HW 3 Answers ☺

7.16

a) Linear Model: $\hat{Y}_t = 10{,}816.04 - 2{,}227.704X_{2i} + 1{,}251.141X_{3i} + 6.283X_{4i} - 197.399X_{5i}$

$$se = (5.988.348)(\ 920.538) \quad (1157021) \quad (29.919) \quad (101.156)$$
$$R^2 = 0.835$$

In this model the slope coefficients measure the rate of change of Y with respect to the relevant variable.

b) Log-Linear Model: $\hat{Y}_t = 0.627 - 1.274lnX_{2i} + 0.937lnX_{3i} + 1.713lnX_{4i} - 0.182lnX_{5i}$

$$se = (6.148)(\ 0.527) \quad (0.659) \quad (1.201) \quad (0.128)$$
$$R^2 = 0.778$$

In this model all the partial slope coefficients are partial elasticities of Y with respect to the relevant variable.

c) The own-price elasticity is expected to be negative, the cross-price elasticity is expected to be positive for substitute goods and negative for complimentary goods, and the income elasticity is expected to be positive, since roses are a normal good.

d) The general formula for elasticity for linear equation is:

$Elasticity = \frac{\partial Y}{\partial X_i} = \frac{\bar{X}_i}{\bar{Y}}$, where X_i is the relevant regressor. That is for a linear model, the elasticity can be computed at the mean values.

e) Both models give similar results. One advantage of the log-linear model is that the slope coefficients give direct estimates of the (constant) elasticity of the relevant variable with respect to the regressor under consideration. But keep in mind that the R^2s of the two models are not directly comparable.

7.18

a) The regression results are:

$$\hat{Y}_t = 19.443 + 0.018X_{2i} - 0.248X_{3i} + 1.343X_{4i} + 6.332X_{5i}$$
$$se = (3.406) \quad (0.006\,) \quad (0.457) \quad (0.259) \quad (3.024)$$
$$R^2 = 0.978;\ \overline{R^2} = 0.972;\ \text{modified } R^2 = 0.734$$

b) A priori, all the slope coefficients are expected to be positive. Except the coefficient for US military sales, all the other variables have the expected signs and are statistically significant at the 5% level.

c) Overall federal outlays and some form of trend variable may be valuable.

7.19

a) (a) Model (5) seems to be the best as it includes all the economically relevant variables, including the composite real price of chicken substitutes, which should help alleviate the multicollinearity problem that may exist in model (4) between the price of beef and price of pork. Model (1) contains no substitute good information, and models (2) and (3) have limited substitute good information.

b) The coefficient of lnX_2 represents income elasticity; the coefficient of lnX_3 +represents own-price elasticity.

c) Model (2) considers only pork as a substitute good, while model (4) considers both pork and beef.

d) There may be a problem of multicollinearity between the price of beef and the price of pork.

e) Yes. This might alleviate the problem of multicollinearity.

f) They should be substitute goods because they compete with chicken as a food consumption product.

g) The regression results of Model (5) are as follows:

$$ln\hat{Y}_t = 2.030 + 0.481 lnX_{2t} - 0.351 lnX_{3t} - 0.061 lnX_{6t}$$
$$se = (0.119) \quad (0.068) \quad (0.079) \quad (0.130)$$
$$R^2 = 0.980;\ \overline{R^2} = 0.977; \text{modified } R^2 = 0.810$$

The income elasticity and own-price elasticity have the correct signs.

h) The consequence of estimating model (2) would be that the estimators are likely to be biased due to model misspecification.

7.22

a) The estimated output/labor and output/capital elasticities are positive, as one would expect.
b) The elasticity of output/labor ratio (i.e., labor productivity) with respect to capital/labor ratio is about 0.68, meaning that if the latter increases by 1%, labor productivity, on average, goes up by about 0.68%. A key characteristic of developed economies is a relatively high capital/labor ratio.

