# Assignment 14

## ARIMA & GARCH Models

From the data set `assign_timeseries.dta`:

Part I

1. Estimate Autoregressive Integrated Moving Average (ARIMA(p,d,q)) model for spot and future – determine the most appropriated order for p, d, and q using SBIC given the maximum lag equals 4.

***According to the unit root test on the last assignment (Assignment13), it suggests that both spot and future are integrated series of order 1, i.e. d = 1***

```
. tsset time

        time variable:  time, 1 to 795

                delta:  1 unit

       . qui arima spot, arima(1,1,1)

. est store arima111

. qui arima spot, arima(1,1,2)

. est store arima112

. qui arima spot, arima(1,1,3)

. est store arima113

. qui arima spot, arima(1,1,4)

. est store arima114

. qui arima spot, arima(2,1,1)

. est store arima211

. qui arima spot, arima(2,1,2)

. est store arima212

. qui arima spot, arima(2,1,3)

. est store arima213

. qui arima spot, arima(2,1,4)

. est store arima214

. qui arima spot, arima(3,1,1)

. est store arima311

. qui arima spot, arima(3,1,2)

. est store arima312

. qui arima spot, arima(3,1,3)

. est store arima313

. qui arima spot, arima(3,1,4)
```

```
. est store arima314
. qui arima spot, arima(4,1,1)
. est store arima411
. qui arima spot, arima(4,1,2)
. est store arima412
. qui arima spot, arima(4,1,3)
. est store arima413
. qui arima spot, arima(4,1,4)
. est store arima414
. est table arima11*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

-------------------------------------------------------------------------------
    Variable |   arima111        arima112        arima113        arima114
-------------+-----------------------------------------------------------------
spot         |
       _cons | -.10471744      -.10468155      -.10474014      -.10717398
-------------+-----------------------------------------------------------------
ARMA         |
          ar |
         L1. |  .43536547*      .32884791      -.21609401       .59490084
             |
          ma |
         L1. | -.35351081      -.25287121       .29330109      -.52209152
         L2. |                  .02661686       .06275321      -.00167449
         L3. |                                  .07084457**     .03360908
         L4. |                                                 -.07201064*
-------------+-----------------------------------------------------------------
sigma        |
       _cons |  8.1399985***    8.1383248***    8.1283318***    8.1204174***
-------------+-----------------------------------------------------------------
Statistics   |
           N |        794             794             794             794
          ll | -2791.4965      -2791.3458      -2790.3816      -2789.5823
        chi2 |  18.312657        16.97941       13.218602       28.319443
         aic |   5590.993       5592.6915       5592.7633       5593.1646
         bic |  5609.7013        5616.077       5620.8258       5625.9042
```

```
------------------------------------------------------------------------------
                                             legend: * p<.1; ** p<.05; *** p<.01

. est table arima21*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

------------------------------------------------------------------------------
    Variable |   arima211        arima212        arima213        arima214
-------------+----------------------------------------------------------------
spot         |
       _cons | -.10453586      -.10767542      -.10609456      -.10747347
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |  .30210361       1.3247307***    .46523894***    1.3085163***
         L2. |  .02452891      -.92390998***   -.83397031***   -.92906275***
             |
          ma |
         L1. | -.22494448      -1.3022249***   -.39216993***   -1.2433923***
         L2. |                  .94779741***    .82697929***    .88679293***
         L3. |                                  .09595475***    .05677728
         L4. |                                                  .00003472
-------------+----------------------------------------------------------------
sigma        |
       _cons |  8.1387104***     8.077039***    8.1149358***    8.0663508***
-------------+----------------------------------------------------------------
Statistics   |
           N |        794             794             794             794
          ll | -2791.3835      -2785.5252      -2789.1057      -2784.5179
        chi2 |  16.085901       3656.4671       241.72156       4791.7034
         aic |  5592.7671       5583.0504       5592.2113       5585.0358
         bic |  5616.1525       5611.1129       5624.9509       5622.4524
------------------------------------------------------------------------------
                                             legend: * p<.1; ** p<.05; *** p<.01

. est table arima31*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)
```

