

1. Perform unit root test of series y and x.

```
. dfuller y, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

----- Interpolated Dickey-Fuller -----				
Test	1% Critical	5% Critical	10% Critical	
Statistic	Value	Value	Value	
Z(t)	1.000	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 1.0000

D.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
L1.	.0001178	.0001179	1.00	0.318	-.0001137	.0003494
LD.	.6997015	.0248993	28.10	0.000	.6507799	.7486231
_trend	2.897751	1.159296	2.50	0.013	.619992	5.175511
_cons	1811.233	147.0426	12.32	0.000	1522.327	2100.139

```
. dfuller x, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

----- Interpolated Dickey-Fuller -----				
Test	1% Critical	5% Critical	10% Critical	
Statistic	Value	Value	Value	
Z(t)	0.601	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 0.9970

D.x	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x						
L1.	.0001061	.0001764	0.60	0.548	-.0002405	.0004526
LD.	.46018	.0349881	13.15	0.000	.3914361	.5289239
_trend	4.166909	1.14105	3.65	0.000	1.924999	6.408818
_cons	2128.626	140.0551	15.20	0.000	1853.449	2403.803

```
. dfuller d.y, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 497

----- Interpolated Dickey-Fuller -----				
Test	1% Critical	5% Critical	10% Critical	
Statistic	Value	Value	Value	
Z(t)	-10.554	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 0.0000

D2.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
D.y						
L1.	-.2787856	.0264156	-10.55	0.000	-.3306866	-.2268845
LD.	-.32127	.0373756	-8.60	0.000	-.3947051	-.2478349
_trend	3.631984	.3708318	9.79	0.000	2.903379	4.36059
_cons	1678.082	154.4788	10.86	0.000	1374.564	1981.6

. dfuller d.x, trend lags(1) regress

Augmented Dickey-Fuller test for unit root Number of obs = 497

Test Statistic	----- Interpolated Dickey-Fuller -----		
	1% Critical Value	5% Critical Value	10% Critical Value
Z (t)	-10.657	-3.980	-3.420

MacKinnon approximate p-value for Z(t) = 0.0000

D2.x	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
D.x						
L1.	-.4007114	.0376021	-10.66	0.000	-.4745915	-.3268312
LD.	-.414172	.0358603	-11.55	0.000	-.4846298	-.3437141
_trend	3.508317	.3506567	10.00	0.000	2.819351	4.197283
_cons	1596.429	147.0971	10.85	0.000	1307.415	1885.444

From the Dickey-fuller unit root test, both x and y are non-stationary and the series are stationary at first difference I~(1) because the p-value of the tests is 0.00 which less than 0.05.

2. Perform cointegration test of series y and x using set up of (i) linear trend; (ii) restricted trend; (iii) unrestricted constant; (iv) restricted constant; and (v) no trend, with one lag term

. vecrank y x, trend(t) lags(1/1) max

Johansen tests for cointegration

Trend: trend Number of obs = 499
Sample: 2 - 500 Lags = 1

				5%	
maximum			trace	critical	
rank	parms	LL	eigenvalue	statistic	value
0	4	-7387.4577	.	790.3357	18.17
1	7	-6992.3441	0.79477	0.1085*	3.74
2	8	-6992.2899	0.00022		

5%

maximum				max	critical
rank	parms	LL	eigenvalue	statistic	value
0	4	-7387.4577	.	790.2272	16.87
1	7	-6992.3441	0.79477	0.1085	3.74
2	8	-6992.2899	0.00022		

. vecrank y x, trend(rt) lags(1/1) max

Johansen tests for cointegration

Trend: rtrend Number of obs = 499
Sample: 2 - 500 Lags = 1

maximum				trace	5% critical
rank	parms	LL	eigenvalue	statistic	value
0	2	-8050.4781	.	2116.3764	25.32
1	6	-7075.2453	0.97993	165.9107	12.25
2	8	-6992.2899	0.28286		

maximum				max	5% critical
rank	parms	LL	eigenvalue	statistic	value
0	2	-8050.4781	.	1950.4657	18.96
1	6	-7075.2453	0.97993	165.9107	12.52
2	8	-6992.2899	0.28286		

. vecrank y x, trend(c) lags(1/1) max

Johansen tests for cointegration

Trend: constant Number of obs = 499
Sample: 2 - 500 Lags = 1

maximum				trace	5% critical
rank	parms	LL	eigenvalue	statistic	value
0	2	-8050.4781	.	2086.8946	15.41
1	5	-7083.8611	0.97923	153.6607	3.76
2	6	-7007.0308	0.26504		

maximum				max	5% critical
rank	parms	LL	eigenvalue	statistic	value
0	2	-8050.4781	.	1933.2339	14.07
1	5	-7083.8611	0.97923	153.6607	3.76
2	6	-7007.0308	0.26504		

. vecrank y x, trend(rc) lags(1/1) max

Johansen tests for cointegration

Trend: rconstant Number of obs = 499
Sample: 2 - 500 Lags = 1

					5%
maximum				trace	critical
rank	parms	LL	eigenvalue	statistic	value
0	0	-8811.3759	.	3607.1930	19.96
1	4	-7092.1887	0.99898	168.8185	9.42
2	6	-7007.7794	0.28703		

					5%
maximum				max	critical
rank	parms	LL	eigenvalue	statistic	value
0	0	-8811.3759	.	3438.3745	15.67
1	4	-7092.1887	0.99898	168.8185	9.24
2	6	-7007.7794	0.28703		

