

Instructions

- (1) Please read the instruction carefully. Also take this habit with you into the exam room.
- (2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at least a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
- (3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

- (1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
- (2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
- (3) Default decimal point is 4.
- (4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
- (5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
- (6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

- (1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
- (2) Login to BE Moodle, head into the course, then the assignment topic.
- (3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

For all questions, answer up to 4 decimal places

Question 1. (15 points) Given this information

$$\begin{aligned}
 n &= 18 & \sum_{i=1}^n X_i &= 388.00 & \sum_{i=1}^n Y_i &= 50.90 \\
 \sum_{i=1}^n (X_i)^2 &= 9,620.00 & \sum_{i=1}^n X_i Y_i &= 1,254.90 \\
 \sum_{i=1}^n (X_i - \bar{X})^2 &= 211.00 & \sum_{i=1}^n (Y_i - \bar{Y})^2 &= 2.5844 \\
 \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) &= 20.58 & \sum_{i=1}^n \hat{u}_i^2 &= 0.5781
 \end{aligned}$$

Use the above sample information to answer all the following questions. Show explicitly all formulas and calculations.

- From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$, **find the estimators** of β_1 and β_2 with OLS method. Interpret the intercept and slope coefficients.
- Compute the value of R^2 and explain its meaning.
- If $X_i = 30$, estimate the value of \hat{Y}_i and explain its meaning.
- Calculate the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$.
- What are the 90-percent confident intervals for β_2 ? Interpret the meaning.
- Test the hypothesis whether the slope coefficients are different from zero at 0.05 level of significance.

- a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$, **find the estimators** of β_1 and β_2 with OLS method. Interpret the intercept and slope coefficients.

$$(A) \hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\hat{\beta}_2 = \frac{20.58}{211.00} = 0.0975$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 2.8278 - (0.0975)(21.5556)$$

$$\hat{\beta}_1 = 0.8278 - 2.1017 = 0.7261$$

$$\hat{Y} = 0.7261 + 0.0975 X_i$$

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{\beta}_1 = 0.7261 \text{ means if } x=0 \text{ } y=0.7261 \text{ on average}$$

$$\hat{\beta}_2 = 0.0975 \text{ means if } x \text{ increase by 1 unit } y \text{ increase by 0.0975 units on average}$$

- b) Compute the value of R^2 and explain its meaning.

$$(B) R^2 = \frac{1 - \sum \hat{U}_i^2}{\sum (y_i - \bar{y})^2}$$

$$= \frac{1 - 0.5781}{2.5844}$$

$$R^2 = 0.7763 \approx 77.63\%$$

The total variation of y can explained by % 77.63%.

- c) If $X_i = 30$, estimate the value of \hat{Y}_i and explain its meaning.

$$\text{SRF; } \hat{Y}_i = 0.7261 + 0.0975 X_i, \quad X=30$$

$$\hat{Y}_i = 0.7261 + 0.0975(30)$$

$$= 3.6511 \#$$

The average of \hat{Y}_i is 3.6511 when $x=30$ #

d) Calculate the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$$D) \text{var}(u_i) = \sigma^2 = \frac{\sum u_i^2}{n-2} = \frac{0.5781}{18-2} = \frac{0.5781}{16} = 0.0361$$

$$\text{var}(u_i) = 0.0361$$

$$\text{var}(\hat{\beta}_2) = \frac{0.036}{211} = 0.0002$$

$$\text{var}(\hat{\beta}_1) = \frac{9620}{18(211)} \cdot 0.0361 = 0.0914 \#$$

e) What are the 90-percent confident intervals for β_2 ? Interpret the meaning.

$$e) \hat{\beta}_2 - \left(+ \left(\frac{0.1}{2} \right) \cdot \text{se}(\hat{\beta}_2) \right) \leq \beta_2 \leq \hat{\beta}_2 + \left(+ \left(\frac{0.1}{2} \right) \text{se}(\hat{\beta}_2) \right)$$

$$0.0975 - (1.740 \cdot \sqrt{0.0002}) \leq \beta_2 \leq 0.0975 + (1.740 \cdot \sqrt{0.0002})$$

$$0.0729 \leq \beta_2 \leq 0.1221$$

upper limit is 0.1221

lower limit is 0.0729



The confident interval 90% error 0.1% real value of β_2
 between 0.0729 and 0.1221

f) Test the hypothesis whether the slope coefficients are different from zero at 0.05 level of significance.

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$$H_0: \beta_2 = 0$$

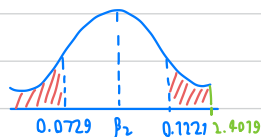
$$H_1: \beta_2 \neq 0$$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{SP(\hat{\beta}_2)}$$

$$= \frac{0.7267 - 0}{0.3023} = 2.4019$$

• upper : 0.7221

lower : 0.0729



Conclusion : Since $t_{cal} = 2.4019$, greater than 0.7221

So there is enough evidence claim to support
that β_2 is not zero at $\alpha = 0.01$

\therefore REJECT $H_0: \beta_2 = 0$

Question 2. Using the 2015 Health and Welfare Survey from the National Statistical Office, a simple linear regression is modeled as follows,

$$outp_i = \beta_1 + \beta_2 age_i + u_i$$

where $outp_i$ is how many times person i has visited hospital in 2015, from 0 to 7 times
 age_i is how old is person i , from 0 to 97 years.

We assume that both $outp_i$ and age_i are continuous, the estimation results in the following table. Answer the following questions and show your work.

