Topic for applications/examples

- Micro-market equilibrium model
  - Single market equilibrium (Partial analysis)
  - Multi-market equilibrium (General analysis)
- Macroeconomic model
  - Keynesian cross
  - IS-LM model

Simple macroeconomic model

- Simple Keynesian Cross
  - Partial equilibrium: <u>good market</u>
  - Financial market is treated as given.
  - Implications: *emphasizing at the role of government sector (fiscal actions)*
- IS-LM model
  - General equilibrium: <u>good and money market</u>
  - Financial market is endogenized. (Determined within the model.)
  - Implications: *emphasizing at the role of fiscal and monetary policy*

Simple macroeconomic model

- Solving for equilibrium solution of endogenous variables.
- Some comparative static analysis: analyze behavior of endogenous variables.
- Policy issues
  - Multipliers
  - Crowding-out effects
  - Policy effectiveness.

A simple Keynesian Cross model

$Y = C + I$ ➔ equilibrium condition

$C = \mathrm{C(Y)} = \mathrm{a} + \mathrm{bY}$ ➔ behavior of consumption

$I = I_0$ ➔ exogenous investment (take it as given.)

Exogenous: I_0.
Endogenous: C and Y.
Parameters: a (autonomous consumption), b (mpc)

A numerical example

$$Y = C + I$$

$$C = \mathrm{C(Y)} = 10 + 0.5 * \mathrm{Y}$$

$$I \quad = 100$$

Exogenous: I_0.

Endogenous: C and Y.

Parameters: $\mathrm{a, b}$

A numerical example: model solution

- Solving for the solution of endogenous variables, we yield that

$$Y^* = \_\_\_\_\_\_220\_\_\_\_\_\_\_\_\_.$$

$$C^* = \_\_\_\_\_\_120\_\_\_\_\_\_\_\_\_.$$

Simple Keynesian Cross model: general solution

- In the general form,

$$Y^* = \frac{a + I_0}{1 - b}$$

$$C^* \quad = a + b\left(\frac{a + I_0}{1 - b}\right)$$

- What does the solution mean?
  - Y and C depend on “a,b” and “the given level of investment.”
  - If any of these change, “Y^*” and “C^*” would change correspondingly.

Simple Keynesian Cross model: Comparative static

- Sensitivity analysis
  - What happen if I_0 increases to I_0' ? Can you compare the change in Y and C, before and after?

$$\Delta Y^* = Y^{*\prime} - Y^* = \frac{1}{1-b} * (I_0' - I_0)$$

$$\Delta C^* = C^{*\prime} - C^* = \frac{b}{1-b} * (I_0' - I_0)$$

Simple Keynesian Cross model: Comparative static

- $I_0' = 110$, $\Delta Y^* = ?$ and $\Delta C^* = ?$

$$\Delta Y^* = {Y^*}' - Y^* = \frac{1}{1-0.5} * (110 - 100)$$

$$\Delta C^* = {C^*}' - C^* = \frac{0.5}{1-0.5} * (110 - 100)$$

Possible extensions

- Endogenous investment
  - Investment also depends on level of income
  - Income-accelerator framework.
- Government sector
  - Typically treated as an exogenous variable.
  - Alternatively, G might be tied to the level of GDP.
- Taxation
  - Consumption depends on disposable income.
  - Income tax.
- Effect of interest rate on private spending
  - Consumption and investment could also depend on income
- Open economy
  - Equilibrium condition must include net export, i.e. X – M.
  - Need to model “X” and “M”.

Extended Keynesian Cross model

$$Y = C + I + G$$ → Equilibrium condition

$$\mathrm{C} = C(Y_d, r)$$ → Consumption function
- Positively vary with disposable income
- Negatively vary with interest rate

$$Y_d = Y - T$$

$$\mathrm{I} = I(Y, r)$$ → Investment function
- Positively vary with income
- Negatively vary with interest rate

$$T = T(Y) \Rightarrow linear\ income\ tax$$

$$G = G_0$$ → Exogenous variable

Example

- Equilibrium (1)

$$Y = C + I + G$$

- Consumption function (2)

$$C = a + bY_d$$

- Disposable income (3)

$$Y_d = Y - T$$

- Tax rule (4)

$$T = T_0 + t * Y$$

- Investment function (5)

$$I = I_0 - I_1 r$$

- Government spending (6)

$$G = G_0$$

Solving for the solution of Endogenous variables

Endo: Y, C, I, T

Exo: G, r

Other than these are Parameters.