8.13

a) The elasticity is –1.34. It is significantly different from zero, for the t value under the null hypothesis that the true elasticity coefficient is zero is:

$$t = \frac{-1.34}{0.32} = -4.1875$$

The p value of obtaining such a t value is extremely low.
However, the elasticity coefficient is not different from one because under the null hypothesis that the true elasticity is 1, the t value is

$$t = \frac{-1.34 - 1}{0.32} = -1.0625$$

This t value is not statistically significant.

b) The income elasticity, although positive, is not statistically different from zero, as the t value under the zero null hypothesis is less than 1.

8.14

a) A priori, salary and each of the explanatory variables are expected to be positively related, which they are. The partial coefficient of 0.280 means, ceteris paribus, the elasticity of CEO salary is a 0.28 percent.
The coefficient 0.0174 means, ceteris paribus, if the rate of return on equity goes up by 1 percentage point (Note: not by 1 percent), then the CEO salary goes up by about 1.07 %. Similarly, ceteris paribus, if return on the firm's stock goes up by 1 percentage point, the CEO salary goes up by about 0.024%.

b) Under the individual, or separate, null hypothesis that each true population coefficient is zero, you can obtain the t values by simply dividing each estimated coefficient by its standard error. These t values for the four coefficients shown in the model are, respectively, 13.5, 8, 4.25, and 0.44. Since the sample is large enough, you can see that the first three coefficients are individually highly statistically significant, whereas the last one is insignificant.

c) To test the overall significance, that is, all the slopes are equal to zero, use the F test

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.283/3}{0.717/205} = 27.02$$

Under the null hypothesis, this F has the F distribution with 3 and 205 df in the numerator and denominator, respectively. The p value of obtaining such an F value is extremely small, leading to rejection of the null hypothesis.

d) Since the dependent variable is in logarithmic form and the roe and ros are in linear form, the coefficients of these variables give semi elasticities, that is, the growth rate in the dependent variable for an absolute (unit) change in the regressor.

8.21

a) The own-price elasticity is -1.274.
b) From the t test, we obtain:

$$t = \frac{1.274 - 0}{0.527} = 2.4174$$

The p value of obtaining such a t statistic under the null hypothesis is about 0.034, which is small. Hence, we reject the hypothesis that the true price elasticity is zero.

c) We obtain:

$$t = \frac{-1.274 - (-1)}{0.527} = -0.5199$$

Since this t value is not statistically significant, we do not reject the hypothesis that the true price elasticity is unity.

d) Both the signs are expected to be positive, although none of these variables is statistically significant.

e) Perhaps our sample size is too small to detect the statistical significance of carnation prices on the demand for roses or that of income on the demand for roses. Moreover, expenditure on roses may be such a small part of total income that one may not notice the impact of income on demand for roses.

8.32

a) In Model I the slope coefficient tells us that per unit increase in the advertising expenditure, on average, retained impressions go up by 0.363 units. In Model II the (average) rate of increase in retained impressions depend on the level of advertising expenditure. Taking the derivative of Y with respect to X, you will obtain:

$$\frac{dY}{dX} = 1.0847 - 0.008X$$

This would suggest that retained impressions increase at a decreasing rate as advertising expenditure increases.

b) And c) We can treat Mode I 1 as the restricted version of Model II and hence can use the restricted least-squares technique to decide between the two models. Since the dependent variable in the two models is the same, we can use the R2 version of the F test. The results are as follows:

$$F = \frac{(0.53 - 0.424)/1}{(1 - 0.53)/18} = \frac{0.106}{0.0261} = 4.0613$$

Under the usual assumptions of the F test, the preceding F value follows the F distribution with 1 df and 18 df in the numerator and denominator, respectively. For these dfs the critical F value is 4.41 (5% level) and 3.01 (10% level).; the p value is 0.0591 or about 6%, which is close to 5%. It seems that we should retain the squared X variable in the model.

c) In part b)

d) As noted in b), there are diminishing returns to advertising expenditure; if the coefficient of the X-squared term were positive, there would have been increasing returns to advertising. Equating the derivative in b) to zero, we obtain:1.0847=0.008 X, which gives X = 135.58. At this value of X, the rate of increase of Y with respect to X is zero. Since X is measured in millions of dollars, we can say that at the level of expenditure of about 136 millions of dollars there is no further gain in retained impressions, which are measured in millions of impressions.