| Variable | arima311 | arima312 | arima313 | arima314 |
|---|---|---|---|---|
| spot | | | | |
| _cons | -.10486338 | -.10606433 | -.10806841 | -.10765191 |
| ARMA | | | | |
| ar | | | | |
| L1. | -.56058829 | .56814277*** | .66749387** | .58444973 |
| L2. | .08569307* | -.89762847*** | -.06995188 | .02671112 |
| L3. | .06374492** | .09752317*** | -.59846871* | -.67269639 |
| | | | | |
| ma | | | | |
| L1. | .63856195 | -.49161073*** | -.61040004* | -.5189438 |
| L2. | | .88783708*** | .0556601 | -.03501278 |
| L3. | | | .65798111** | .71899171 |
| L4. | | | | .02307553 |
| sigma | | | | |
| _cons | 8.1302906*** | 8.1136284*** | 8.0655263*** | 8.0640793*** |
| Statistics | | | | |
| N | 794 | 794 | 794 | 794 |
| ll | -2790.5308 | -2788.9397 | -2784.4054 | -2784.2793 |
| chi2 | 21.334447 | 289.47751 | 1722.9017 | 1770.384 |
| aic | 5593.0617 | 5591.8793 | 5584.8108 | 5586.5587 |
| bic | 5621.1242 | 5624.6189 | 5622.2274 | 5628.6524 |

legend: * p<.1; ** p<.05; *** p<.01

```
. est table arima41*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)
```

| Variable | arima411 | arima412 | arima413 | arima414 |
|---|---|---|---|---|
| spot | | | | |

| | | | | |
|---|---|---|---|---|
| | _cons | -.10781888 | -.10752733 | -.10764238 | -.10766283 |
| ARMA | | | | | |
| | ar | | | | |
| | L1. | .6019371* | 1.3677506*** | .61524226 | .8391139 |
| | L2. | -.00196743 | -1.0093802*** | .00960034 | -.13065856 |
| | L3. | .02357655 | .05753428 | -.67663352 | -.67731235 |
| | L4. | -.074501** | -.00157074 | .02301592 | .1703585 |
| | ma | | | | |
| | L1. | -.52565375 | -1.3026494*** | -.54889172 | -.77179962 |
| | L2. | | .964438*** | -.01961375 | .10834369 |
| | L3. | | | .72233737 | .72448925 |
| | L4. | | | | -.15754229 |
| sigma | | | | | |
| | _cons | 8.1178063*** | 8.0662434*** | 8.0638126*** | 8.0634989*** |
| Statistics | | | | | |
| | N | 794 | 794 | 794 | 794 |
| | ll | -2789.3297 | -2784.5185 | -2784.2686 | -2784.2263 |
| | chi2 | 30.742143 | 5306.7984 | 1879.7454 | 2719.397 |
| | aic | 5592.6594 | 5585.037 | 5586.5371 | 5588.4525 |
| | bic | 5625.399 | 5622.4537 | 5628.6309 | 5635.2234 |