Solve for the solution, one variable at a time.

Example

- Substitute (2), (5), (6) into (1), this yields us
$$\mathrm{Y} = \mathrm{a} + \mathrm{bY_d} + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0} \quad (7)$$
- Substituting (4) into (3), this yields us
$$\mathrm{Y_d} = Y - T_0 - t * Y \quad (8)$$
- Substituting (8) into (7), this yields us
$$\mathrm{Y} = \mathrm{a} + \mathrm{b}(Y - T_0 - t * Y) + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0}$$
$$= \mathrm{a} - \mathrm{bT_0} + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0} + \mathrm{b}(1 - t)Y \quad (9)$$
- Solving for "Y" in terms of other variables,
$$\mathrm{Y} = \frac{1}{1 - b(1 - t)}[\mathrm{a} - \mathrm{bT_0} + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0}]$$
- Explaining the general form of endogenous level of "Y" for given values of parameters and exogenous variables, aka, the IS equation.

Example

- What can we play with the solution?
  - "Y" responds to changes of whatever on the right!
  - Sensitivity analysis

$$\mathrm{Y} = \frac{1}{1-b(1-t)}[\mathrm{a} - \mathrm{bT_0} + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0}]$$

Sensitivity analysis

- a=1 (old value) ➔ a' =2 (new value)
- Old equilibrium

  $$-Y^{old} = \frac{1}{1-b(1-t)}[1 - bT_0 + I_0 - I_1r + G_0]$$

- New equilibrium

  $$-Y^{new} = \frac{1}{1-b(1-t)}[2 - bT_0 + I_0 - I_1r + G_0]$$

- Change in Y = $\frac{1}{1-b(1-t)}$(2-1)

Sensitivity analysis

- That is, one unit change in "a" causes an increase in "y" by $\frac{1}{1-b(1-t)} > 1$ units.
- This means that "a" has a multiplicative effect of "Y".
- We therefore call $\frac{1}{1-b(1-t)}$ as the multiplier of "a", the autonomous consumption.

Multiplier for I_0?

- Do the same, we yield that multiplier of I_0 (autonomous investment) is equal to,

$$\frac{1}{1-b(1-t)}$$

- Coefficient attached to the variable I_0 and a.

Multiplier of government variables

$$\mathrm{Y} = \frac{1}{1 - b(1 - t)}[\mathrm{a} - \mathrm{bT_0} + \mathrm{I_0} - \mathrm{I_1 r} + \mathrm{G_0}]$$

$$\Delta\mathrm{Y} = \frac{1}{1 - b(1 - t)}[\Delta\mathrm{a} - \Delta(\mathrm{bT_0}) + \Delta\mathrm{I_0} - \Delta(\mathrm{I_1 r}) + \Delta\mathrm{G_0}]$$

- Multiplier of T_0: $\frac{\Delta Y}{\Delta T_0} = \frac{-\mathrm{b}}{1-b(1-t)}$
- Multiplier of G_0: $\frac{\Delta Y}{\Delta G_0} = \frac{1}{1-b(1-t)}$

Multiplier of government variables

- Notice something?
  - In the absolute term, tax has smaller size of multiple effect than government purchase.
  - Why?
  - Intuitively, tax cut doesn’t increase output in the first round. Tax cut indirectly increases output through an increase in consumption.
  - On the other hand, government spending increases output in the first round, suddenly right after the increase in G.

What about “b”?

- No formula can be given for now. Need to find by using derivative. (Will come back to its closed-form formula later)
- Just live with the *Brute-force* method:
  - You must compare the two values “y”.
  - New Y (with new “b”) – Initial Y (with initial “b”).
- Intuitively, the change should be positive when “b = MPC” increases.