```
. vecrank y x, trend(n) lags(1/1) max
```

Johansen tests for cointegration

```
Trend: none                                Number of obs =    499
Sample:  2 - 500                            Lags =          1
```

maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical value
0	0	-8811.3759	.	3204.1193	12.53
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3163	0.00974		

					5%
maximum				max	critical
rank	parms	LL	eigenvalue	statistic	value
0	0	-8811.3759	.	3199.2333	11.44
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3163	0.00974		

3. Perform cointegration test of series y and x using set up of linear trend with (i) one lag term; (ii) two lag terms; and (iii) three lag terms.

```
. vecrank y x, trend(t) lags(1/1) max
```

Johansen tests for cointegration

```
Trend: trend                                     Number of obs =      499
Sample:  2 - 500                                Lags =              1
```

maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical value
0	4	-7387.4577	.	790.3357	18.17
1	7	-6992.3441	0.79477	0.1085*	3.74
2	8	-6992.2899	0.00022		

maximum				max	5% critical
rank	parms	LL	eigenvalue	statistic	value
0	4	-7387.4577	.	790.2272	16.87

1	7	-6992.3441	0.79477	0.1085	3.74
2	8	-6992.2899	0.00022		

. vecrank y x, trend(t) lags(2) max

Johansen tests for cointegration

Trend: trend Number of obs = 498
Sample: 3 - 500 Lags = 2

5%					
maximum				trace	critical
rank	parms	LL	eigenvalue	statistic	value
0	8	-6795.5337	.	187.5771	18.17
1	11	-6703.3826	0.30932	3.2749*	3.74
2	12	-6701.7451	0.00655		

5%					
maximum				max	critical
rank	parms	LL	eigenvalue	statistic	value
0	8	-6795.5337	.	184.3022	16.87
1	11	-6703.3826	0.30932	3.2749	3.74
2	12	-6701.7451	0.00655		

. vecrank y x, trend(t) lags(3) max

Johansen tests for cointegration

Trend: trend Number of obs = 497
Sample: 4 - 500 Lags = 3

5%					
maximum				trace	critical
rank	parms	LL	eigenvalue	statistic	value
0	12	-6756.658	.	140.0434	18.17
1	15	-6688.113	0.24106	2.9534*	3.74
2	16	-6686.6363	0.00592		

5%					
maximum				max	critical
rank	parms	LL	eigenvalue	statistic	value
0	12	-6756.658	.	137.0900	16.87
1	15	-6688.113	0.24106	2.9534	3.74
2	16	-6686.6363	0.00592		

For the linear trend model with one lag term, we rejected model with rank 0 ($H_0: r = 0$) because the critical value is higher than statistic test ($790.3357 > 18.17$), hence we continue to rank one test ($H_0: r = 1$). We failed to reject the model with rank1 with a critical value equal to 0.185 (< 3.74), therefore there is one cointegration equation.

From the linear trend model with two lag terms, we rejected model with rank 0 ($H_0: r = 1$) because the critical value is higher ($187.5771 > 18.17$), so we continue to rank one test ($H_0: r = 1$). We failed to reject the model with rank1 with a critical value equal to 3.2749 (< 3.74), hence there is one cointegration equation.

For the linear trend model with three lag terms, we rejected model with rank 0 ($H_0: r = 0$) since the critical value is higher ($140.0434 > 18.17$), so we continue to rank one test ($H_1: r = 0$). We failed to reject the model with rank 1 with a critical value equal to 2.9534 (< 3.74), therefore there is one cointegration equation.

4. From (2) & (3), determine the most appropriated set up and make the conclusion of the test whether y and x are cointegrated series? If yes, how many cointegrating equations.

From (2) & (3), y and x are cointegrated series since the rank of the matrix is not 0, and there is 1 cointegrating equation from the maximum rank equal to 1.

5. Estimated VECM models of y and x using (i) one lag term; (ii) two lag terms; and (iii) three lag terms. Determine the optimal lags. Specify cointegrating equation and speed of adjustment parameters.