legend: * p<.1; ** p<.05; *** p<.01

***From the result, the best ARIMA model (ARIMA(p,d,q)) for spot is ARIMA(1,1,1) since it gives the lowest value of BIC among any other combination of (p,1,q), $1 \leq p,q \leq 4$***

```
. drop _est_arima111 _est_arima112 _est_arima113 _est_arima114 _est_arima211 _est_arima212
_est_arima213 _est_arima214 _est_arima311 _est_arima312 _est_arima313 _est_arima314 _est_arima411
_est_arima412 _est_arima413 _est_arima414

. qui arima future, arima(1,1,1)

. est store arima111

. qui arima future, arima(1,1,2)

. est store arima112

. qui arima future, arima(1,1,3)

. est store arima113

. qui arima future, arima(1,1,4)

. est store arima114

. qui arima future, arima(2,1,1)

. est store arima211

. qui arima future, arima(2,1,2)

. est store arima212

. qui arima future, arima(2,1,3)

. est store arima213

. qui arima future, arima(2,1,4)

. est store arima214

. qui arima future, arima(3,1,1)

. est store arima311

. qui arima future, arima(3,1,2)

. est store arima312

. qui arima future, arima(3,1,3)

. est store arima313

. qui arima future, arima(3,1,4)

. est store arima314

. qui arima future, arima(4,1,1)

. est store arima411

. qui arima future, arima(4,1,2)

. est store arima412

. qui arima future, arima(4,1,3)

. est store arima413

. qui arima future, arima(4,1,4)

. est store arima414
```

```
. est table arima11*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

-------------------------------------------------------------------------------
    Variable |   arima111        arima112        arima113        arima114
-------------+-----------------------------------------------------------------
future       |
       _cons | -.11424575      -.11408115       -.1139183      -.11370744
-------------+-----------------------------------------------------------------
ARMA         |
          ar |
         L1. | -.36850497       .30224152       .07471074      -.26441554
             |
          ma |
         L1. |  .33119538      -.33885872      -.11128313       .23060131
         L2. |                  .04456173        .0345267       .02935071
         L3. |                                   .03319847       .04074154
         L4. |                                                   .03843721
-------------+-----------------------------------------------------------------
sigma        |
       _cons |  9.5427695***    9.5390832***    9.5360585***     9.532552***
-------------+-----------------------------------------------------------------
Statistics   |
           N |        794             794             794             794
          ll |  -2917.724      -2917.4236       -2917.166      -2916.8767
        chi2 |  3.3320137       3.4714053       5.5726249       7.6343946
         aic |  5843.4479       5844.8473       5846.3321       5847.7535
         bic |  5862.1563       5868.2327       5874.3946        5880.493
-------------------------------------------------------------------------------
                                        legend: * p<.1; ** p<.05; *** p<.01

. est table arima21*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

-------------------------------------------------------------------------------
    Variable |   arima211        arima212        arima213        arima214
-------------+-----------------------------------------------------------------
future       |
```

```
       _cons | -.11399264      -.11468686      -.11518755        -.114745
-------------+------------------------------------------------------------
ARMA         |
          ar |
         L1. |  .26006773        1.2835845***    .39680958***    .29779715***
         L2. |  .03927244      -.90741041***   -.83492246***   -.89274399***
             |
          ma |
         L1. | -.29537225      -1.2846838***   -.43709178***   -.33588403***
         L2. |                  .95026799***    .86844397***     .9442071***
         L3. |                                  .02125732       .00060118
         L4. |                                                  .05136959*
-------------+------------------------------------------------------------
sigma        |
       _cons |  9.5398841***     9.4618855***    9.4808276***     9.473342***
-------------+------------------------------------------------------------
Statistics   |
           N |        794             794             794             794
          ll | -2917.4896      -2911.1516      -2912.6217      -2912.0146
        chi2 |  2.8388146       6764.7279       566.77521       690.44183
         aic |  5844.9792       5834.3032       5839.2433       5840.0292
         bic |  5868.3646       5862.3657       5871.9829       5877.4458
------------------------------------------------------------------------------
                                        legend: * p<.1; ** p<.05; *** p<.01

.
. est table arima31*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

------------------------------------------------------------------------------
    Variable |   arima311        arima312        arima313        arima314
-------------+----------------------------------------------------------------
future       |
       _cons | -.11395314      -.11434855      -.11482479      -.11489771
-------------+----------------------------------------------------------------
ARMA         |
          ar |
```

```
         L1. |  .06416111       1.3204613***   -.56192851***    .01972589
         L2. |  .02920804      -.90163025***   -.42025281***   -.77499053***
         L3. |  .02294871      -.05791248**     -.8338931***   -.25249026
             |
          ma |
         L1. | -.10003354      -1.3735497       .51793325      -.05826621
         L2. |                  1.0000008       .40777617       .81652449***
         L3. |                                   .8898383       .26256083
         L4. |                                                  .05469308*
-------------+----------------------------------------------------------------
sigma        |
       _cons |  9.5380895***    9.4185571       9.4577904       9.4722994***
-------------+----------------------------------------------------------------
Statistics   |
           N |        794             794             794             794
          ll | -2917.3418      -2909.7635      -2911.5297      -2911.9403
        chi2 |  3.8262796        32923.58       7392.6231       480.87615
         aic |  5846.6836       5833.5271       5839.0595       5841.8806
         bic |  5874.7461       5866.2666       5876.4761       5883.9743
------------------------------------------------------------------------------
                                       legend: * p<.1; ** p<.05; *** p<.01

.
. est table arima41*, star(0.1 0.05 0.01) stat(N ll chi2 aic bic)

------------------------------------------------------------------------------
    Variable |   arima411        arima412        arima413        arima414
-------------+----------------------------------------------------------------
future       |
       _cons | -.11336618      -.11461152      -.11590868      -.11376705
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. | -.26142231        .3181852***    1.7676398***   -.41944632***
         L2. |  .01694481      -.85050807***     -1.50548*      .36718541***
         L3. |  .03124469       .00130972       .41843716      -.35522863***
```