A numerical example

- Equilibrium

$$Y = C + I + G$$

- Consumption

$$C = 48 + 0.8Y$$

- Investment

$$I = 98 - 75r$$

- Government

$$G = G_0$$

A numerical example

Questions:

a. Derive the IS equation

b. Solve for equilibrium output when r = 0.1 (=10%)

c. Calculate multiplier of autonomous investment.

Derive the IS equation

$$Y = 48 + 0.8Y \ + 98 - 75r \ + G_0$$

$$0.2Y = 146 - 75r \ + G_0$$

$$Y = 5(48 + 98 - 75r \ + G_0)$$

Solve for equilibrium output when r = 0.1 (=10%)

$$Y = 5(48 + 98 - 7.5 \ + G_0)$$

Calculate multiplier of autonomous investment.

- $Y = \mathbf{5}(48 + 98 - 7.5 + G_0)$

Autonomous C ↑ (48)

Autonomous I ↑ (98)

  - Multiplier G = 5
  - Multiplier autonomous investment = 5
  - Multiplier autonomous consumption = 5

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IS-LM model

- So far, we have seen a simple Keynesian cross model that "r" is exogenously given.
  - An increase in "r" reduces aggregate expenditure, and hence the level of income in the equilibrium.
- Is the assumption that "r" an exogenous variable sensible?
- IS-LM model is an extension of the Keynesian cross.
  - It endogenizes the value of interest rate by modeling how interest rate is determined in the equilibrium.

IS-LM model

- Conceptually, interest rate is determined from the equilibrium in money market, where money demand is equal to money supply.
- In economics, we typically assume that money demand depends on (i) income and (ii) interest rate.
- Since money demand also depends on income, equilibrium level of interest rate in the money market also depends on the level of income.
- As a result, income and interest rate will have to be simultaneously determining each other in the general equilibrium.

Money market

- Equilibrium

$$M^d = M^s$$

- Money demand equation

$$M^d = \mathrm{L}_0 + \mathrm{L}_1\mathrm{Y} - \mathrm{L}_2\mathrm{r}$$

- Money supply

$$M^s = M_0^s$$

Equilibrium in the money market

- Putting everything into the equilibrium condition

$$L_0 + L_1 Y - L_2 r = M_0^S$$

- Suppose that "Y" is given for now. Write "r" in terms of "Y" and "M_0^S", we yield the LM equation,

$$r = \frac{1}{L_2}(L_0 + L_1 Y - M_0^S)$$

IS-LM (general) equilibrium

- (Y*,r*) that is the simultaneous solution of the system of equations that is characterized by
  - IS equation

$$Y = \frac{1}{1 - b(1 - t)}[a - bT_0 + I_0 - I_1 r + G_0]$$

  - LM equation

$$r = \frac{1}{L_2}(L_0 + L_1 Y - M_0^S)$$

To get a better sense, consider the following numerical example

- $C = 0.8(Y - T)$
- $T = 1{,}000$
- $I = 800 - 20r$
- $G = 1{,}000$
- $Y = C + I + G$
- $M^d = 0.4Y - 40r$
- $M^S = 1{,}200$

Example

- Write numerical formula for IS equation
- Write numerical formula for LM equation
- Solve for endogenous equilibrium solution of “Y”, “C”, “I” and “saving”.
- What happen to equilibrium “Y” when G increases by 200.
- Back to when G is equal to 1000. What happen to equilibrium “Y” when money supply increases by 200.
- Measuring in term of the change in output, which policy is more effective? Explain the reason.

Write numerical formula for IS equation

$C = 0.8(Y - T)$

$T = 1{,}000$

$I = 800 - 20r$

$G = 1{,}000$

$Y = C + I + G$

$Y = 0.8(Y - 1{,}000) + 800 - 20r + 1{,}000$

$\Rightarrow Y = 5*\{800 - 0.8(1{,}000) + 1{,}000 - 20r\}$

$= 5000 - 100r$

Write numerical formula for LM equation

$M^d = 0.4Y - 40r$

$M^S = 1{,}200$

$0.4Y - 40r = 1200$

$r = 0.01Y - (1200/40)$

$= 0.01Y - 30$

Solve for endogenous equilibrium solution of “Y”, “C”, “I” and “saving”.

