FN452: Asset management and portfolio analysis

Black-Litterman model

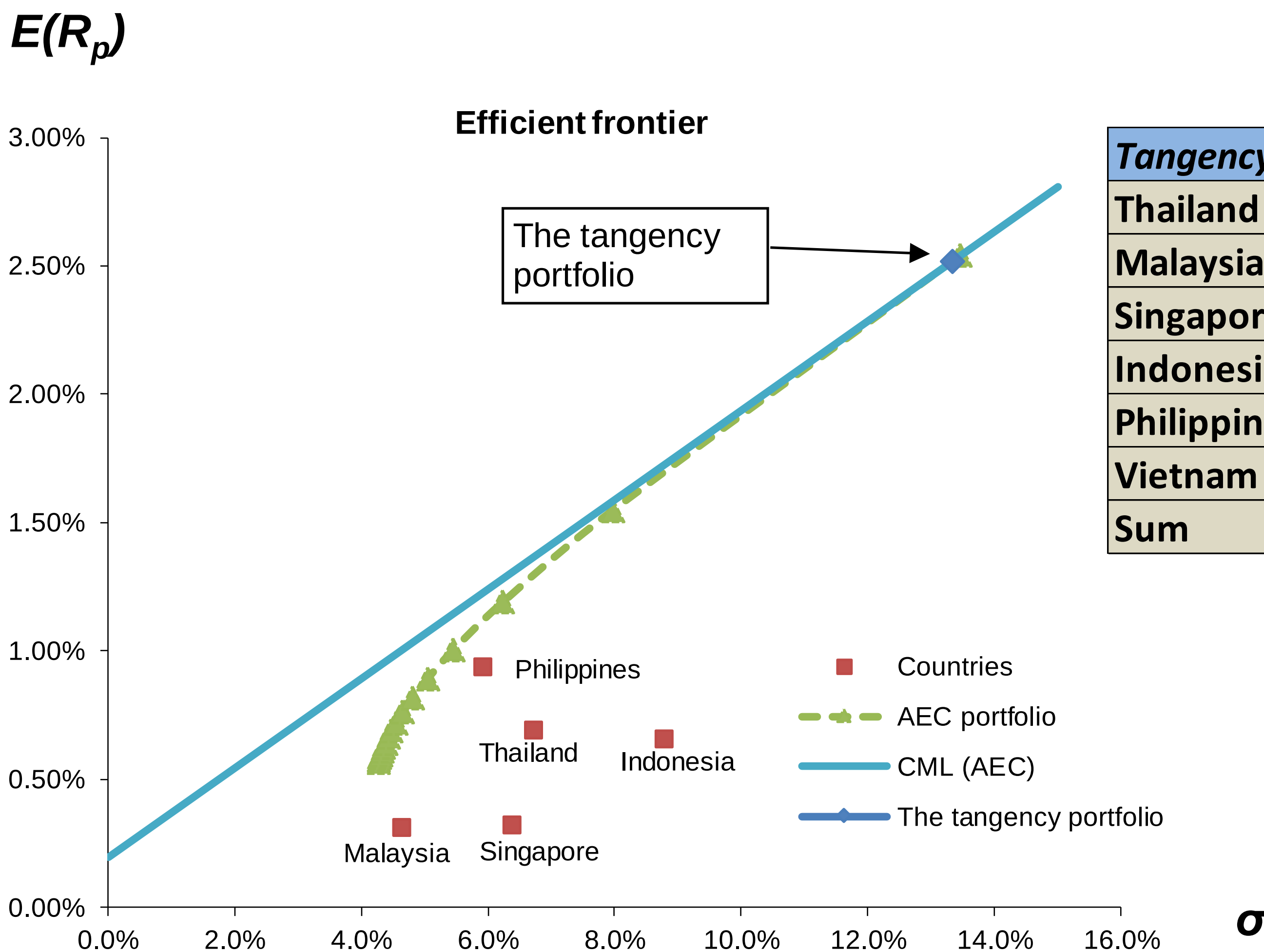
Sakkakom Maneenop

Thammasat Business School

Practical issues in modern portfolio theory

- MPT produces unrealistic portfolio allocation with extreme long/short positions in stocks
- With short-sale constraint, this problem can be avoided but this will restrict investors from allocating their positions in asset universe
- Also, the previous result assumes that past risks/returns represent those of the future
- This leads to the outperformance of an equal-weighted portfolio over the optimal risky portfolio

