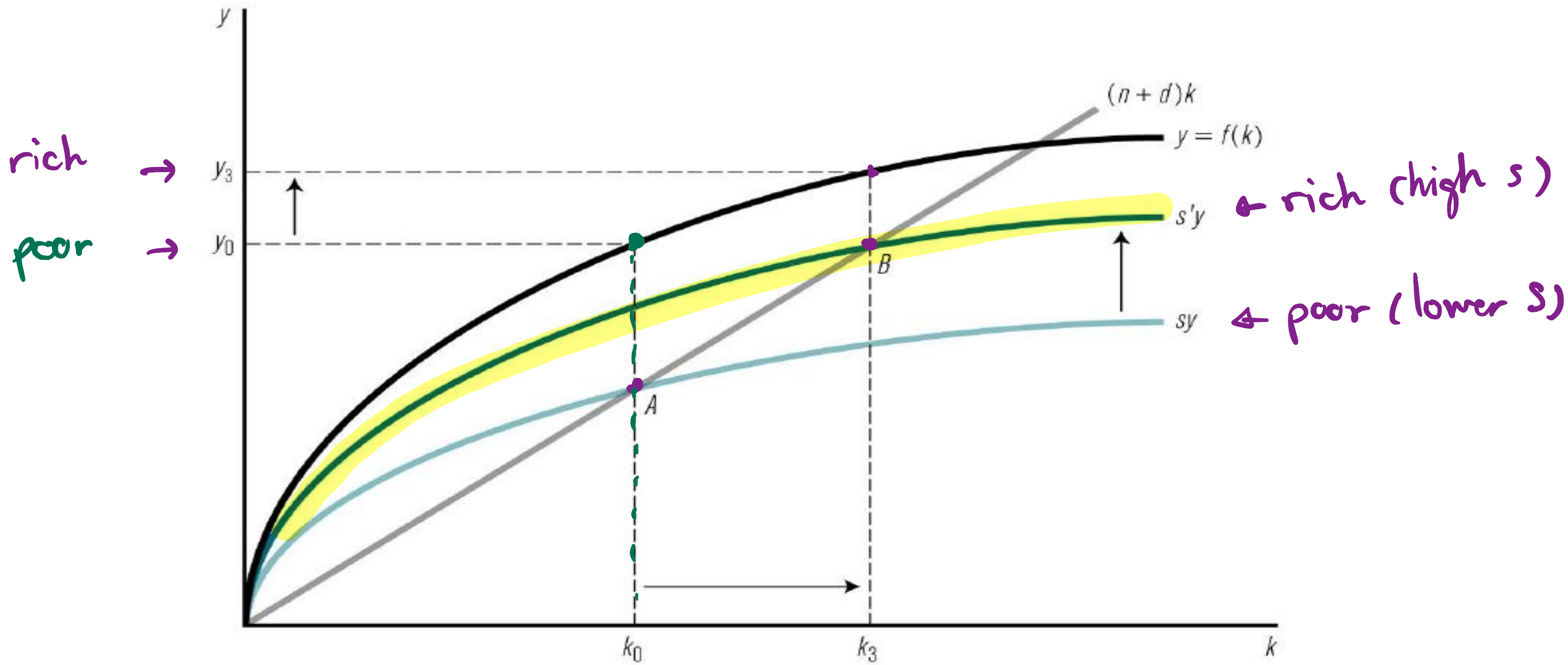
$\frac{\Delta k}{k} = 0$

$\Rightarrow \frac{\Delta y}{y} = 0$ s.s.

$\frac{\Delta y}{y} \xrightarrow{} 0 = \frac{\Delta Y}{Y} - \frac{\Delta L}{L} \xrightarrow{} n$

