Jidapa 62046863

1.a.) Find $\hat{\beta}_1$ & $\hat{\beta}_2$

- $y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i \Rightarrow \hat{u}_i = y_i - \hat{\beta}_1 - \hat{\beta} X_i$

So min $\hat{u}_i$, ^2 to prevent the ⊖ value

$$\Sigma \hat{u}_i^2 = \boxed{\Sigma (y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2}$$

$$\frac{\partial \Sigma (y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2}{\partial \beta_1} = 0$$

$$2\Sigma (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0 \Rightarrow \hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X} \quad -(1)$$

$$\frac{\partial \Sigma (y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2}{\partial \beta_2} = 0$$

$$2\Sigma (y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)(-X_i) = 0 \quad -(2)$$

(1) plug in (2) $\Rightarrow$ $\cancel{-2}\Sigma x_i (y_i - \bar{y} + \hat{\beta}_2 \bar{X} - \hat{\beta}_2 x_i) = 0$

$$\Sigma x_i [(y_i - \bar{y}) - \hat{\beta}_2 (x_i - \bar{x})] = 0$$

$$\frac{\Sigma x_i (y_i - \bar{y})}{\Sigma x_i (x_i - \bar{x})} = \hat{\beta}_2$$

note
$$\Sigma x_i (y_i - \bar{y}) = \Sigma (x_i - \bar{x})(y_i - \bar{y})$$
$$= \Sigma x_i (y_i - \bar{y}) - \bar{x} \boxed{\Sigma (y_i - \bar{y})}$$ - sum of derivative from mean = 0
$$= \Sigma x_i (y_i - \bar{y})$$

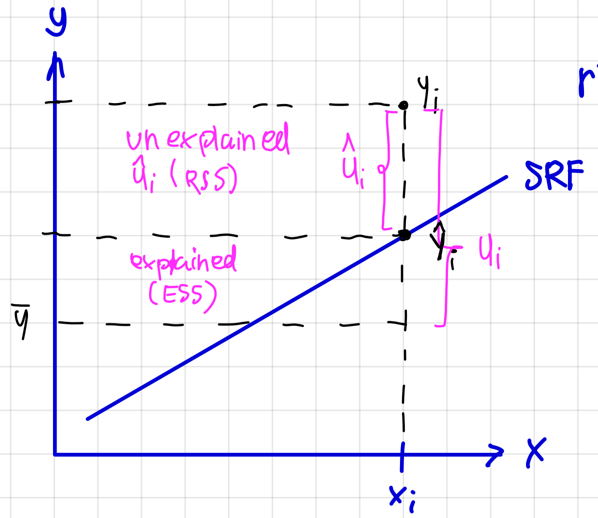
$$\therefore \hat{\beta}_2 = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})(x_i - \bar{x})} = \frac{-174.2}{1098.8} \approx \boxed{-0.1585} \#$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 21.03 + 0.1585(12.2) = \boxed{22.9637} \#$$

- This model is for estimating parameters that we don't know in the linear regression model. After we know the β_1 & β_2 so we can minimize the error term. #

Jidapa 62046863

1.b)



$$r^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$= 1 - \frac{\sum(\overbrace{y_i - \hat{y}_i}^{\hat{u}_i})^2}{\sum(y_i - \bar{y})^2}$$

$$= 1 - \frac{\sum \hat{u}_i^2}{\sum(y_i - \bar{y})^2}$$

$$= 1 - \frac{873.14}{882.97}$$

$$= \boxed{0.01113}$$

r^2 is a summary measure that tells how well the sample regression line fits the data. If r^2 is low, it mean error is high.

