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1. Suppose that there are two countries, X and Y, that differ in both their rates of investment and their population growth rates. In country X, investment is 20% of GDP, and the population grows at 0% per year. In country Y, investment is 5% of GDP, and the population grows at 4% per year. The two countries have the same levels of productivity, A. In both countries, the rate of depreciation is 5%. Use the Solow model to calculate the ratio of their steady-state levels of income per capita, assuming that capital share = 1/3.

Answer

$$\Delta k = \gamma f(k) - (\delta + n)k$$

In steady-state, $\Delta k = 0$ so that $\gamma f(k) = (\delta + n)k$.

Plug the Cobb-Douglas production function $y = Ak^{\propto}$ in the steady-state condition above,

$$\gamma A k^{\propto} = (\delta + n)k$$

$$k_{ss} = \left(\frac{\gamma A}{n + \delta}\right)^{\frac{1}{1-\propto}}$$

Plug this back in the production function to obtain

$$y_{ss} = (A)^{\frac{1}{1-\propto}}\left(\frac{\gamma}{n + \delta}\right)^{\frac{\propto}{1-\propto}}$$

Thus, $\frac{y_{ss}^{x}}{y_{ss}^{Y}} = \frac{(A)^{\frac{1}{1-\propto}}\left(\frac{\gamma_X}{n_X+\delta}\right)^{\frac{\propto}{1-\propto}}}{(A)^{\frac{1}{1-\propto}}\left(\frac{\gamma_Y}{n_Y+\delta}\right)^{\frac{\propto}{1-\propto}}} = \left(\frac{0.2(0.04+0.05)}{0.05(0+0.05)}\right)^{\frac{1/3}{2/3}} = \sqrt{7.2} = 2.68$

2. You are given the following data for a country (all per capita):
   1980: Output 200, capital 300, human capital 2.
   2005: Output 600, capital 1000, human capital 3.

The country's production function is Cobb-Douglas with capital share 1/3. Compute the average annual growth rate of per-capita output for 1980-2005. Determine how much capital, human capital, and productivity have each contributed to per-capita output growth.

Answer

From Solow growth model:

$$Y = AK^{\frac{1}{3}}(hL)^{\frac{2}{3}}$$

Per capita: $y = Ak^{\frac{1}{3}}h^{\frac{2}{3}}$

Taking natural log

$$lny = lnA + \left(\frac{1}{3}\right) lnk + \left(\frac{2}{3}\right) lnh$$

Total differentiating

$$\hat{y} = \hat{A} + \left(\frac{1}{3}\right)\hat{k} + (2/3)\hat{h}$$

Output growth =[(600/200)^(1/25)]-1 (or 1/25*[ln(600)-ln(200)]) =4.49%,
Capital growth = [(1000/300)^(1/25)]-1 (or 1/25*[ln(1000)-ln(300)])=4.93; growth contribution is 1/3 => 1.64%
Human capital growth = [(3/2)^(1/25)]-1 (or 1/25*[ln(3)-ln(2)]=1.64%; growth contribution is 2/3 => 1.09%
Productivity growth = output growth – factor contribution = 4.49%-1.64%-1.09% = 1.76%

3. Krugman finds that Asian countries' rapid growth rate of output largely depends on a huge investment in physical capital, the growth in the employed share of the population and improved education standards of the work force. Explain why according to Paul Krugman Asia's "miracle" growth was a myth.

Answer

Krugman points out that the newly industrializing countries of Asia have achieved rapid growth in large part through an astonishing mobilization of resources. Consider the growth accounting. It says that growth rate of output is the sum of growth rate of productivity and growth rate of factors of production. Krugman finds that Asian countries' rapid growth of output largely depends on growth rates of factors of production, not on growth rate of productivity. For example, in case of Singapore, their per capita income grew at a 6.6 percent rate between 1966 and 1990. But this rapid growth is based on the growth in the employed share of the population from 27 to 51 percent, improved education standards of the work force, and a huge investment in physical capital. But there is no sign at all of increased efficiency. Because of diminishing returns, an increase in physical capital and human capital cannot sustain the long-run growth. For the long-run growth, improvement in productivity through improving technology and efficiency is needed. This is why Krugman argues that Asia's miracle growth was a myth.

4. Are developing and developed countries facing the same problem of population? How may population in these countries affect output?

Answer

In many poor countries, they are having high population growth rate. According to the Solow growth model, this dilutes per capita capital stock more quickly, and so lowers the steady state level of output per capita. In developed countries, however, they are having a decline in fertility which eventually produces a rise in the fraction of the population that is of elderly and a fall in the fraction of the population that is of working age. Ageing society consequently leads to a lower growth rate of GDP per capita. It is interesting to note that, once fertility rates drop, countries will experiencing a period with an increase in labor force fraction, meaning higher output. This is called a demographic "gift".

5. Consider a country described by the one-country model. Suppose that the country ***temporarily*** raises its level of γ_A. Draw graphs showing how the time paths of output per worker (y) and productivity (A) will compare under this scenario with what would have happened if there had been no change in γ_A.

