1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

			x_i	y_i	$x_i y_i$	x_i^2		actual - estimate		
Student	Y_i	X_i	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})(Y-\bar{Y})$	$(X-\bar{X})^2$	$\hat{Y}_i$	$\hat{u}_i$	$\hat{u}_i^2$	$\hat{X}_i^2$
1	2.8	63	-14.625	-0.4125	6.0328	213.8906	2.7938	0.0862	0.0074	3969
2	3.4	72	-5.625	0.1875	-1.0547	31.6406	3.0207	0.3793	0.1439	5184
3	3.0	78	0.375	-0.2125	-0.0797	0.1406	3.2253	-0.2253	0.0508	6084
4	3.5	81	3.375	0.2875	0.9703	11.3906	3.3276	0.1724	0.0207	6561
5	3.6	87	9.375	0.3875	3.6328	87.8906	3.5322	0.0678	0.0050	7569
6	3.0	75	-2.625	-0.2125	0.5578	6.8906	3.123	-0.123	0.0151	5625
7	2.7	75	-2.625	-0.5125	1.3453	6.8906	3.123	-0.123	0.1789	5625
8	3.7	90	12.375	0.4875	6.0328	153.1406	3.6345	0.0655	0.0043	8100
Σ	25.7	621	0		17.4374	511.875		0.0001	0.4347	48,717
Mean	3.2125	77.625			2.1797	63.9844				

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} \qquad \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= \frac{17.7374}{511.8748} = 0.0341 \qquad = 0.5655$$

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$$

$$= 0.5655 + 0.0341 X_i$$

interpret If total econometrics exam point is 0, then GPA of BE student would be 0.5655.

If total econometrics increase by 1 point, on average GPA of BE student would be more by 0.0341.

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} \qquad \text{var}(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2 \qquad \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2}$$

$$= \frac{0.0725}{511.875} \qquad = \frac{0.0725(48,717)}{8(511.875)} \qquad = \frac{0.4347}{6}$$

$$= 0.000142 \qquad = 0.000142 \qquad = 0.0725$$

2.

X_i	Y_i	x_i: $X-\bar{X}$	y_i: $Y-\bar{Y}$	$x_i y_i$: $(X-\bar{X})(Y-\bar{Y})$	x_i^2: $(X-\bar{X})^2$	$\hat{Y}_i$	actual - estimate: $\hat{u}_i$	$\hat{u}_i^2$	$\hat{X}_i^2$
10	0	-10	-9.1	91	100	0.145	-0.145	0.0210	100
12	2	-8	-7.1	56.8	64	1.936	0.064	0.0041	144
14	5	-6	-4.1	24.6	36	3.727	1.273	1.6205	196
16	6	-4	-3.1	12.4	16	5.518	0.482	0.2323	256
18	7	-2	-2.1	4.2	4	7.309	-0.309	0.0954	324
22	10	2	0.9	1.8	4	10.891	-0.891	0.7939	484
24	10	4	0.9	3.6	16	12.682	-2.682	7.1931	576
26	15	6	5.9	35.4	36	14.473	0.527	0.2778	676
28	16	8	6.9	55.2	64	16.264	-0.264	0.6970	784
30	20	10	10.9	109	100	18.055	1.945	3.7830	900
Σ 200	91			394	440		0	14.0909	4440
Mean 20	9.1								

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} \qquad\qquad \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= \frac{394}{440} = 0.8955 \qquad\qquad = 9.1 - (0.8955)(20) = -8.81$$

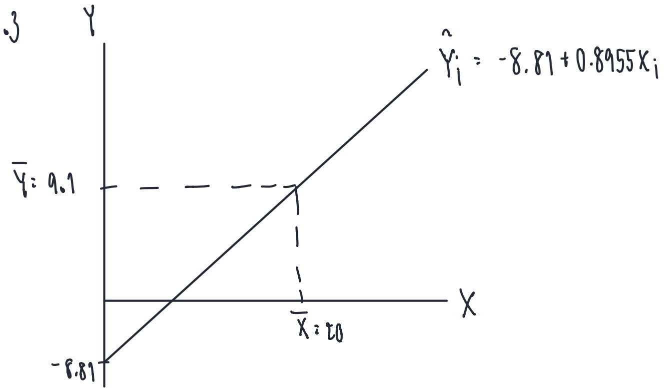
$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$$

$$= -8.81 + 0.8955X$$

interpret: if $X = 0$, Y_i would be -8.81

if X increase by 1 unit, on average Y will be more by 0.8955.

2.3



$$\hat{Y}_i = -8.81 + 0.8955X_i$$

$$= -8.81 + 0.8955(20) = 9.1$$

Hence, the regression line passes through $\bar{X}, \bar{Y}$

2.4 if $X_i = 78$

$$\hat{Y}_i = -8.87 + 0.08955(78) = 7.309$$

2.5 $\text{var}(\hat{\beta}_2) = \dfrac{\sigma^2}{\sum x_i^2}$ $\text{var}(\hat{\beta}_1) = \dfrac{\sum X_i^2}{n \sum x_i^2} \sigma^2$ $\hat{\sigma}^2 = \dfrac{\sum \hat{u}_i^2}{n-2}$

$= \dfrac{1.7614}{440}$ $= \dfrac{(1.7614)(4440)}{10(440)}$ $= \dfrac{14.0909}{8} = 1.7614$

$= 0.0004$ $= 1.7774$

3. $\bar{Y} = \beta_1 + \beta_2 \bar{X} + u$ — ①

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$ — ②

sub ① into ② ; $\hat{\beta}_1 = \beta_1 + \beta_2 \bar{X} + \bar{u} - \hat{\beta}_2 \bar{X}$

$$\hat{\beta}_1 = \beta_1 + (\beta_2 - \hat{\beta}_2) \bar{X} + u$$

$$E(\hat{\beta}_1 | X) = E(\beta_1 | X) + \bar{X} E[(\beta_2 - \hat{\beta}_2) | X] + E(u | X)$$

$$E(\hat{\beta}_1 | X) = \beta_1 + \bar{X}(\beta_2 - E(\hat{\beta}_2 | X))$$

$$E(\hat{\beta}_1 | X) = \beta_1$$