

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$(X_i - \bar{X})^2$	$\hat{Y}_i$	$\hat{u}_i$
1	2.8	63	-14.625	-0.4125	6.03	213.89	2.7313	0.0687
2	3.4	72	-5.625	0.1875	-1.05	31.64	3.0274	0.3726
3	3	78	0.375	-0.2125	-0.08	0.14	3.2248	-0.2248
4	3.5	81	3.375	0.2875	0.97	11.39	3.3235	0.1765
5	3.6	87	9.375	0.3875	3.63	87.89	3.5209	0.0791
6	3.0	75	-2.625	-0.2125	0.56	6.89	3.1261	-0.1261
7	2.7	75	-2.625	-0.5125	1.35	6.89	3.1261	-0.4261
8	3.7	90	12.375	0.4875	6.03	153.14	3.6196	0.0804

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\bar{X} = 77.625$$

$$\bar{Y} = 3.2125$$

$$\hat{\beta}_1 = 0.0329$$

$$\hat{\beta}_0 = 0.6586 \#$$

1.2 For each observation i , find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=1}^N \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \rightarrow 0.6586 + 0.0329 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i \rightarrow Y_i - \hat{Y}_i$$

$$\hat{\beta}_1 = 0.0329$$

$$\hat{\beta}_0 = 0.6586$$

Y_i	X_i	$\hat{Y}_i$	$\hat{u}_i$
2.8	63	2.7313	0.0687
3.4	72	3.0274	0.3726
3	78	3.2248	-0.2248
3.5	81	3.3235	0.1765
3.6	87	3.5209	0.0791
3.0	75	3.1261	-0.1261
2.7	75	3.1261	-0.4261
3.7	90	3.6196	0.0804

$$= 0$$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$\hat{\beta}_1 = \beta_1 + \left(\sum_{i=1}^n u_i k_i \right)$$

$$var(\hat{\beta}_1) = var(\beta_1) + var\left(\sum_{i=1}^n u_i k_i\right)$$

2. Data is listed in the table

X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$\hat{y}_i$	$\hat{u}_i$
		-10	-9.1	91	100	0.145	-0.145
10	0	-8	-7.1	56.8	64	1.936	0.064
12	2	-6	-4.1	24.6	36	3.727	1.273
14	5	-4	-3.1	12.4	16	5.518	0.482
16	6	-2	-2.1	4.2	4	7.309	-0.309
18	7	2	0.9	1.8	4	10.891	-0.891
22	10	4	0.9	3.6	16	12.682	-2.682
24	10	6	5.9	35.4	36	14.473	0.527
26	15	8	6.9	55.2	64	16.264	-0.264
28	16	10	10.9	109	100	18.055	1.945
30	20						

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\begin{aligned} \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \longrightarrow \hat{\beta}_0 = 0.8955 \\ \hat{\beta}_1 &= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \longrightarrow \hat{\beta}_1 = -8.81 \end{aligned}$$

$$\bar{x} = 20$$

$$\bar{y} = 9.1$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

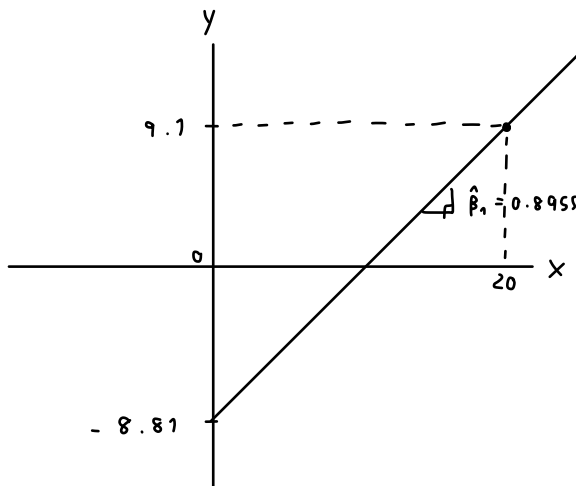
$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \rightarrow 0.8955 - 8.81x_i, \quad \hat{\beta}_0 = 0.8955$$

$$\hat{u}_i = y_i - \hat{y}_i \quad \hat{\beta}_1 = -8.81$$

X_i	Y_i	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.736	0.064
14	5	3.727	1.273
16	6	<u>5.518</u>	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945

} $\oplus = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$= 0.8955 - 8.81x_i$$

2.4 If $X_i = 16$, what is the predicted Y?

$$x_i = 16, \quad \hat{y} = 5.518$$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

suppose $X_i = x_i - \bar{x}$ $K_i = \frac{x_i}{\sum_{i=1}^n x_i^2}$

$$\begin{aligned} \beta_1 &= \sum_{i=1}^n (y_i - \bar{y}) K_i \quad \text{or} \quad \hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} \\ &= \sum_{i=1}^n (\hat{\beta}_0 - \beta_0 + u_i - \beta_0 + \beta_0) K_i \\ &= \sum_{i=1}^n \beta_1 (x_i - \bar{x}) K_i + \sum_{i=1}^n u_i K_i \\ &= \beta_1 \sum_{i=1}^n X_i K_i + \sum_{i=1}^n u_i K_i \\ &= \beta_1 \sum_{i=1}^n X_i \frac{X_i}{\sum_{i=1}^n X_i^2} + \sum_{i=1}^n u_i K_i \\ &= \beta_1 \frac{\sum_{i=1}^n X_i^2}{\sum_{i=1}^n X_i^2} + \sum_{i=1}^n u_i K_i \end{aligned}$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_{i=1}^n u_i K_i]$$

$$\text{SLR 4: } E(u_i | x_i) = 0$$

$$E(\beta_1) = \beta_1 + \sum_{i=1}^n K_i E(u_i)$$

$\uparrow_0$

$$\therefore E(\hat{\beta}_1) = \beta_1$$