Answer

(Just for your understanding. You don't need to write this down.) With no change in the fraction of workers devoted to research and development, productivity and output per worker would continue to grow indefinitely at the previous rate. That is,

$$\hat{y} = \hat{A} = \frac{\gamma L}{\mu},$$

where γ denotes the fraction of workers devoted to R&D. With an increase in γ to γ', we know from the following equation that,

$$\gamma < \gamma' \text{ implies } \hat{y} = \hat{A} = \frac{\gamma L}{\mu} < \hat{y}' = \hat{A} = \frac{\gamma' L}{\mu}.$$

Therefore, at the time of change, denoted as t_1 in the graph above, the growth rate of output per worker and productivity, $\hat{y}'$ and $\hat{A}'$ respectively, will be greater than before. However, the increase in the rate of growth will be accompanied by a decrease in the *level* of output per worker. Simply put,

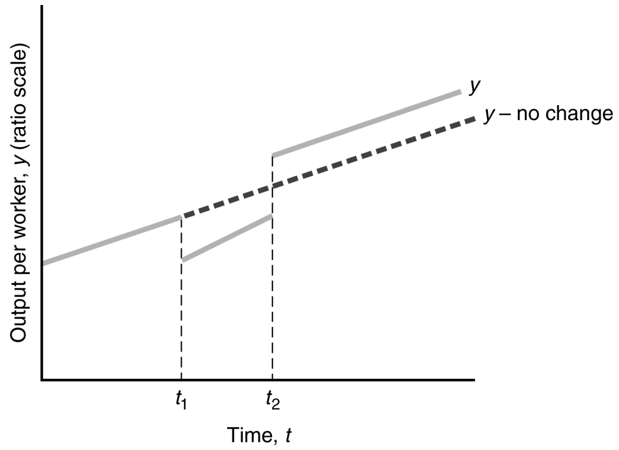
$$y = A(1-\gamma) > y' = A(1-\gamma').$$

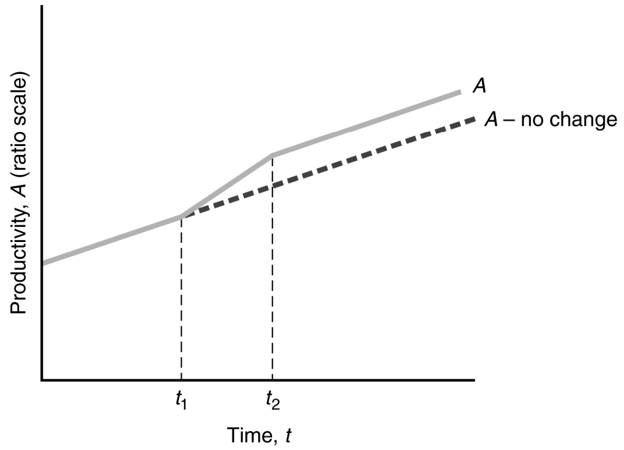
This amounts to an increase in the slope of, and a drop in, the level of output per worker. For productivity, there will not be a drop in the level of *A*, but the slope will rise at the time of change.

Because the change in γ to γ' is temporary, at the time t_2, that γ' returns to the original fraction γ, the process will reverse itself. Output per worker will jump up to a new level as workers move out of the R&D sector. But the jump up is larger in magnitude than the jump down. Intuitively, the level of productivity has risen during the temporary increase in the number of workers devoted to R&D. Thus, with the same number of people moving to the non-R&D sector as that moving to the R&D sector from before, the same number of workers can now be more productive. Mathematically, the first and second jumps are given as

$$\textbf{At } t_1\textbf{:}\ \Delta y = y - y' = A(1-\gamma) - A(1-\gamma') = A(\gamma' - \gamma) = A(\Delta\gamma),$$
$$\textbf{At } t_2\textbf{:}\ \Delta y = y'' - y' = A'(1-\gamma) - A'(1-\gamma') = A'(\gamma' - \gamma) = A'(\Delta\gamma).$$

Because the change in γ is the same and $A' > A$, we know that the absolute difference in the jump up is greater than the jump down. The level of A' is dependent on the length of the temporary increase in the fraction of workers devoted to R&D. Regardless of the level, the new growth path of output per worker will be the same rate before the temporary change, but it will start out at a higher level than if no change had occurred. As for productivity, the change in γ will return the growth rate of productivity to its original level, without any jumps. The figures are given below.





6. In the correlation diagram above, predicted GDP per capita relative to the U.S.=h_i/h_{us}, where h_i is based on the average years of schooling in country i.

1. Compare for Brazil the actual and predicted income relative to the U.S. Which one is larger?

Answer

Predicted income of Brazil relative to the United States is larger than actual income of Brazil relative to the United States.

2. Explain why *h* is an imperfect measure of human capital? How can this help explain the imperfect fit between actual and predicted income per capita relative to the U.S.?

Answer

h is based on the average years of schooling in country, but this is an imperfect measure of human capital for the following reasons.

1. Quality and quantity differences in education: For example, student/teacher ratio is larger in developing countries, teacher is not as well trained, and textbooks etc. are scarce. Those are quality differences. Additionally, number of days of schooling is higher in developed countries. This leads to quantity differences.

2. Externalities: There are positive externalities related to education, since giving one person more education not only raises her own human capital but also the human capital of those around her. But our proxy for human capital measures only the private return to education and it does not take into account positive externalities.

Hence, we tend to overstate human capital in Brazil, and this leads to higher predicted income of Brazil.

3. Does the human capital data lead to an understatement or overstatement of the role of productivity differences in explaining output differences?

<u>Answer</u>

Overstate human capital in Brazil =>Understate variation in human capital => understate role of factor accumulation => overstate role of productivity differences