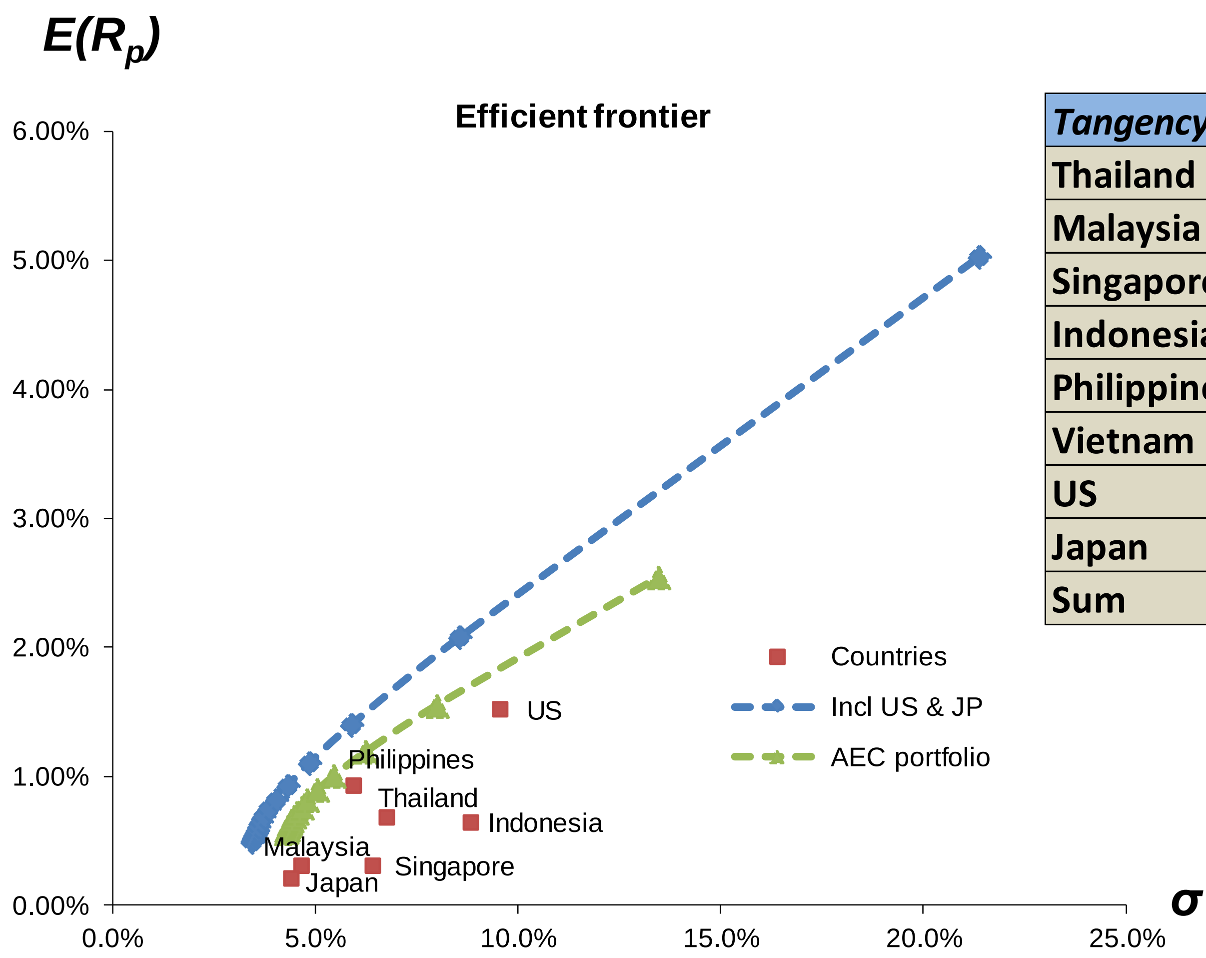
Practical issues in modern portfolio theory



Tangency port	AEC
Thailand	55.10%
Malaysia	-20.40%
Singapore	-116.96%
Indonesia	3.54%
Philippines	252.99%
Vietnam	-74.27%
Sum	**100.00%**

