Exercise 8

Techniques of Integration: Substitution rule/Integration by parts/Improper Integrals

1. Evaluate the integrals.

   (a) $\int \sin^3(x)\cos^2(x)\ dx$ Ans: $-\frac{\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C$

   (b) $\int_0^{\pi/2} \cos^5(x)\ dx$ Ans: $\frac{8}{15}$

2. Evaluate the integrals (integration by parts).

   (a) $\int x\ln(x)\ dx$ Ans: $\frac{1}{2}x^2\ln(x) - \frac{x^2}{4}dx + C$

   (b) $\int xe^{2x}\ dx$ Ans: $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.$

   (c) $\int \sin^{-1}(x)\ dx$ Ans: $x\sin^{-1}(x) + \sqrt{1-x^2} + C.$

   (d) $\int x^2\cos(3x)\ dx$ Ans: $\frac{1}{3}x^2\sin(3x) + \frac{2}{3}\cos(3x) - \frac{2}{27}\sin(3x) + C$

   (e) $\int \sin(\ln(x))\ dx$ Ans: $\frac{1}{2}x\sin(\ln(x)) - \frac{1}{2}x\cos(\ln(x)) + C$

   (f) $\int e^{2t}\sin(3t)\ dt$ Ans: $\frac{4}{13}\left[\frac{1}{2}e^{2t}\sin(3t) - \frac{3}{4}e^{2t}\cos(3t)\right] + C$

   (g) $\int_0^{\pi/2} x\cos(2x)\ dx$ Ans: $\frac{1}{4}(-1-1) = -\frac{1}{2}$

   (h) $\int_0^1 (x^2+1)e^{-x}\ dx$ Ans: $-6e^{-1} + 3$

   (i) $\int_1^4 \ln(\sqrt{x})\ dx$ Ans: $x\ln(x) - x + C.$

3. Evaluate each given improper integral or show that it diverges.

   (a) $\int_{-\infty}^{3} e^{2x}dx$ Ans: $\dfrac{e^6}{2}$

   (b) $\int_1^{\infty} \dfrac{\ln(x)}{x}dx$ Ans: Diverges

   (c) $\int_e^{\infty} \dfrac{1}{x(\ln(x))^3}dx$ Ans: $\frac{1}{2}$

   (d) $\int_{-\infty}^{\infty} \dfrac{x}{(x^2+1)^{3/2}}dx$ Ans: 0

   (e) $\int_{-1}^{\infty} \dfrac{1}{x^2+2x+2}dx$ Ans: $\frac{\pi}{2}$

   (f) $\int_0^{\infty} e^{-x}\sin(x)dx$ Ans: $\frac{1}{2}$

   (g) $\int_{1/2}^{\infty} \dfrac{x+1}{x^3}dx$ Ans: 4

   (h) $\int_0^2 \dfrac{1}{\sqrt{2-x}}dx$ Ans: $2\sqrt{2}$

(i) $\displaystyle\int_{-1}^{1} \frac{1}{x^{5/3}} dx$ Ans: Diverges

(j) $\displaystyle\int_{0}^{\pi} \frac{\sin(x)}{1+\cos(x)} dx$ Ans: Diverges