$Y = 5000 – 100r$

$r = 0.01Y – 30$

$Y^* = 8000/2 = 4000$

$r^* = 10$ (=1000%)

$C^* = 2400$

$I^* = 600$

What happen to equilibrium “Y” when G increases by 200.

- G = 1000 ➔ G = 1200

old

$$Y = 5*\{800 - 0.8(1{,}000) + 1{,}000 - 20r\}$$

new

$$Y = 5*\{800 - 0.8(1{,}000) + 1200 - 20r\}$$

$$= 6000 - 100r$$

$$= 6000 - 100*(0.01Y - 30)$$

$$Y* = 9000/2 = 4500.$$

Back to when G is equal to 1000. What happen to equilibrium "Y" when money supply increases by 200.

old: $r = 0.01Y – (1200/40)$

new: $r = 0.01Y – (1400/40)$

$Y = 5000 – 100r$ (the same IS)

$= 5000 – 100(0.01Y-35)$

$= 8500/2 = 4250.$

Measuring in term of the change in output, which policy is more effective? Explain the reason.

	Change in output
M increase = 200	250 (1.25)
G increase = 200	500 (2.5)

G is more effective.

Why??

Back to general form

- (Y*,r*) that is the simultaneous solution of the system of equations that is characterized by
  - IS equation

$$Y = \frac{1}{1-b(1-t)}[a - bT_0 + I_0 - I_1 r + G_0]$$

  - LM equation

$$r = \frac{1}{L_2}(L_0 + L_1 Y - \ M_0^S)$$

A bit tedious…but worth digging out

$$Y = \frac{1}{1 - b(1 - t)} [a - bT_0 + I_0 - I_1 r + G_0]$$

$$Y = \frac{1}{1 - b(1 - t)} \left[a - bT_0 + I_0 - I_1 \left(\frac{1}{L_2} (L_0 + L_1 Y - M_0^S) \right) + G_0 \right]$$

- Suppose that $\mathrm{x} = \frac{1}{1-b(1-t)}$: the multiplier of G_0 in the simple Keynesian cross model.

A bit tedious…but worth digging out

$$Y = x\left[a - bT_0 + I_0 - I_1\left(\frac{1}{L_2}(L_0 + L_1 Y - M_0^s)\right) + G_0\right]$$

$$Y^* = \frac{x}{1 + x\left(I_1 \frac{L_1}{L_2}\right)}\left[a - bT_0 + I_0 - \frac{I_1}{L_2}L_0 + \frac{I_1}{L_2}M_0^s + G_0\right]$$

Solution when both good and money market are cleared together

Multiplier of G: Keynesian cross and IS-LM

Multiplier of G in the IS-LM is $\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)}$

