

Chp5 Optimization with constraint

1. Solve the optimization problem of the following model
 - a) $Z = -2x^2 + y^2$ subject to $2x - y = 1$
 - b) $Z = xy$ subject to $15x + 5y - 150 = 0$
 - c) $Z = 7 - y + x^2$ subject to $x + y = 0$
 - d) $Z = x + 2y + 3w + xy - yw$ subject to $x + y + 2w = 10$
 - e) $Z = xy$ subject to $x^2 + y^2 = 16$ where $x, y > 0$
2. Find the equilibrium of a consumer when the utility function and constraint are given as
 $U(x, y) = xy$ subject to $x + y = 200$
3. The firm produces 2 kinds of goods which are x and y , if their total cost function is $TC = 8x^2 - xy + 12y^2$. Note that the ability of this firm in production is limited not more than 42 units. Find the quantity of x and y that minimizes the total cost of the firm.
4. Given the utility function as $U = XY + X$ and budget constraint is $6X + 2Y = 110$. Find the value of X and Y that maximizes the utility.
5. A long-run production function is given as $Q = 4(KL)^{0.5}$
Given prices of capital and labor are 2 and 1 baht per unit, respectively, find the optimal level of K and L that a producer can minimize the production costs from producing 120 units
6. Given a utility function as $U = (x+2)(y+1)$
And $P_x = 4$, $P_y = 6$ and $I = 130$
 - a) Write the Lagrangian function
 - b) Find the optimal level of purchase of x and y
 - c) Is the second order sufficient condition for maximum satisfied?

7. Given $U = \sqrt{xy}$ where U is utility and x, y are goods purchased. If the consumer's budget is 100 baht and price of x and y are 5 and 10 baht respectively, find (with proof) the levels of x and y purchased that yield maximum utility.
8. Given the total cost of two firms are $TC_1 = 10 + 15Q_1 + Q_1^2$ and $TC_2 = 5 + 5Q_2 + 4Q_2^2$. If the total quantity of goods to be produced is 100 units, find quantity to produce in each firm in order to minimize their total cost of production.
9. Given $P = -2X^2 - 3Y^2 + 30X + 40Y$ that P is the production per hour. If X and Y are two raw materials used in production, then the prices of X and Y are 10 baht and 20 baht respectively. If the budget in production is 410 baht per hour and the firm would like to maximize the production per hour (P),
- Write Lagrangian Function
 - Find the solution
10. If two types of goods are sold and their supply function are
- $$X = 72 - 0.5 P_x$$
- $$Y = 120 - P_y$$

The total cost of producing two goods are summed up to be

$TC = X^2 + XY + Y^2 + 35$ and $X + Y = 40$. Find the level of production to maximize profit.