

TU 152 Practice Problems

Mathematical induction

1. Prove that for every positive integer n

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

2. Let x be any positive real numbers. Prove that for every positive integer n

$$(1+x)^n \geq 1+x^n$$

3. Prove that for every positive integer n that is greater than 4,

$$4^n > n^4.$$

4. For every nonnegative integer n ,

$$3 \mid (2^{2n} - 1)$$

Inequalities

Solve the following inequalities.

1. $x+1 > x-2$

2. $\frac{2x}{3} - 3 \geq 4 + \frac{2x}{3}$

3. $1 - \frac{x}{2} \geq 2 - \frac{3x}{2}$

4. $3x < \frac{x}{3} - 2$

5. $x^2 - 5x + 4 \geq 0$

6. $x^3 + 3x^2 + 2x > 0$

7. $(4x^2 - 2x + 3)(2x^2 - 3x - 20) \leq 0$

8. $\frac{x^2}{x^2 - 3x + 2} \geq 1$

9. $\frac{2x^2 - 3x + 5}{x+1} \leq x$

10. $1 < \frac{5}{x^2 - x + 3}$

11. $\frac{-x^2 + 2x}{(x-2)(4x^2 - 2x + 1)} < 0$

12. $\frac{2x^2 - 4x + 16}{x+2} \leq 4$

13. $\frac{5x^2 + 7x - 1}{x+1} < x+2$

14. $\frac{x^3 + 2x^2 + 5x}{x^2 + 2x - 3} \geq 0$