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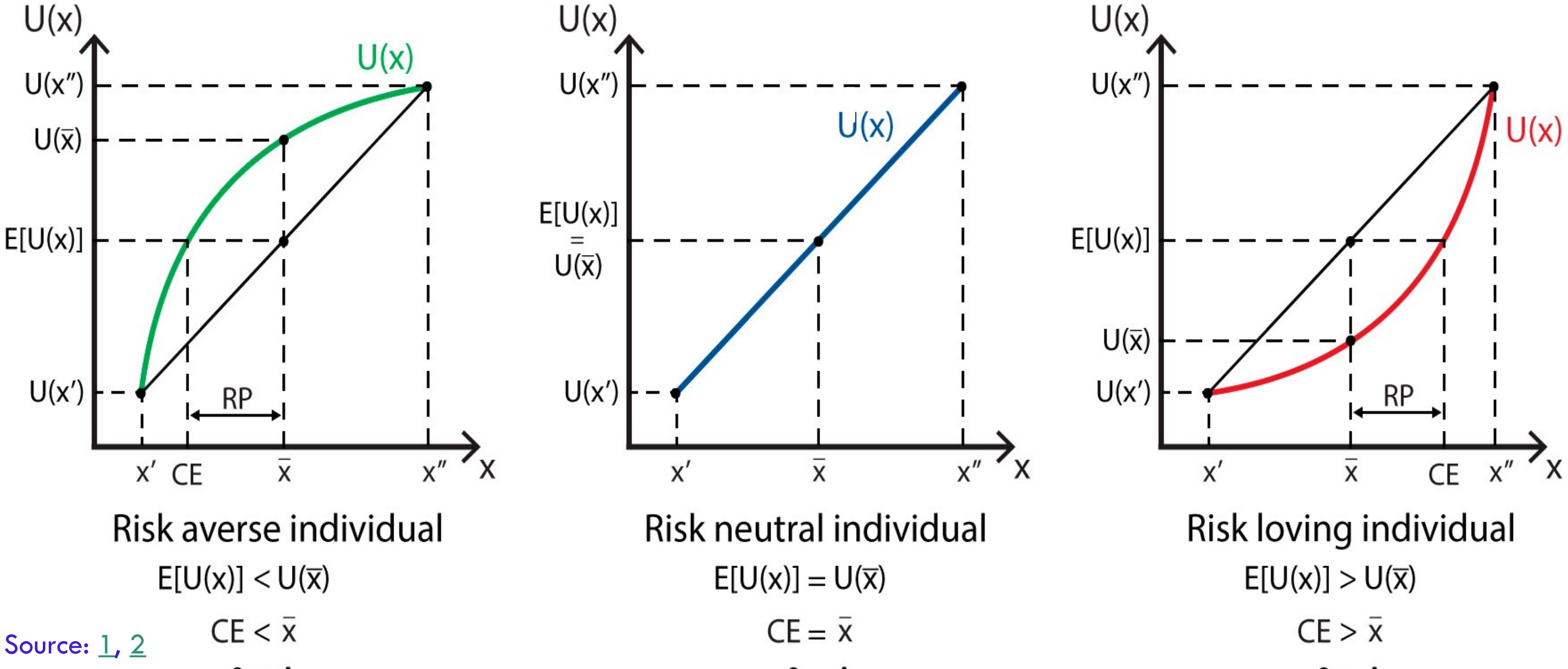
Risk Preferences

PROSPECT THEORY PART2:

EDITING STAGE & EVALUATION STAGE'S VALUE FUNCTION

Recap:Risk aversion&Certainty equivalent $\boxed{U(CE) = E[U(x)]}$

The **certainty equivalent** is an amount of money that provides equal utility to the random payoff of the gamble. The certainty equivalent is labeled *CE* in the figure. Note that *CE* is less than the expected outcome, if the person is risk averse. This is because risk-averse individuals prefer the expected outcome to the risky outcome.



Source: 1, 2

Recap

Suppose you have initial wealth w,
Consider lotteries $X = (x, p; 0,1-p)$, i.e. $X = (x, p)$
How will you evaluate lottery X ?

Expected Value Theory

$EV = px + w$

Expected Utility Theory

$EU = pU(w + x) + (1-p)U(w)$

Prospect theory

$V(X, p) = \pi(p)v(x)$, $v(.)$ at the reference point is usually normalized to 0.

Recap

Suppose you have initial wealth w,
Consider two lotteries $X = (x, p; 0, 1-p)$ and $\mathrm{Y} = (y, q; 0, 1-q)$
You will prefer lottery X over lottery Y IF….