1.c) $\hat{y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$; $X_0 = 5$

$$\hat{y}_0 = 22.9637 - 0.1585(5)$$

$$= \boxed{22.1712}$$

note : k = 2

1.d) Estimator of $Var(u_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k} = \frac{873.14}{30-2} = \boxed{31.1836}$

$$Var(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\sum X_i^2}{n \sum x_i^2} \hat{\sigma}^2 = \frac{5{,}564}{30(1098.8)}(31.1836) = \boxed{5.2635}$$

$$Var(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2} = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{31.3836}{1098.8} = \boxed{0.0286}$$

Jidapa 62046863

1.e) ※1 State H.

$H_0 : \beta_2 = 0$ - Null H.

$H_a : \beta_2 \neq 0$ - Alternative_H.

β_2

Jidapa 62046863

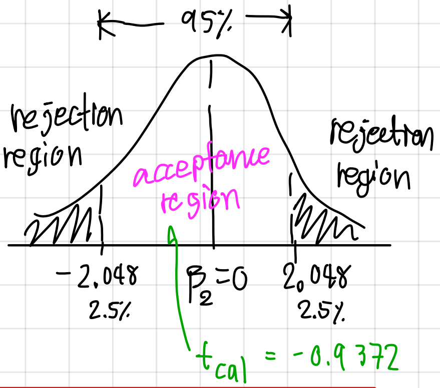
※2 T_{cal}

$$= \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{\sqrt{0.0286}} = -0.9372$$

※3 Decision Rule

- $\alpha = 0.05$, df = 28
- Lower bound : $t_{\frac{\alpha}{2}} = t_{0.025} = -2.048$

upper bound : $t_{\frac{\alpha}{2}} = t_{0.025} = 2.048$



- If t_{cal} lies within any boundary of acceptance region, so we can't reject the Null Hypothesis at significance level of 95%. we aren't sure that β_2 is not zero 95 out of 100 times when we sample. #

1.e) ※1 State H.

$H_0 : \beta_1 = 0$ - Null H.

$H_a : \beta_1 \neq 0$ - Alternative_H.

β_1

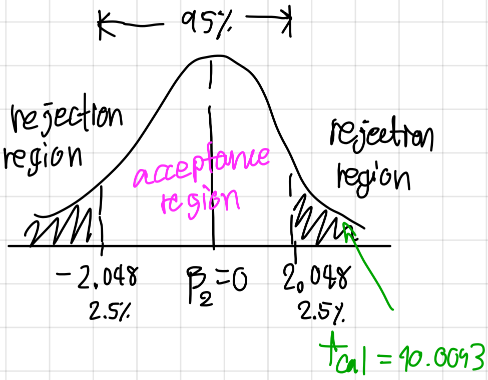
※2 T_{cal}

$$= \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \frac{22.9637 - 0}{\sqrt{5.2635}} = 10.0093$$

※3 Decision Rule

- $\alpha = 0.05$, df = 28
- Lower bound : $t_{\frac{\alpha}{2}} = t_{0.025} = -2.048$

upper bound : $t_{\frac{\alpha}{2}} = t_{0.025} = 2.048$



- If t_{cal} lies beyond any boundary of rejection region, so we can reject the Null Hypothesis at significance level of 95%. we are sure that β_1 is not zero 95 out of 100 times when we sample. #

Jidapa 62046863

1.f) ※1 State H_0

$H_0 : \beta_2 \geq 0$ - Null H.

$H_a : \beta_2 < 0$ - Alternative_H.

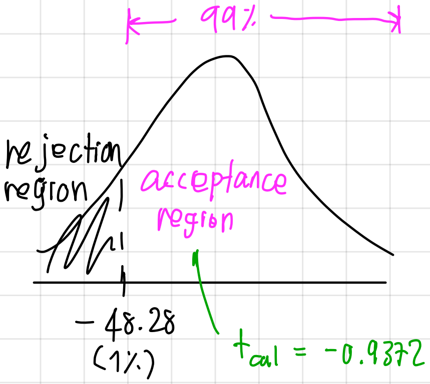
β_2

※2 T_{cal}

$$= \frac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{\sqrt{0.0286}} = -0.9372$$

※3 Decision Rule

- $\alpha = 0.01$, $df = 28$
- Lower bound : $t_\alpha = -48.28$



• If t_{cal} lies within any boundary of acceptance region, so we cannot reject the Null Hypothesis at significance level of 99%.
We are not sure that β_2 is less than 99 out of 100 times when we sample. #

※1 State H_0

$H_0 : \beta_1 \geq 0$ - Null H.