| | | | | |
|---|---|---|---|---|
| | L4. | .0307881 | .04773204 | .00058669 | -.86090936*** |
| | | | | | |
| | ma | | | | |
| | L1. | .22521179 | -.35750674*** | -1.8167691*** | .38743681*** |
| | L2. | | .90090477*** | 1.6207448* | -.30501342*** |
| | L3. | | | -.4995254 | .42560261*** |
| | L4. | | | | .8658178*** |
| sigma | | | | | |
| | _cons | 9.5350812*** | 9.4739792*** | 9.4436415*** | 9.4058783*** |
| Statistics | | | | | |
| | N | 794 | 794 | 794 | 794 |
| | ll | -2917.0927 | -2912.0811 | -2909.6096 | -2906.4848 |
| | chi2 | 5.4882963 | 661.06102 | 4207.3322 | 932.73412 |
| | aic | 5848.1853 | 5840.1622 | 5837.2193 | 5832.9696 |
| | bic | 5880.9249 | 5877.5789 | 5879.313 | 5879.7404 |

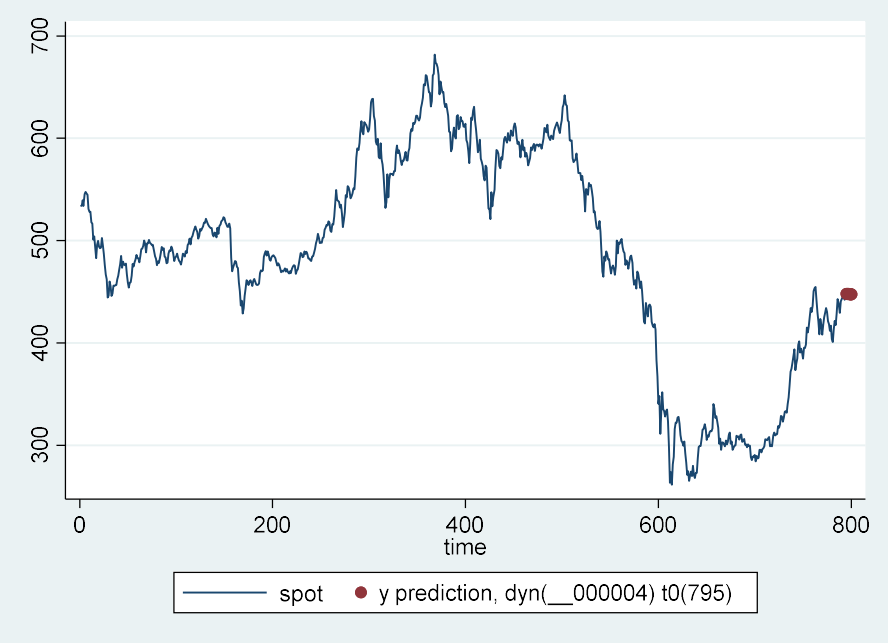
legend: * p<.1; ** p<.05; *** p<.01

***From the result, the best ARIMA model (ARIMA(p,d,q)) for future is ARIMA(1,1,1) since it gives the lowest value of BIC among any other combination of (p,1,q), $1 \leq p,q \leq 4$***