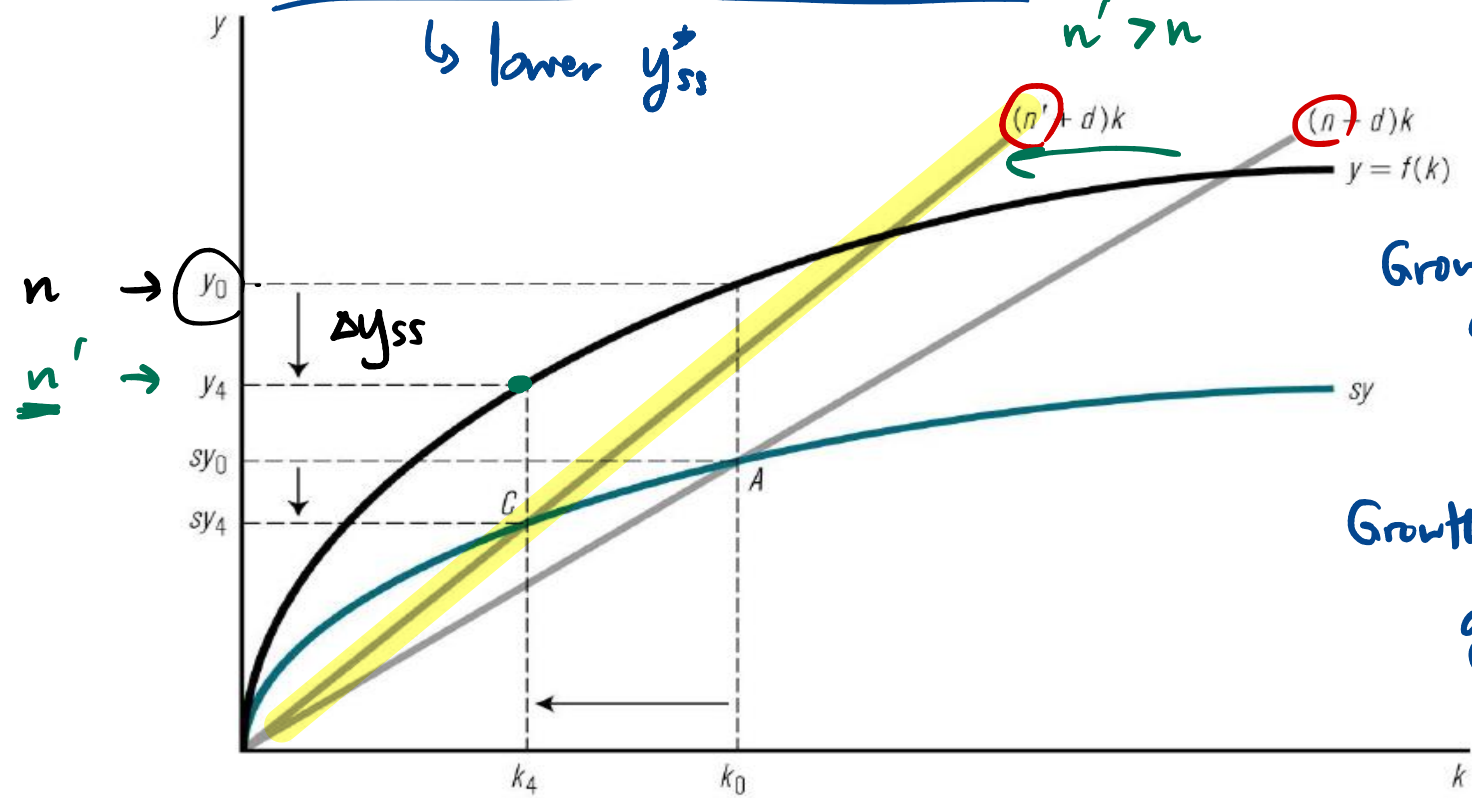
$\therefore \frac{\Delta Y}{Y}^{s.s.} = n$

Comparative Statics :
An Increase in the Saving Rate in the Solow Model



- An increase in the saving rate results in an upward shift in the capital deepening curve. ➔ capital per worker increases.
- Higher saving rate leads to a *temporary* increase in the economic growth, but the long-run output growth rate remains at n. Why?

Comparative Statics :
An Increase in in Population Growth in the Solow Model



Growth of GDP/capita:

$g_y^{ss} = 0$

($\because$ At s.s., $\Delta k = 0$)

Growth of GDP:

$g_Y^{ss} = n$

- ➢ An increase in the pop growth rate causes the capital widening curve to rotate to the left. ➔ capital per worker decreases.
- ➢ Higher population growth rate (n' > n) leads to higher growth rate of total income (Y), whereas the per capita income decreases (y_0 to y_4).