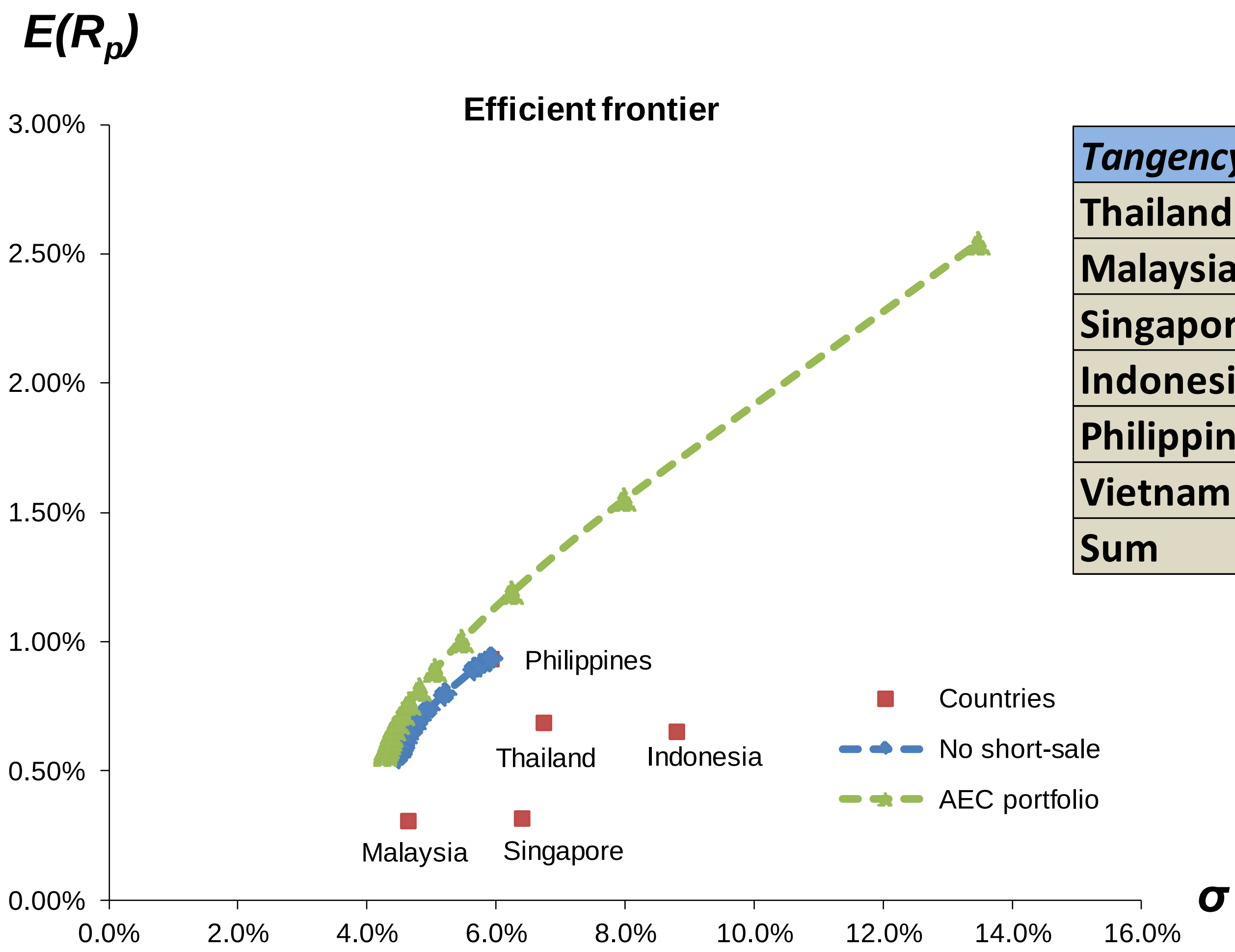
Practical issues in modern portfolio theory

When adding the U.S. and Japan indices



Tangency port	Incl US & JP	AEC
Thailand	19.88%	55.10%
Malaysia	-112.18%	-20.40%
Singapore	-145.35%	-116.96%
Indonesia	-16.75%	3.54%
Philippines	314.63%	252.99%
Vietnam	-99.54%	-74.27%
US	170.53%	
Japan	-31.21%	
Sum	**100.00%**	**100.00%**

Practical issues in modern portfolio theory



Tangency port	AEC
Thailand	0.00%
Malaysia	0.00%
Singapore	0.00%
Indonesia	0.00%
Philippines	100.00%
Vietnam	0.00%
Sum	**100.00%**

Black-Litterman approach

- **Black and Litterman (1991)** assume that an investor chooses his optimal portfolio from among a given group of assets
- This group is defined as a **benchmark portfolio** which **cannot be outperformed**
- BL approach assumes that a given portfolio is optimal and investors can **derive the expected returns** of the benchmark components
- Asset returns can be interpreted as the market's information about the future returns of each asset in the benchmark portfolio
    - If the investor agrees with the market, he can buy this benchmark portfolio
    - If not, the investor can **incorporate his opinions** into the optimization problem to produce a better portfolio

Black-Litterman approach

What does the market think?