## 2. Make dynamic forecast for period time = 796 to 800.

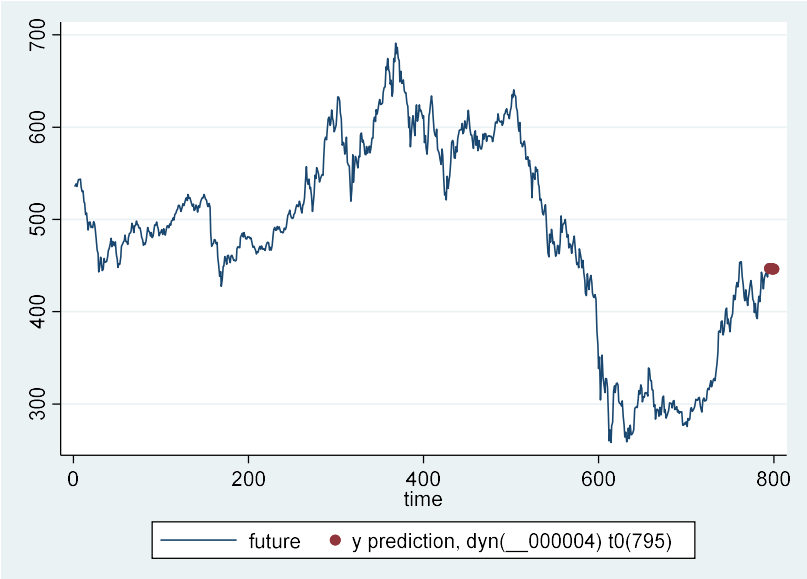
```
. arima spot, arima(1,1,1)

(setting optimization to BHHH)
Iteration 0:   log likelihood =  -2791.825
. . .
Iteration 7:   log likelihood = -2791.4965

ARIMA regression

Sample:  2 - 795                                Number of obs     =        794
                                                Wald chi2(2)      =      18.31
Log likelihood = -2791.496                      Prob > chi2       =     0.0001

------------------------------------------------------------------------------
             |                 OPG
      D.spot |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
spot         |
       _cons |  -.1047174   .3525647    -0.30   0.766    -.7957316    .5862967
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .4353655   .2343856     1.86   0.063    -.0240219    .8947528
             |
          ma |
         L1. |  -.3535108    .242659    -1.46   0.145    -.8291138    .1220922
-------------+----------------------------------------------------------------
      /sigma |   8.139999   .1507405    54.00   0.000     7.844553    8.435444
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided
      confidence interval is truncated at zero.

. set obs 800
number of observations (_N) was 795, now 800
```

```
. replace time = _n
(5 real changes made)

. predict yhat, y dynamic(.) t0(795)
Note: beginning dynamic predictions in period        4.
(794 missing values generated)

. twoway (line spot time, sort) (scatter yhat time, sort)
```




```
. arima future, arima(1,1,1)

(setting optimization to BHHH)
Iteration 0:   log likelihood = -2918.0578
. . .
Iteration 6:   log likelihood =  -2917.724

ARIMA regression

Sample:  2 - 795                                Number of obs     =        794
                                                Wald chi2(2)      =       3.33
Log likelihood = -2917.724                      Prob > chi2       =     0.1890
```

```
------------------------------------------------------------------------------
             |                 OPG
   D.future  |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
future       |
       _cons |  -.1142458   .3326737    -0.34   0.731    -.7662743    .5377828
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   -.368505   .5847749    -0.63   0.529    -1.514643    .7776328
             |
          ma |
         L1. |   .3311954   .5949511     0.56   0.578    -.8348873    1.497278
-------------+----------------------------------------------------------------
      /sigma |    9.54277   .1687756    56.54   0.000     9.211975    9.873564
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided
      confidence interval is truncated at zero.

.  predict yhat2, y dynamic(.) t0(795)
Note: beginning dynamic predictions in period
(794 missing values generated)

. twoway (line future time, sort) (scatter yhat2 time, sort)
```