Technological Change in the Solow Model

- Introduce the labor-augmenting technological change (T).
- The new production function with technological progress is:

$$Y = F(K, T \times L)$$

T x L = amount of effective units of labor $L_e = TL$ $\quad g_{L_e} = \frac{\Delta L_e}{L_e} \approx \frac{dL_e}{L_e}$

➔ The growth rate of effective supply of labor = n + θ (why?) → See next page

- Output per effective worker is defined as: $y_e = Y/(T \times L)$
- Capital per effective worker is defined as: $k_e = K/(T \times L)$

- Then, the capital accumulation equation changes to:

$$\Delta k_e = sy_e - (n+d+\theta)k_e$$

> To keep constant, *saving per effective worker* must be equal to the amount of *new capital needed to compensate for changes in the size of labor force, depreciation, and technological change*.

Consider $Y = F(K, TL)$ where $L_e = T \times L$.

From $L_e = TL$, $\ln(L_e) = \ln(T) + \ln(L)$

Totally differentiate: $\frac{dL_e}{L_e} = \frac{dT}{T} + \frac{dL}{L} \iff \frac{\Delta L_e}{L_e} = \underbrace{\frac{\Delta T}{T}}_{=\theta} + \underbrace{\frac{\Delta L}{L}}_{=n}$

$\therefore \boxed{\frac{\Delta L_e}{L_e} = \theta + n}$

To derive $\Delta k_e = ?$, start with $k_e = \frac{K}{TL}$.

$\Rightarrow \ln(k_e) = \ln(\underbrace{K/L}_{=k}) - \ln(T)$

$\Rightarrow \frac{dk_e}{k_e} = \frac{dk}{k} - \cancel{\frac{dT}{T}}^{=\theta}$

or $\frac{\Delta k_e}{k_e} = \frac{\Delta k}{k} - \theta$ —— (*)

But $\Delta k = sy - (n+d)k$

Plug in Δk in (*):

$\frac{\Delta k_e}{k_e} = \frac{[sy - (n+d)k]}{k} - \theta$

$\frac{\Delta k_e}{k_e} = \frac{sy}{k} - (n+d+\theta)$

$\frac{y}{k} \times k_e = \frac{y}{\cancel{k}} \cdot \frac{\cancel{k}}{TL} = \frac{y}{TL} = y_e$

