EE431 Economics of Financial Markets and Institutions
Exercise 4: Capital Asset Pricing Model (CAPM)

1. You own 50 shares of stock A, which has a price of $12 per share, and 100 shares of stock B, which has a price of $6 per share. What is the portfolio weight for stock B in your portfolio?

   ANSWER.
   $$\begin{aligned} \text{Investment in Stock A} &= (50 \times 12) &= 600 \\ \text{Investment in Stock B} &= (100 \times 6) &= 600 \\ Total &= 600 + 600 &= 1,200 \\ \text{weight of stock B} &= \frac{\text{value of stock B}}{\text{total value}} &= \frac{600}{1,200} \\ & &= 0.5 \end{aligned}$$

   weight of stock B in the portfolio is 0.5.

2. Use the following information to answer all parts. Assume that all assumptions of CAPM hold. An investor want 4 % rate of return

Assets	Expected Return (%)	Standard Deviation (%)
Market Portfolio	7	10
Risk-free Asset	1	

   - Market port folio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

   (a) What are the portfolio weights the investor put on risk free asset and the market portfolio?
   ANSWER. Let a is weight put on the market portfolio and $b = (1 - a)$ is weight put on the risk free asset.
   $$\begin{aligned} E(R_p) &= (1-a)R_f + aE(R_m) \\ 0.04 &= (1-a)(0.01) + a(0.07) \\ 0.04 &= 0.01 - 0.01a + 0.07a \\ 0.03 &= 0.06a \\ a &= \frac{0.03}{0.06} \\ &= 0.5 \\ (1-a) &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$
   Therefore, weight put on risk free asset is equal to 0.5 and weight put on the market portfolio is equal to 0.5.

   (b) What is the standard deviation of the portfolio in quation (a)?
   ANSWER. The standard deviation of the portfolio in equation (a) is equal to $\sigma_p = a\sigma_m = 0.5(0.10) = 0.05 = 5\%$.

3. If the covariance of stock 1 with stock 2 is -.0065, then what is the covariance of stock 2 with stock 1?

   Answer. -0.0065. $COV(X,Y) = COV(Y,X)$.
   $$\begin{aligned} COV(X,Y) &= E(X - E(X))(Y - E(Y)) \\ &= E(Y - E(Y))(X - E(X)), \text{ (commutative property)} \\ &= COV(Y,X) \end{aligned}$$

4. Company X has a beta of 1.45. The expected risk-free rate of interest is 2.5% amd the expected return on the market as a whole is 10%. Using the CAPM, what is ABC's expected return?

   According to CAPM, $ER_i = R_f + \beta(E(R_m) - R_f)$

   $R_i = 2.5\% + 1.45\%(10\% - 2.5\%) = 13.375\%$

5. Suppose CAPM holds. Given the following data, calculate the betas and the expected rates of reuturns of the two securities.

	Expected rate of returns (%)	Standard deviation (%)	Correlation with the market portfolio
Security 1	ER_1?	20	0.9
Security 2	ER_2?	9	0.8
Market portfolio	12	12	1
Risk free asset	5	0	0

   The security market line (SML) is $ER_i = R_f + \beta_i(ER_m - R_f)$.

   SML : $ER_i = 5\% + \beta_i(12\% - 5\%) = 5\% + 7\%\beta_i$

   Security 1. $\beta_1 = \dfrac{\sigma_{1m}}{\sigma_m^2} = \dfrac{r_{im}\sigma_1\sigma_m}{\sigma_m^2} = \dfrac{0.9 \times 20\% \times 12\%}{12\% \times 12\%} = 1.5$. $ER_1 = 5\% + 1.5 \times 7\% = 15.5\%$.

   Security 2. $\beta_2 = \dfrac{\sigma_{2m}}{\sigma_m^2} = \dfrac{r_{im}\sigma_2\sigma_m}{\sigma_m^2} = \dfrac{0.8 \times 9\% \times 12\%}{12\% \times 12\%} = 0.6$. $ER_1 = 5\% + 0.6 \times 7\% = 9.2\%$.