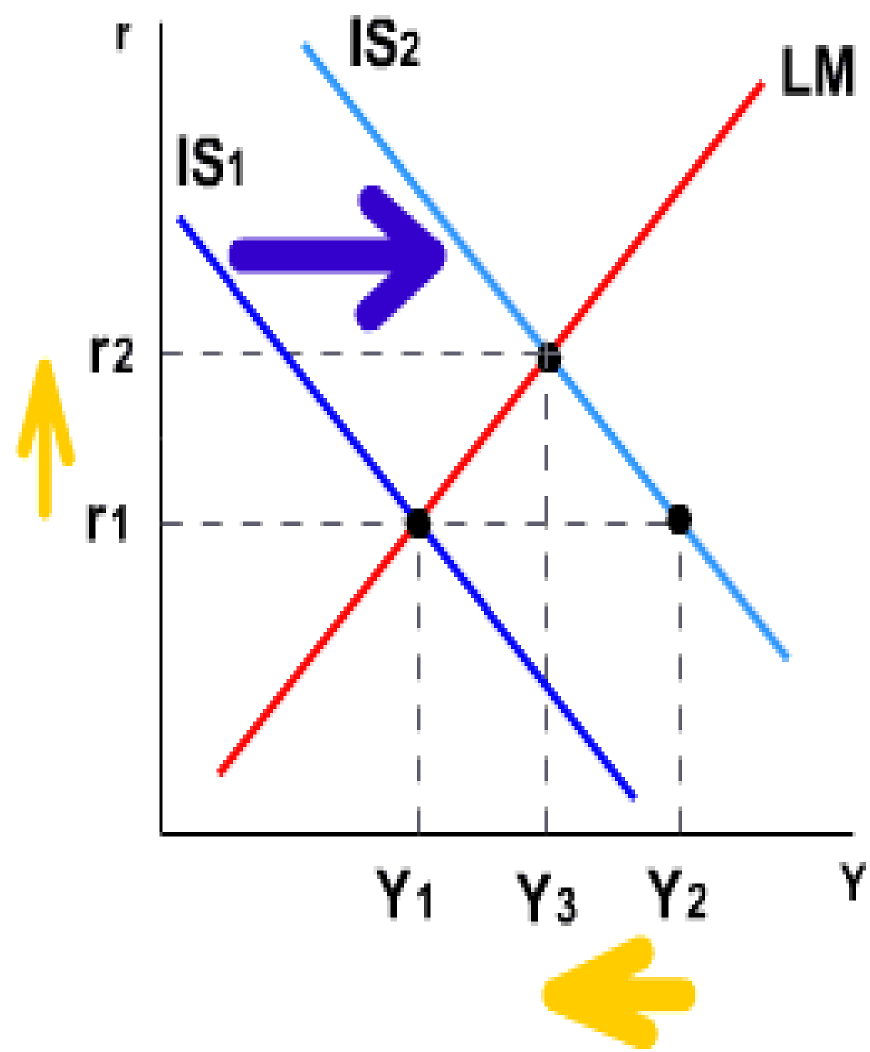
Since $\left(I_1\frac{L_1}{L_2}\right) > 0$, this implies that

$$\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)} < x$$

Compare with the previous case that "r" is given, multiplier effect of G is now getting smaller!

Multiplier of G: Keynesian cross and IS-LM

- The crowding-out effect
  - Obviously, an increase in "G" causes an increase in "y".
  - If interest rate were kept fixed, effect of "G" would be very strong. (Y1Y2)
  - However, as "y" increases, interest rate tends to move along. (following the LM equation)
  - This would lower the investment, and thus reducing aggregate expenditure.
  - So, it partially offsets the effect of "G". (Y3Y2)
  - Income wouldn't be changing that much, comparing to when you don't have this feedback effect. (Y1Y3 vs Y1Y2)



A bit tedious…but worth digging out

$$Y = x\left[a - bT_0 + I_0 - I_1\left(\frac{1}{L_2}(L_0 + L_1 Y - M_0^s)\right) + G_0\right]$$

$$Y^* = \frac{x}{1 + x\left(I_1 \frac{L_1}{L_2}\right)}\left[a - bT_0 + I_0 - \frac{I_1}{L_2}L_0 + \frac{I_1}{L_2}M_0^s + G_0\right]$$

Solution when both good and money market are cleared together

Multiplier of other exogenous variables

- Multiplier of tax is $-\text{b}\left[\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)}\right]$

- Multiplier of I_0 is the same as multiplier of G

- Multiplier of M is $\frac{x}{1+x\left(I_1\frac{L_1}{L_2}\right)}\frac{I_1}{L_2}$

Relative effectiveness of G and M

- Which policy is more effective, between an increase in government and an increase in money supply?
- To be more precise, which one can increase income/GDP more, for the same amount of the initial change?
- Compare multipliers the two.

Relative effectiveness of G and M

- Compare the multipliers of M and G

$$\frac{multiplier\ of\ M}{multiplier\ of\ G} = \frac{I_1}{L_2}$$

- $\mathrm{I}_1 > L_2$ ➔ "M" is better than "G". (vice versa)
- Intuition:
  - High value of I_1 implies that investment is very sensitive to interest rate.
  - High value of L_2 implies that demand money is less sensitive to income. Crowding out effect is not strong.