- Recall the previous tangency portfolio formula

$$\mathrm{w} = \frac{\Sigma^{-1}\left(\mu - R_f 1\right)}{1^{\mathrm{T}}\Sigma^{-1}\left(\mu - R_f 1\right)}$$

$$\mathrm{w}_B \cdot \mathrm{Norm} = \Sigma^{-1}\left(\mu - R_f 1\right), \mathrm{Norm} = 1^{\mathrm{T}}\Sigma^{-1}\left(\mu - R_f 1\right) \text{ and } w_B =$$ Benchmark proportion

- Therefore, we can write the **market implied expected returns** as

$$\mu_E = \Sigma \cdot \mathrm{w}_B \cdot \mathrm{Norm} + R_f 1, \quad \mu_E =$$ Expected portfolio returns

- And it can be shown that

$$\mathrm{Norm} = 1^{\mathrm{T}}\Sigma^{-1}\left(\mu - R_f 1\right)$$

$$= \frac{\mathrm{w}_B^{\mathrm{T}}\left(\mu - R_f 1\right)}{\mathrm{w}_B^{\mathrm{T}}\Sigma \mathrm{w}_B}$$

$$= \frac{\mu * - R_f}{\mathrm{w}_B^{\mathrm{T}}\Sigma \mathrm{w}_B},$$ where $\mathrm{w}_B^{\mathrm{T}}\mu = \mu *, \mathrm{w}_B^{\mathrm{T}} 1 = 1, \text{and}$ $\mu^* =$ Benchmark portfolio return

Black-Litterman approach

What does the market think?

$$\mu_E = \Sigma \cdot \mathrm{w}_B \cdot \mathrm{Norm} + R_f 1,$$

$=$ Market implied expected returns

Incorporating investor opinions

- Adjust opinion based on expected benchmark return
- Then calculate the opinion-adjusted optimized portfolio based on the earlier formula

$$\mathrm{w}_{BL} = \frac{\Sigma^{-1}\left(\mu_{Opn} - R_f 1\right)}{1^{\mathrm{T}}\Sigma^{-1}\left(\mu_{Opn} - R_f 1\right)}$$

- Also, if an investor is not 100% certain of his opinions, he can adjust the optimized portfolio based on degree of his confidence level
- For simplicity, such proportions can be shown as:

$$\boldsymbol{w}_{adj} = \left(1-\gamma\right)\boldsymbol{w}_B + \gamma \boldsymbol{w}_{BL}, \quad \gamma = \text{Confidence level}$$

Black-Litterman approach

What does the market think?

Expected benchmark return (μ^*)	0.75%					
Risk-free rate	0.20%					
Normalizing factor (Norm)	1.82					
	Thailand	**Malaysia**	**Singapore**	**Indonesia**	**Philippines**	**Vietnam**
Market cap (Billion USD)	348,798	382,977	639,956	353,271	238,820	51,877
Market cap (Billion THB)	12,567,192	13,798,649	23,057,611	12,728,353	8,604,682	1,869,128
Benchmark proportion (w_B)	17.30%	19.00%	31.75%	17.53%	11.85%	2.57%
Average return	0.69%	0.31%	0.32%	0.65%	0.93%	-0.25%
Benchmark return	0.50%					

$$\text{Norm} = 1^{\mathrm{T}}\Sigma^{-1}\left(\mu - R_f 1\right)$$

$$= \frac{\mathrm{w}_B^{\mathrm{T}}\left(\mu - R_f 1\right)}{\mathrm{w}_B^{\mathrm{T}}\Sigma \mathrm{w}_B}$$

$$= \frac{\mu * - R_f}{\mathrm{w}_B^{\mathrm{T}}\Sigma \mathrm{w}_B}, \mathrm{w}_B^{\mathrm{T}}\mu = \mu *, \mathrm{w}_B^{\mathrm{T}} 1 = 1, and\ \mu^* =$$ Benchmark portfolio return