Source	SS	df	MS	Number of obs	=	27,886
Model	77.5444409	1	77.5444409	F(1, 27884)	=	186.96
Residual	11565.0627	27,884	.414756231	Prob > F	=	0.0000
				R-squared	=	0.0067
				Adj R-squared	=	0.0066
Total	11642.6072	27,885	.417522223	Root MSE	=	.64402

outp	Coefficient	Std. err.	t	P> t	[95% conf. interval]
age	.0031338	.0002292			.0026846 .003583
_cons	.4279898	.0140339			.4004828 .4554969

- Test if both parameters are significantly different from zero or not. Use $\alpha = 0.05$.
- Interpret the meaning of $\hat{\beta}_2$. Does the sign of $\hat{\beta}_2$ make economic sense? Explain.
- If $outp_i$ is turned into natural logarithmic scale (ln), how would you reinterpret the relationship between $\hat{\beta}_2$ and \widehat{outp}_i , assumed that the given coefficient given in the table above can be used to interpret this new functional form.
- If age_i variable is divided by 10, how does it affect both the coefficients, standard errors, and confidence intervals? Answer the changes of both the constant and slope (if there is).
- Find the confidence interval of mean prediction at the age of 50 years old, given that $var(\hat{Y}_0) = 0.00002$ and $\alpha = 0.01$.

Question 3. Discuss in a short paragraph why the confidence interval for both the mean prediction and individual prediction get larger as the X_0 is further away from \bar{X} .

a) Test if both parameters are significantly different from zero or not. Use $\alpha = 0.05$.

A) TEST IF $\hat{\beta}_1 = 0.4279898$

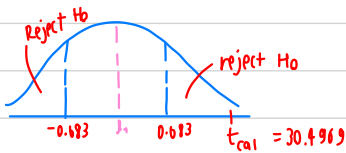
$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

$$\alpha = 0.05$$

$$t = \frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_1)}$$

$$= \frac{0.4279898 - 0}{0.0140339} = 30.49685$$



$t_{cal} = 30.4969$ is greater than $t = \frac{\alpha}{2}, n-k = 0.683$

reject H_0 , so $\hat{\beta}_1$ is difference from zero

means X affected y

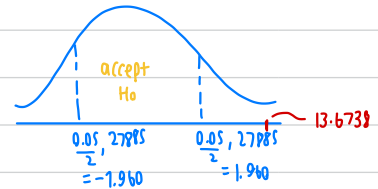
TEST IF $\hat{\beta}_2 = 0.0031338$ is different from zero

$$H_0: \hat{\beta}_2 = 0$$

$$H_1: \hat{\beta}_2 \neq 0$$

$$t_{cal} = \frac{0.0031338 - 0}{0.0002292} = 13.6738$$

decision



$\therefore t_{cal} = 13.6738$ is greater than

$t = \frac{\alpha}{2}, n-k = 1.960$ reject H_0 , so $\hat{\beta}_2$ is difference from zero means that X affected y

b) Interpret the meaning of $\hat{\beta}_2$. Does the sign of $\hat{\beta}_2$ make economic sense? Explain.

$\therefore \hat{\beta}_2 = 0.003 \rightarrow$ suggesting that age of people grow up by 1 years. This might cause citizens to need to visit the hospital with the rate around 0.003 times per year.

\therefore when the age increase, people will tend to visit hospital more frequently.

- c) If $outp_i$ is turned into natural logarithmic scale (\ln), how would you reinterpret the relationship between $\hat{\beta}_2$ and \widehat{outp}_i , assumed that the given coefficient given in the table above can be used to interpret this new functional form.

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$$(c) \ln outp_i = \beta_1 + \beta_2 age_i$$

$$\ln outp_i = 0.4279898 + 0.0031338 age_i$$

\therefore This means that an increasing 1 year on average level, about 0.31% number of time person visit hospital increase.

- d) If age_i variable is divided by 10, how does it affect both the coefficients, standard errors, and confidence intervals? Answer the changes of both the constant and slope (if there is).

$$\text{coefficient} = 0.0031338 \times 10 = 0.031338$$

$$\text{Std. Error} = 0.0002292 \times 10 = 0.002292$$

$$\begin{aligned} \text{Confident interval} &= (0.0026846 \times 10, 0.003583 \times 10) \\ &= (0.026846, 0.03583) \end{aligned}$$

\therefore This means that the coefficient, SE and CI by 10 times from its old value.

- e) Find the confidence interval of mean prediction at the age of 50 years old, given that $\text{var}(\hat{y}_0) = 0.00002$ and $\alpha = 0.01$. age $x=50$

$$\hat{y} = 0.4279898 + 0.0031338(50) = 0.5897$$

$$\hat{y} - (t_{\frac{\alpha}{2}}, 27, 985 \cdot SE(\hat{y}_0)) \leq y \leq \hat{y} + (t_{\frac{\alpha}{2}}, 27, 985 \cdot SE(\hat{y}_0))$$

$$0.5897 - 10.576 \cdot \sqrt{0.00002} \leq y \leq 0.5897 + 10.576 \cdot \sqrt{0.00002}$$

$$0.58212 \leq y \leq 0.5973$$

Question 3. Discuss in a short paragraph why the confidence interval for both the mean prediction and individual prediction get larger as the X_0 is further away from \bar{X} .

when x has a bigger gap from $\bar{x} \rightarrow$ high Variance

high number of variance $\rightarrow t_{cal} \frac{\hat{\beta}_1 - \beta}{SE(\hat{\beta}_1)}$

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high variance \uparrow high SE

$t_{cal} \downarrow \rightarrow$ hard to reject H_0 . This will cause β or mean prediction (\hat{y}) insignificant has a wider gap to x .