```
. vec y x , lags(1/1)
```

Vector error-correction model

```
Sample: 2 - 500
Log likelihood = -7083.861
Det(Sigma_ml) = 7.34e+09
Number of obs = 499
AIC = 28.41227
HQIC = 28.42883
SBIC = 28.45448
```

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	2	331.479	0.9988	399343.6	0.0000
D_x	2	430.726	0.9953	105050.2	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
D_y	_cel					
	L1.	-1.303404	.0095397	-136.63	0.000	-1.322102 -1.284707
	_cons	-107.843	69.40373	-1.55	0.120	-243.8718 28.18583
D_x	_cel					
	L1.	-.8364345	.0123959	-67.48	0.000	-.8607301 -.812139
	_cons	168.0502	90.1839	1.86	0.062	-8.706964 344.8074

Cointegrating equations

Equation	Parms	chi2	P>chi2
_cel	1	1.71e+10	0.0000

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	4	213.415	0.9995	964110.3	0.0000
D_x	4	263.876	0.9982	280728.2	0.0000

Cointegrating equations

Equation	Parms	chi2	P>chi2
cel	1	8.09e+08	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

	beta	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_ce1							
	y	1	.	.	.	.	.
	x	-1.500176	.0000527	-2.8e+04	0.000	-1.50028	-1.500073
	_cons	-577.9622	.	.	.	.	.

```
. vec y x , lags(3)
```

Vector error-correction model

Sample: 4 - 500	Number of obs	=	497
	AIC	=	27.1245
Log likelihood = -6727.439	HQIC	=	27.16771
Det(Sigma_ml) = 1.96e+09	SBIC	=	27.23459

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	6	211.673	0.9995	980003.4	0.0000
D_x	6	253.098	0.9984	305162.2	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
D_y							
	_ce1						
	L1.	-.3193909	.058155	-5.49	0.000	-.4333726	-.2054092
	_y						
	LD.	.3553209	.0559202	6.35	0.000	.2457194	.4649225
	L2D.	.0174858	.0312998	0.56	0.576	-.0438607	.0788323
_x							
LD.	.6042668	.0763396	7.92	0.000	.4546439	.7538896	
L2D.	.0521968	.0634683	0.82	0.411	-.0721988	.1765925	
_cons		181.5838	45.38519	4.00	0.000	92.63049	270.5372
D_x							
	_ce1						

Equation	Parms	chi2	P>chi2
_ce1	1	6.64e+08	0.0000

Johansen normalization restriction imposed

	beta	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
_ce1						
	y	1	.	.	.	.
	x	-1.500049	.0000582	-2.6e+04	0.000	-1.500163 -1.499935
	_cons	-92.13439	.	.	.	.

Because the model with the number of lag equal to 3 has the lowest SBIC, the most appropriated model is model with 3 lag term. The cointegrating equation is $y = 1.500049x + 92.13439$, and the speed of adjustment parameter: $\delta_1 = -.3193909$ and $\delta_2 = .4434064$