Expected Value Theory

$$w + px > w + qy$$

Expected Utility Theory

$$pU(w+x) + (1-p)U(w) > qU(w+y) + (1-q)U(w)$$

Prospect theory

$$\pi(p)v(x) > \pi(q)v(y)$$

Editing Stage

Evaluation Stage

Prospect Theory

Prospect Theory: Editing Stage

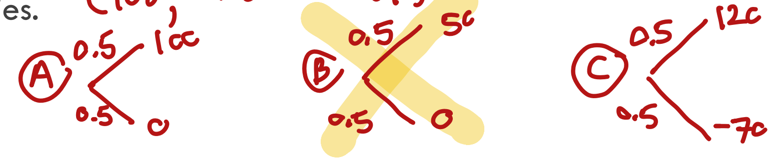
organize & reformulate the problem

What is going on in editing stage? Taking an objective prospect $(\hat{x}_1, \hat{p}_1; \ldots; \hat{x}_m, \hat{p}_m)$ and transforming it into an object for evaluation $(x_1, p_1; \ldots; x_n, p_n)$ by:

- **Coding:** code outcomes as gains & losses relative to reference point.
- **Combination:** e.g., (100, .5; 100, .5) replaced with (100, 1).
- **Segregation:** e.g., (100, .5; 200, .5) replaced with 100 for sure plus (0, .5; 100, .5).
- **Cancellation:** discard shared components.
- **Simplification:** rounding of probabilities.
- **Eliminating dominated alternatives.**

(100; 0.5)

(100; 0.53998764) & 0 otherwise



Prospect theory: Evaluation stage

A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.

Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

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p x

1−p = q y

or if p+q < 1

p x

q y

1−p−q 0

A person evaluates a prospect $(x, p; y, q)$ according to the functional

"Prospect-theory value"

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

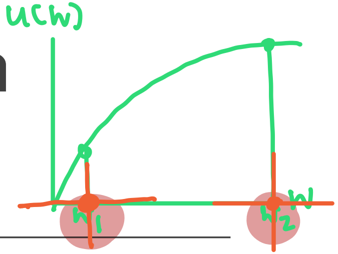
Note: Nowadays, it might be best to take Prospect theory as a theory for "simple prospects" with at most two non-zero outcomes.

ex. (200, 0.5 ; 50, 0.25 ; 0, 0.25)

(200, 0.5 ; 0, 0.5)

(150, 0.5 ; −70, 0.5) etc.

Prospect Theory: Value Function

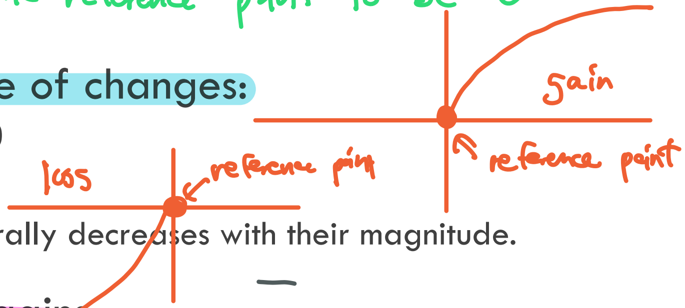


Three key features of the value function $v(.)$:

- **The carriers of value are changes in wealth, instead of final wealth.**
  - The value function is defined on deviations from **the reference point**.
  - $v(0) = 0$

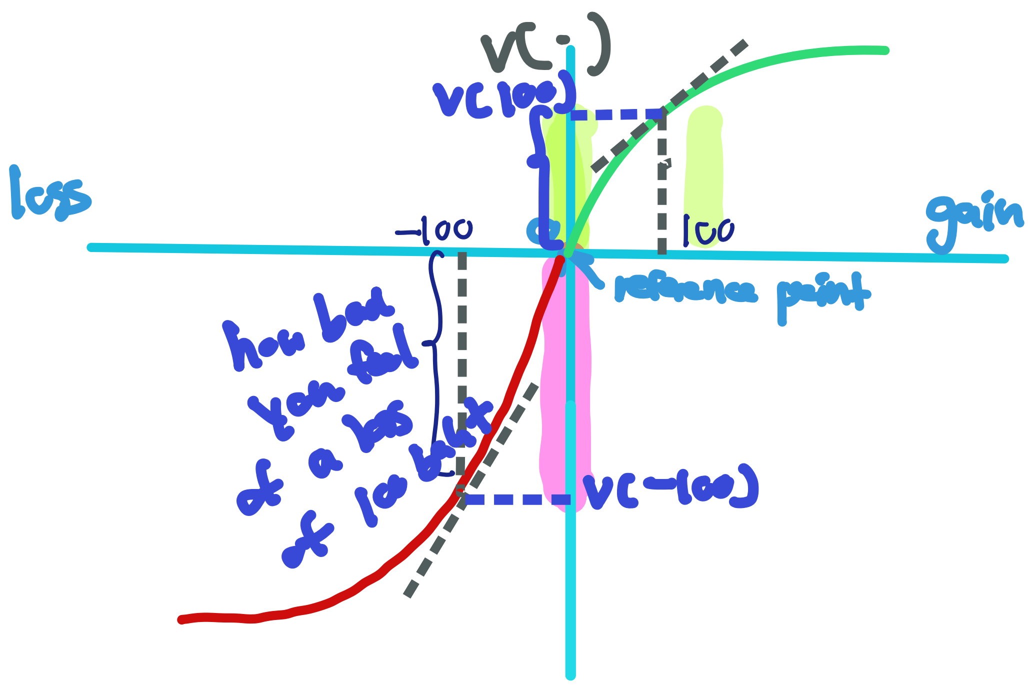
We usually normally the reference point to be 0
& v(0)=0