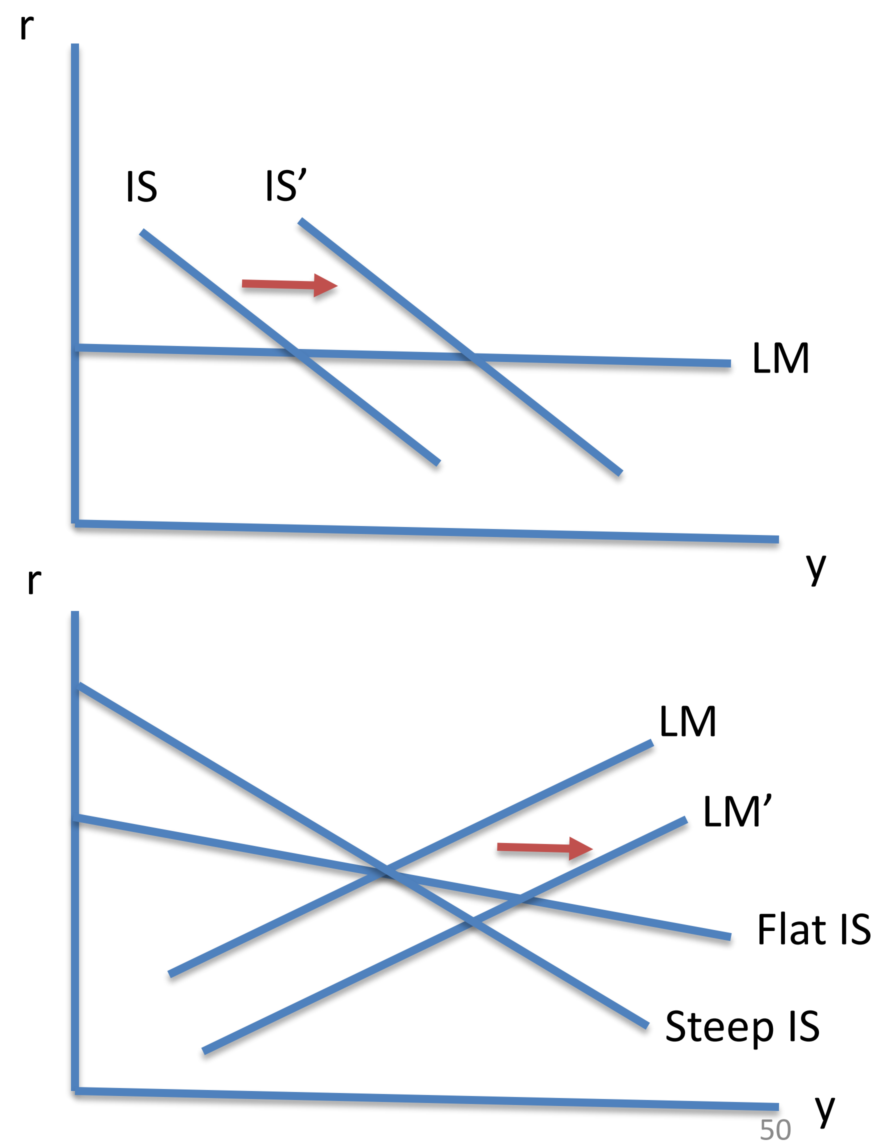
Relative effectiveness of G and M

- Two key channels of influence:
  - The flatter is the IS curve, the more effective is monetary policy in influencing output.
    - Aggregate expenditure is highly sensitive to interest rate. (high I_1)
    - $Y = \frac{1}{1-b(1-t)}[a - bT_0 + I_0 - I_1 r + G_0]$
  - The flatter is the LM curve, the more effective is fiscal policy in influencing output.
    - Crowding effect is weak. (high L_2)

      $r = \frac{1}{L_2}(L_0 + L_1 Y - M_0^S)$

Relative slope of IS and LM curve

- Which policy is more effective depends on the relative slope of IS and LM curve.
  - LM relatively flatter ➔ Fiscal policy
  - IS relatively flatter ➔ Monetary policy
- Next, we will show this.



Back to the previous example

- Does it make sense for the answer that you have solved for?
  - Yes $I_1 = 20$ and L_2=40
  - So, $\frac{I_1}{L_2} = \frac{1}{2} < 1$
  - $\frac{multiplier\ of\ M}{multiplier\ of\ G} < 1$