Black-Litterman approach

	Benchmark proportion (w_B)	Expected benchmark returns (μ_E)
Thailand	17.3%	0.76%
Malaysia	19.0%	0.56%
Singapore	31.7%	0.79%
Indonesia	17.5%	0.97%
Philippines	11.8%	0.64%
Vietnam	2.6%	0.58%

$$\mu_E = \Sigma \cdot \mathrm{w}_B \cdot \mathrm{Norm} + R_f 1,$$

$=$ Expected portfolio returns

Black-Litterman approach

Incorporating investor opinions

- Adjust opinion based on expected benchmark return
- Then calculate the opinion-adjusted optimized portfolio based on the earlier formula

$$w_{BL} = \frac{\Sigma^{-1}(\mu_{Opn} - R_f 1)}{1^T \Sigma^{-1}(\mu_{Opn} - R_f 1)}$$

	Benchmark proportion (w_B)	Expected benchmark returns (μ_E)	Opinion (μ_{Opn})	Opinion-adjusted optimized portfolio (w_{BL})
Thailand	17.3%	0.76%	**1.00%**	**78.09%**
Malaysia	19.0%	0.56%	0.56%	**13.22%**
Singapore	31.7%	0.79%	0.79%	**-1.73%**
Indonesia	17.5%	0.97%	0.97%	**-1.90%**
Philippines	11.8%	0.64%	0.64%	**-4.46%**
Vietnam	2.6%	0.58%	**0.70%**	**16.79%**

Black-Litterman approach

Incorporating investor confidence level

- Also, if an investor is not 100% certain of his opinions, he can adjust the optimized portfolio based on degree of his confidence level
- For simplicity, such proportions can be shown as:

$$\boldsymbol{w}_{adj} = (1-\gamma)\boldsymbol{w}_B + \gamma \boldsymbol{w}_{BL}, \quad \gamma = \text{Confidence level}$$

Adjusted with confidence level

	Benchmark proportion $(\boldsymbol{w}_B)$	Expected benchmark returns (μ_E)	Opinion (μ_{Opn})	Opinion-adjusted optimized portfolio $(\boldsymbol{w}_{\tan})$	Opinion-adjusted optimized portfolio $(\boldsymbol{w}_{adj})$
Thailand	17.3%	0.76%	**1.00%**	**78.09%**	**62.89%**
Malaysia	19.0%	0.56%	0.56%	**13.22%**	**14.66%**
Singapore	31.7%	0.79%	0.79%	**-1.73%**	**6.64%**
Indonesia	17.5%	0.97%	0.97%	**-1.90%**	**2.95%**
Philippines	11.8%	0.64%	0.64%	**-4.46%**	**-0.39%**
Vietnam	2.6%	0.58%	**0.70%**	**16.79%**	**13.23%**

Black-Litterman approach

THAMMASAT BUSINESS SCHOOL

Incorporating investor confidence level with constraint on short sale

- Also, if an investor is not 100% certain of his opinions, he can adjust the optimized portfolio based on degree of his confidence level
- For simplicity, such proportions can be shown as:

$$\boldsymbol{w}_{adj} = (1-\gamma)\boldsymbol{w}_B + \gamma \boldsymbol{w}_{BL}, \quad \gamma = \text{Confidence level}$$

No short sale

Opinion confidence level	75%					No short sales
	Benchmark proportion $(\boldsymbol{w}_B)$	Expected benchmark returns (μ_E)	Opinion (μ_{Opn})	Opinion-adjusted optimized portfolio $(\boldsymbol{w}_{\tan})$	Opinion-adjusted optimized portfolio $(\boldsymbol{w}_{adj})$	Opinion-adjusted optimized portfolio $(\boldsymbol{w}_{adj})$
Thailand	17.3%	0.76%	**1.00%**	**78.54%**	**63.23%**	**73.62%**
Malaysia	19.0%	0.56%	0.56%	**12.72%**	**14.29%**	**7.95%**
Singapore	31.7%	0.79%	0.79%	**-3.19%**	**5.54%**	**0.00%**
Indonesia	17.5%	0.97%	0.97%	**-2.24%**	**2.70%**	**0.00%**
Philippines	11.8%	0.64%	0.64%	**-5.75%**	**-1.35%**	**0.00%**
Vietnam	2.6%	0.58%	**0.75%**	**19.93%**	**15.59%**	**18.43%**