- **Diminishing sensitivity to the magnitude of changes:**
  - Concave in the gain domain: $v''(x) < 0$ for $x > 0$
  - Convex in the loss domain: $v''(x) > 0$ for $x < 0$
  - The marginal value of both gains and losses generally decreases with their magnitude.



- **Loss aversion: losses loom larger than gains.**

$$\frac{v'(x|x<0)}{v'(x|x>0)} > 1, \text{ or } (y, 0.5; -y, 0.5) \succ (x, 0.5; -x, 0.5) \text{ if } 0 \leq y < x$$



$0 < y < x$

fair gamble : expected value = 0

$0.5y + 0.5(-y) = 0$

$V(y, 0.5\ ;\ -y, 0.5)$

$= \pi(0.5)v(y) + \pi(0.5)v(-y)$

$V(x, 0.5\ ;\ -x, 0.5)$

$= \pi(0.5)v(x) + \pi(0.5)v(-x)$

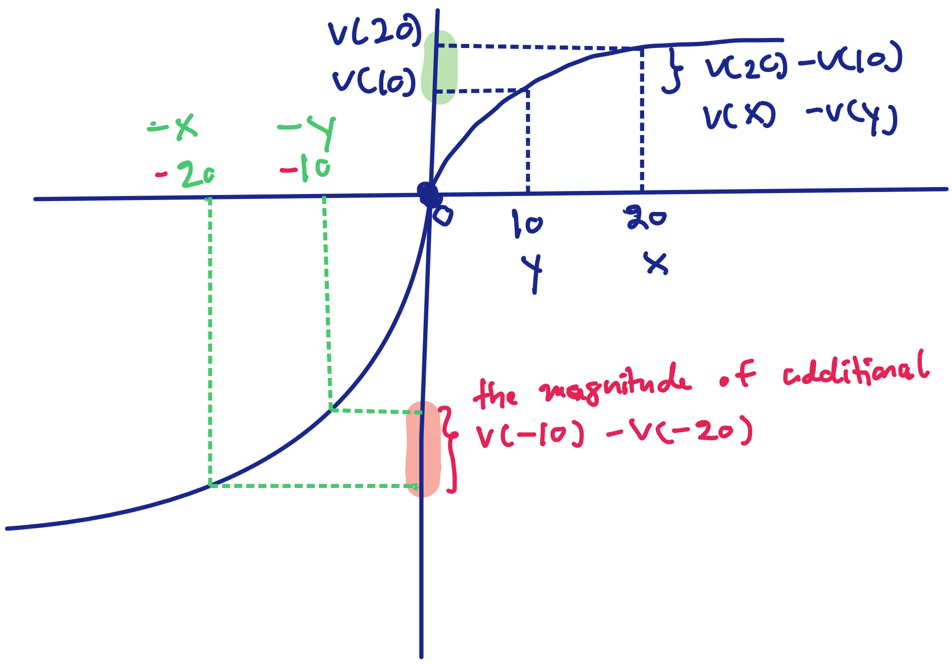
$$(y, 0.5; -y, 0.5) \succ (x, 0.5; -x, 0.5)$$

$$\cancel{\pi(0.5)}v(y) + \cancel{\pi(0.5)}v(-y) > \cancel{\pi(0.5)}v(x) + \cancel{\pi(0.5)}v(-x)$$

$$v(y) + v(-y) > v(x) + v(-x) \quad ; 0 < y < x$$

$$; \text{eg } 0 < 10 < 20$$

$$v(-y) - v(-x) > v(x) - v(y)$$



The reference point

- The reference point is usually where you are (the status quo), but can be where you would like to be, or think you should be.
  - For example, in terms of wealth, reference point can be the previous level, the aspired level, the wealth of other players, etc.
- We do not yet fully understand how reference points are determined, but they should be subject to phenomena such as adaptation and social pressure (social norms), etc. social comparison aspiration

Value function's diminishing sensitivity

Prospect theory attempts to explain risk aversion in gains and risk loving in losses by positing that the utility function is fundamentally wrong from biological principles as follows:

Value function's diminishing sensitivity

- The nervous system is set up to primarily detect differences, not absolute levels.
- A gain is perceived as a pleasurable change from the status quo (reference point) and the nervous system shows a decreasing response both to the intensity and duration of *pleasurable* stimuli.
- A loss is perceived as a painful change from the status quo (reference point) and the nervous system shows a decreasing response both to the intensity and duration of *painful* stimuli.