Part II

The following GARCH model:

Mean Equation: $$rfuture_t = \alpha + \beta rspot_t + \varepsilon_t \quad (1)$$

Variance Equation: $$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \delta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (2)$$

3. Estimate model (1) using OLS by employing future return (rfuture) as dependent variable and spot return (rspot) as explanatory variable, and determine whether ARCH-effect significantly occurs.

```
. gen rspot = (spot-l.spot)/l.spot
(6 missing values generated)
. gen rfuture = (future-l.future)/l.future
(6 missing values generated)
. reg rfuture rspot

      Source |       SS           df       MS      Number of obs   =       794
-------------+----------------------------------   F(1, 792)       =   6189.46
       Model |  .338771077         1  .338771077   Prob > F        =    0.0000
    Residual |  .043348982       792  .000054734   R-squared       =    0.8866
-------------+----------------------------------   Adj R-squared   =    0.8864
       Total |  .382120059       793  .000481866   Root MSE        =     .0074

------------------------------------------------------------------------------
     rfuture |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rspot |   1.108778   .0140935    78.67   0.000     1.081113    1.136443
       _cons |   .0000525   .0002626     0.20   0.841    -.0004629    .0005679
------------------------------------------------------------------------------
. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-------------------------------------------------------------------------
    lags(p)  |          chi2              df                 Prob > chi2
-------------+-----------------------------------------------------------
       1     |         43.545              1                   0.0000
-------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

***From the test, there exists ARCH effect since p-value of the test =0.00 < 0.05***

4. Estimate GARCH(p,q) for future return (rfuture) using spot return (rspot) as explanatory variable for mean equation (model (1) and (2)) – determine the most appropriated order p and q for variance equation using SBIC given the maximum lag equals to 2.

```
. qui arch rfuture rspot, garch(1/1) arch(1/1)
. est store garch11
. qui arch rfuture rspot, garch(1/1) arch(1/2)
. est store garch12
. qui arch rfuture rspot, garch(1/2) arch(1/1)
. est store garch21
. qui arch rfuture rspot, garch(1/2) arch(1/2)
. est store garch22
.  estimates table garch11 garch12 garch21 garch22, star(0.1 0.05 0.01) stat(aic bic ll)
```

| Variable | garch11 | garch12 | garch21 | garch22 |
|---|---|---|---|---|
| rfuture | | | | |
| rspot | 1.066779*** | 1.0686232*** | 1.0662306*** | 1.0681952*** |
| _cons | .00003064 | .00002254 | .00002879 | .00002246 |
| ARCH | | | | |
| arch | | | | |
| L1. | .0846231*** | .2121659*** | .11369069*** | .21614027*** |
| L2. | | -.14954562*** | | -.17303686*** |
| | | | | |
| garch | | | | |
| L1. | .89700601*** | .92239359*** | .54514899 | 1.1227156*** |
| L2. | | | .3148726 | -.17547447 |
| | | | | |
| _cons | 9.850e-07*** | 7.744e-07*** | 1.380e-06** | 5.016e-07** |
| Statistics | | | | |
| aic | -5702.2841 | -5708.9307 | -5702.54 | -5707.938 |
| bic | -5678.8987 | -5680.8682 | -5674.4775 | -5675.1984 |
| ll | 2856.1421 | 2860.4654 | 2857.27 | 2860.969 |

legend: * p<.1; ** p<.05; *** p<.01

***According to the result, the most appropriated order of GARCH is GARCH(1,2) since it gives the lowest BIC.***

5. From (6), predict the variance of future return (rfuture).

```
. predict sigmahat, v

. line sigmahat time
```