$\therefore \Delta k_e = sy_e - (n+d+\theta)k_e$

The Solow Model with Technical Change

$y_e \equiv Y/TL = \frac{(Y/L)}{T} = \frac{y}{T}$

$\ln(y_e) = \ln(y) - \ln(T)$

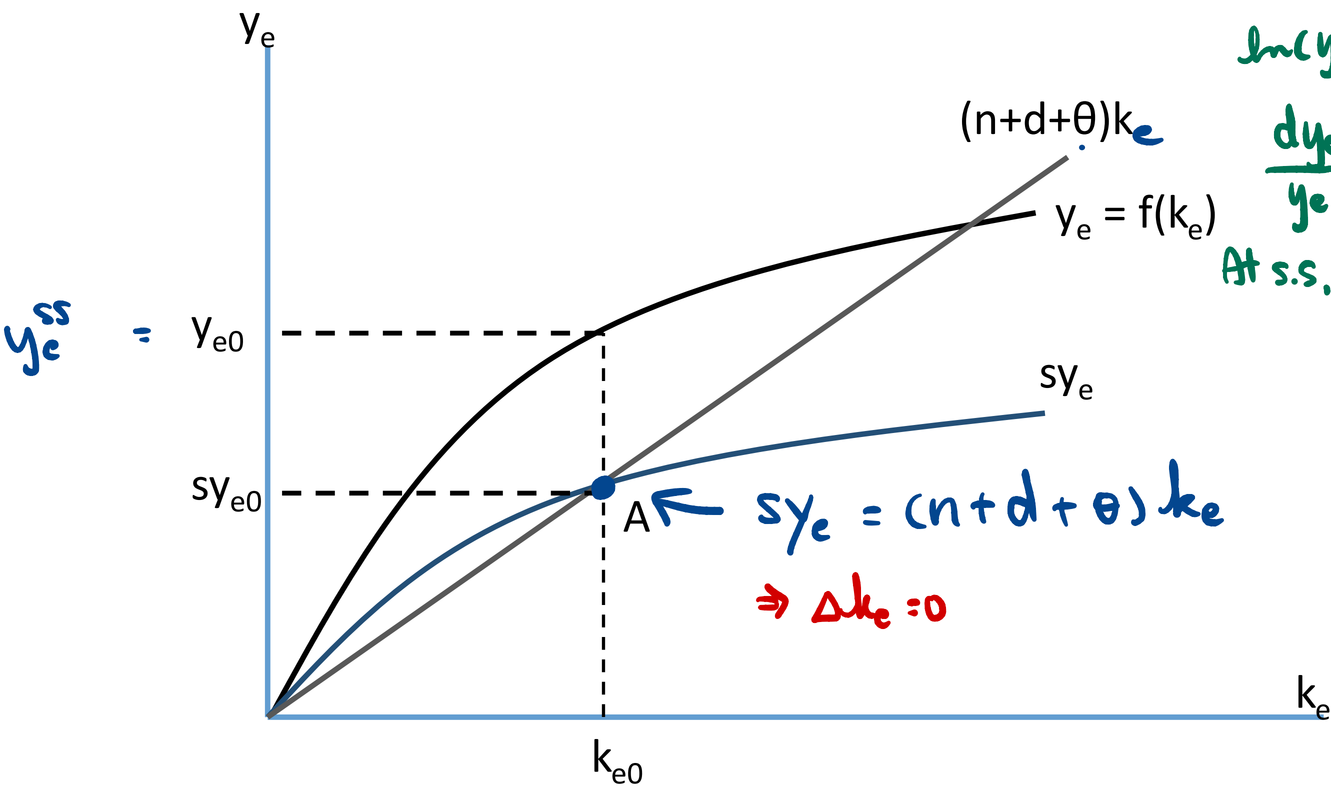
$\frac{dy_e}{y_e} = \frac{dy}{y} - \frac{dT}{T}$

At s.s., $\frac{dy_e}{y_e} = g_{y_e} = 0$

$\Rightarrow \boxed{g_y = \frac{dy}{y} = \frac{dT}{T} = \theta}$

$g_Y = \frac{dY}{Y} = \frac{dy}{y} + \frac{dL}{L}$

$\therefore g_Y = \theta + n$



$g_{y_e} = 0$

- At the steady state, output per effective worker (y_e) is constant.
- Total output grows at the rate $n+\theta$, so that the income per capita increases at rate θ. $g_y = \theta$

Steady State in The Solow Model with Technology (For own practice!)

- Let $Y = F(K, TL) = K^{\alpha}(TL)^{1-\alpha}$.
- Define: $\tilde{k} \equiv \frac{K}{TL} = k/T$ and $\tilde{y} \equiv \frac{Y}{TL} = y/T$.
- Find the new steady state.