6. Consider an economy with just two assets. The details of these are given below.

Stock	Number of shares	Price ($)	Expected return (%)	Standard deviation (%)
A	100	1.5	15	15
B	150	2	12	9

   The correlation coefficient between the returns on the two assets is $\frac{1}{3}$ and there is also a risk free asset. Assume the CAPM model is satisfied.

   (a) What is the expected rate of return on the market portfolio?

   Total Value of Stock A $= 100 \times \$1.5 = \$150$

   Total Value of Stock B $= 150 \times \$2 = \$300$

   Total Value of Market $= \$150 + \$300 = \$450$

   Weight of stock A in the market portfolio is equal to $w_A = \dfrac{150}{450} = \dfrac{1}{3}$.

   Weight of stock B in the market portfolio is equal to $w_B = \dfrac{300}{450} = \dfrac{2}{3}$.

   Hence, the return on the market portfolio is equal to

$$\begin{aligned} ER_m &= w_A ER_A + w_B ER_B, \\ &= \frac{1}{3} \times 15\% + \frac{2}{3} \times 12\%, \\ &= 13\%. \end{aligned}$$

   (b) What is the standard deviation of the market portfolio?

$$\begin{aligned} \sigma_m^2 &= w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B r_{AB}\sigma_A\sigma_B, \\ &= \left(\frac{1}{3}\right)^2 \times (15\%)^2 + \left(\frac{2}{3}\right)^2 \times (9\%)^2 + 2 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(15\%)(9\%) \\ &= 81 \\ \sigma_m &= 9\% \end{aligned}$$

(c) What is the beta of stock A?

By definition, $\beta_A = \frac{r_{A,m}\sigma_A\sigma_m}{\sigma_m^2} = \frac{COV(R_A, R_m)}{\sigma_m^2}$.

Find $COV(R_A, R_m)$.

$$
\begin{aligned}
COV(R_A, R_m) &= E([(R_A - E(R_A)(R_m - E(R_m))] \\
&= E([(R_A - E(R_A)(w_A R_A + w_B R_B - w_A ER_A - w_B ER_B)] \\
&= E([(R_A - E(R_A)\{(w_A R_A - w_A ER_A) + (w_B R_B - w_B ER_B)\}] \\
&= E\left[w_A(R_A - E(R_A)^2 + w_B(R_A - ER_A)(R_B - ER_B)\right] \\
&= w_A\sigma_A^2 + w_B COV(R_A, R_B) \\
&= w_A\sigma_A^2 + w_B r_{AB}\sigma_A\sigma_B \\
&= \left(\frac{1}{3}\right)(15\%)^2 + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)(15\%)(9\%) \\
&= 105 \\
\beta_A &= \frac{COV(R_A, R_m)}{\sigma_m^2} \\
&= \frac{105}{9^2} \\
&= 1.2963
\end{aligned}
$$

(d) What is the risk free rate of return?

The risk-free return is derived from the the security market line. The security market line gives

$$
\begin{aligned}
ER_A &= R_f + \beta_A(ER_m - R_f), \\
15\% &= R_f + 1.2963(13\% - R_f), \\
15\% &= (1 - 1.2963)R_f + (1.2963 \times 13\%), \\
0.2963R_f &= 16.8519\% - 15\%, \\
0.2963R_f &= 1.8519, \\
R_f &= \frac{1.8519}{0.2963}, \\
&= 6.25\%.
\end{aligned}
$$

(e) Construct the capital market line and the security market line.

The capital market line

$$
\begin{aligned}
ER_p &= R_f + \left(\frac{ER_m - R_f}{\sigma_m}\right)\sigma_p, \\
&= 6.25\% + \left(\frac{13\% - 6.25\%}{9\%}\right)\sigma_p, \\
&= 6.25\% + 0.75\sigma_p.
\end{aligned}
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The security market line

$$
\begin{aligned}
ER_i &= R_f + \beta_i(ER_m - R_f),, \\
&= 6.25\% + (13\% - 6.25\%)\,\beta_i, \\
&= 6.25\% + 6.75\beta_i.
\end{aligned}
$$

7. Imagine you have a portfolio of two risky stocks which turns out to have no diversification. The reason you have no diversification is ....the returns move perfectly with one another.....