Risk aversion over gain & Risk seeking over loss

In a situation where a sure gain is compared to a gamble with (averagely or highly) probable larger gain, diminishing sensitivity cause risk aversion.

In a situation where a sure loss is compared to a gamble with (averagely or highly) probable larger loss, diminishing sensitivity cause risk seeking.

At the same level of wealth, an individual can both be risk averse and risk seeking.

Prospect Theory: Value Function

Two common functional forms for the value function:

Tversky & Kahneman (1992)

gain domain

$$v(x) = \begin{cases} x^{\alpha} & if\ x > 0 \\ -\lambda(\underbrace{-x}_{\text{the magnitude of loss}})^{\beta} & if\ x < 0 \end{cases}$$

loss domain

,where $\alpha, \beta \in (0,1]$ and $\lambda \geq 1$

Two-part linear

$$v(x) = \begin{cases} x & if\ x > 0 \\ \lambda x. & if\ x < 0 \end{cases}$$

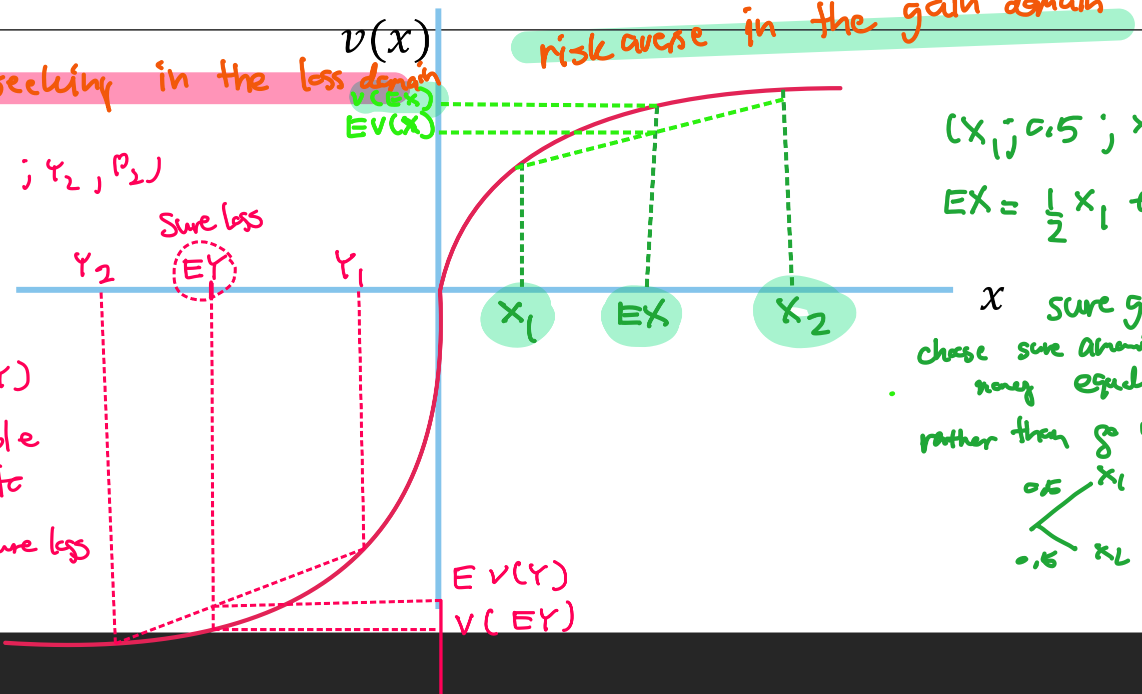
,where $\lambda \geq 1$

⇒ diminishing sensitivity is not here.

⇒ highlight loss aversion

λ is the coefficient of loss aversion.

Value function



$Y = (Y_1, p_1; Y_2, p_2)$

$EY = p_1 Y_1 + p_2 Y_2$

$E\,v(Y) > v(EY)$

∴ taking gamble is prefered to taking the sure loss

$(x_1; 0.5; x_2; 0.5)$

$EX = \frac{1}{2} x_1 + \frac{1}{2} x_2$

sure gain

chose sure amount of money equal to EX rather than go with

0.5 x_1

0.5 x_2

Utility function vs. Value function

Utility function

Evaluation of choices is made over final states of wealth.

x-axis: final wealth

y-axis: utility

Value function

Evaluation of choices is made relative to a reference point.

x-axis: gain, loss

y-axis: value

Utility function vs. Value function