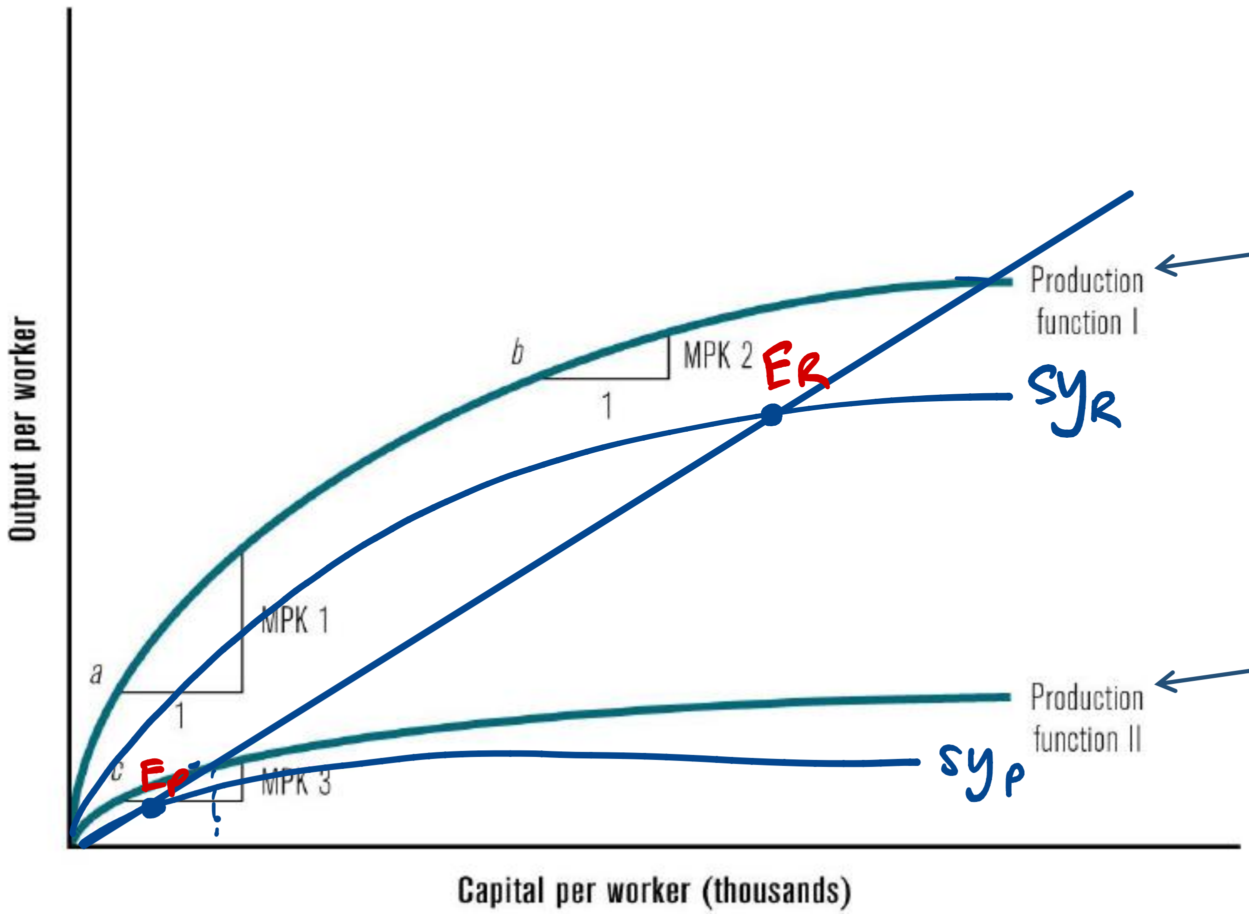
Strengths and Weaknesses of Solow Model

- *Strength:*
  - It allows for substitution between inputs (need not be fixed).
  - Provides good insights about the role of technology and productivity growth in the growth process.
- Weaknesses:
  - It specifies productivity growth as *exogenous*.
    - Didn't specify how it takes place, or how the growth process itself might affect productivity.
  - One sector approach, factors that drive steady state, and assumes saving rate, population growth , and technical change as given.
    - It does not explain how these parameters change over time

Diminishing Returns and the Production Function

- Three implications of diminishing MP of capital:
  1. Poor countries have a *potential* to grow more rapidly than do rich countries (b/c they face capital scarcity).
  2. Richer countries with capital abundance grow slowly.
  3. Since poor countries have more *potential* to grow faster than do rich countries, they can *catch up* and close the gap in relative income.
- However, the above implications rest on the assumption that all else is equal between the two countries (i.e. same *s, n, d*, etc.).
- But, if the two countries do not have the same technology then the predictions for rich and poor countries might not hold.
  - Ex. If the production function is flatter, then poor countries might not grow faster than rich countries and may never catch up.

