Discussion handout 4

Question 1**: Production function with three factor inputs**

Consider a production function that takes the form:

$$Q = K^{\frac{1}{3}} * L^{\frac{1}{2}} * E^{\frac{1}{4}}$$

where K is the level of capital, L is the number of labor hired and E is the level of electricity usage. In the order of the way they are introduced and defined, suppose that prices per unit of each factor input are given by r, w and m.

a) Does the production exhibit a return to scale technology? If yes, state the degree of return of scale?
b) Derive the marginal rate of transformation
c) Derive the optimal bundle of K, L and E that maximizes the profit when p is the unit price of output.
d) Verify your answer in (c) with the second-order conditions.
e) Show that your demand equation has constant elasticity.

Question 2: **Multi-product problem**

A business conglomerate produces three different types of products, namely A, B and C. Each product has the market demand equation given by:

Product A: $P_A = 36 - Q_A^2;$

Product B: $P_B = 35 - 2Q_B^2 + Q_C;$

Product C: $P_C = 20 + Q_B - Q_C;$

Suppose that the cost function for producing the three types of products is given by:

$$TC(Q_A, Q_B, Q_C) = 8 + 2Q_A^3 - 2Q_B^3 + 3Q_B Q_c + 30Q_B + 12Q_C + \frac{1}{2}Q_B^2,$$

and the conglomerate is the sole supplier in each of the three markets.

Consider the following problems

a) Write down the profit function of this conglomerate firm.
b) Using the first-derivative test, solve for the profit-maximizing level of output for each of the three products.
c) Confirm your answer in (b) with the second-order derivative test.
d) Determine the level of maximized profit.