$H_a : \beta_1 < 0$ - Alternative_H.

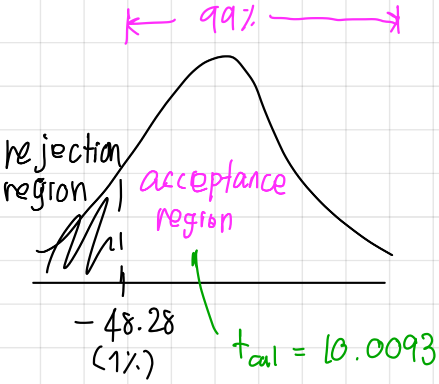
β_1

※2 T_{cal}

$$= \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{22.9637 - 0}{\sqrt{5.2635}} = 10.0093$$

※3 Decision Rule

- $\alpha = 0.01$, $df = 28$
- Lower bound : $t_\alpha = -48.28$



• If t_{cal} lies within any boundary of acceptance region, so we cannot reject the Null Hypothesis at significance level of 99%.
We are not sure that β_1 is less than 99 out of 100 times when we sample. #

Jidapa 62046863

2.a) x is the age of car and $\hat{\beta}_2$ is telling us when the age of car is longer by 1 year, how market price of a car will be decreased. $\hat{\beta}_2$ is involved with the estimation of car price, so it can affect the economy.

2.b) $\hat{Y}_i = 7{,}836 - 502.4X_i$; $X_i = 5$

$\hat{Y}_i = 7{,}836 - 502.4(5)$; $\hat{Y}_i = 5{,}324$

> $n-k = 11-2 = 9$; $\bar{X} = 7.45$; $\hat{\sigma}^2 = 212{,}877$; $\Sigma(X_i-\bar{X})^2 = 78.73$

> $1-\alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow t_{\frac{\alpha}{2}} = t_{0.025} = 2.262$

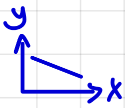
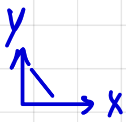
> $\text{var}(\hat{Y}_i) = \sigma^2\left[\frac{1}{n} + \frac{(X_0-\bar{X})^2}{\Sigma(X_i-\bar{X})^2}\right] = 212{,}877\left[\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right] = 35{,}582.5345$

> Calculate lower & Upper bound. $\Rightarrow \hat{Y}_i \pm t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_i}$

- Lower bound $= 5{,}324 - 2.262\sqrt{35{,}582.5345} = \boxed{4{,}897.3114}$
- Upper bound $= 5{,}324 + 2.262\sqrt{35{,}582.5345} = \boxed{5{,}750.6886}$

- The confidence interval is over the mean value which means 95 out of 100 ties the CI will cover true value of $E(Y|X_0)$

2.c) $\hat{Y}_i = 7{,}836 - 5024X_i$

the slope of SRF will be steeper from  $\Rightarrow$ 

original : $X_i \uparrow$ by 1 year $\Rightarrow \hat{Y}_i \downarrow$ 502.4 USD

New : $X_i \uparrow$ by 1 year $\Rightarrow \hat{Y}_i \downarrow$ 5024 USD

2.d) $X_i = 10 \Rightarrow \hat{Y}_i = 7836 - 5024 = 2812$

Elasticity of market price $\Rightarrow \frac{\Delta Y}{\Delta X}\left(\frac{X}{Y}\right) = -5024\left(\frac{10}{2812}\right)$

$= \boxed{-17.866}$

Jidapa 62046863