Diminishing Marginal Product of Capital



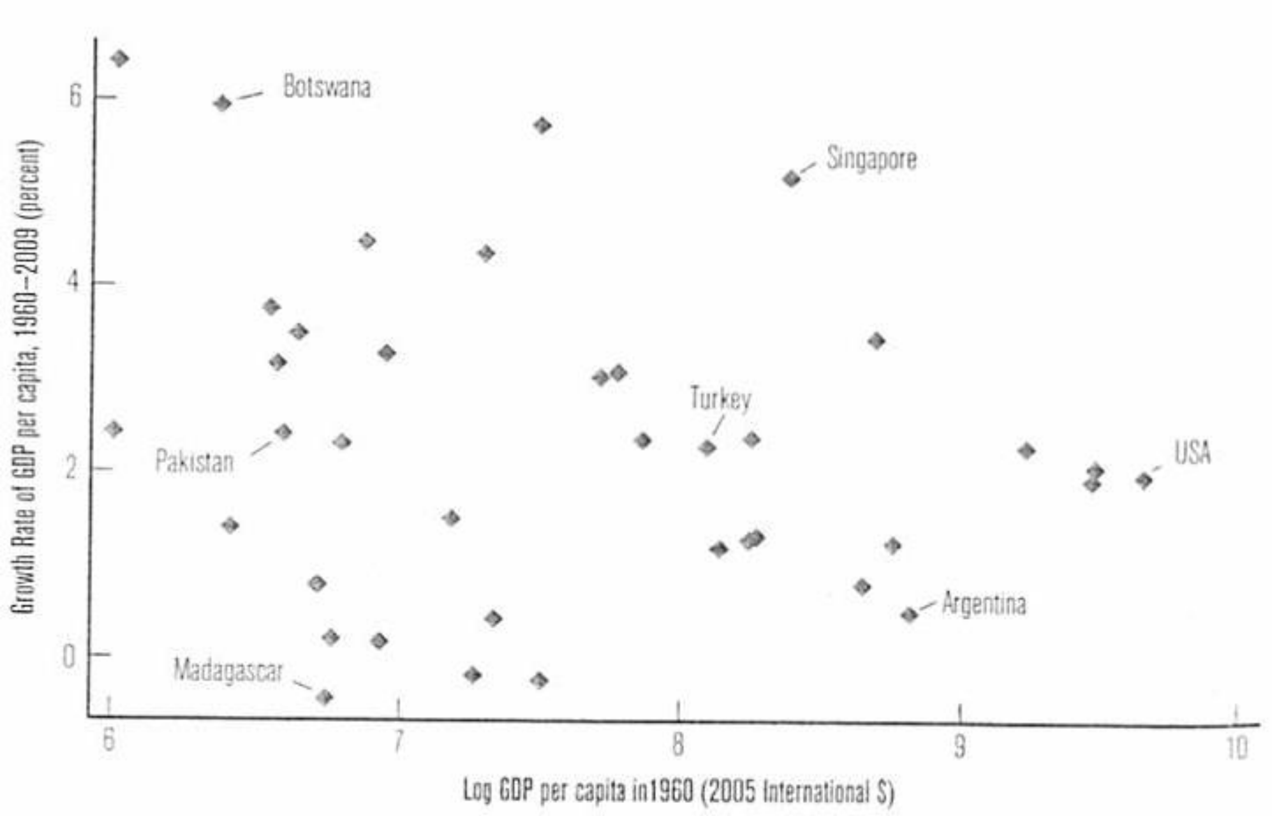
Production function for rich countries

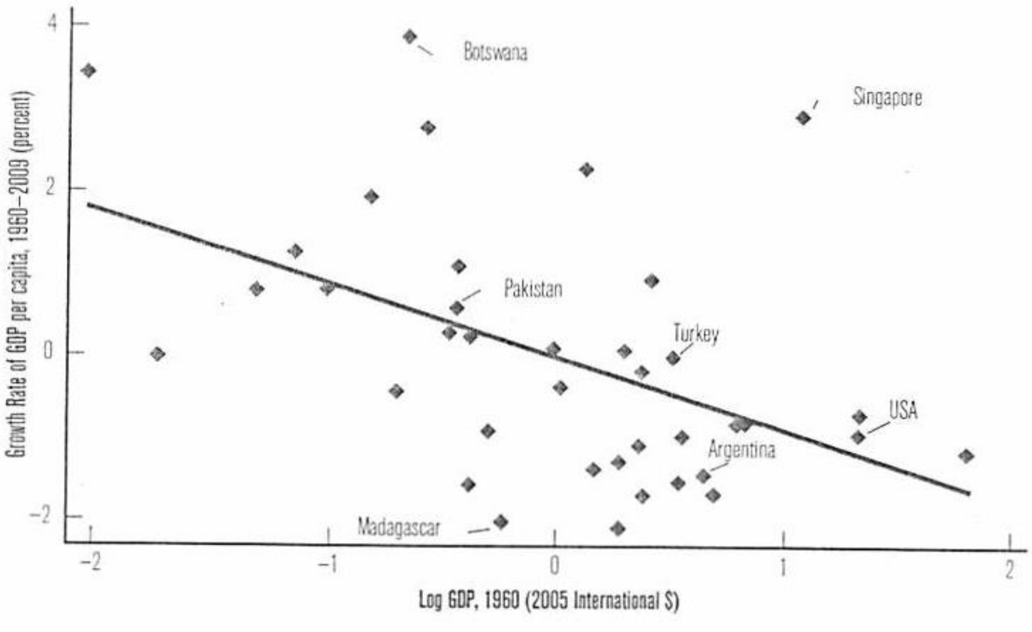
Production function for poor countries

The Convergence Debate

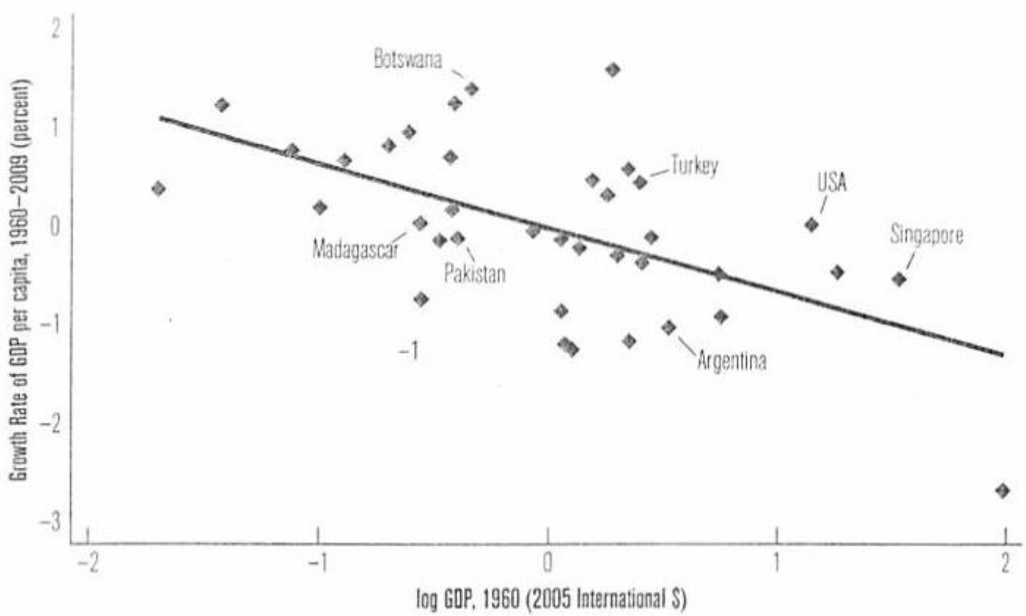
- Question: Has convergence actually happened?
  - Yes, but only for some countries that share the same features.
  - Ex. Japan Other?
- In general, there is no "unconditional convergence."
  - *Unconditional* – assumption that all countries share the same key parameters (pop growth, saving rate, depreciation).
- But there may be *"conditional convergence"* across rich and poor countries.
  - By allowing countries to have their own steady states, which are conditional upon the countries' population growth rate (and other characteristics), the growth rates tend to be higher for poor countries and smaller for rich countries.

GDP Growth, Unconditional





GDP growth, conditional on population growth



GDP growth, conditional on openness, savings, and population growth

Beyond the Solow Model: New Approaches to Growth

- The Solow model assumes exogenously fixed saving rate, growth rate of labor supply, and the pace of technological change.
- Recent works provides models where these variables are determined within or endogenously in the model.
- These new models allow for increasing returns to scale and positive externalities.
  - The impact of investment in K or L would be larger than suggested by Solow.
  - New knowledge may have larger contribution to economic growth.
  - Economies do not necessary reach a steady-state level of income.
- They are called *endogenous growth models*, which we will study in the next topic.