

MULTICOLLINEARITY

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THE NATURE OF MULTICOLLINEARITY

Multiple regression

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_k X_{ki} + u_i$$

$X_1 = 1$ for all observations to allow for the intercept term, an exact linear relationship is said to exist if the following condition is satisfied:

$$\lambda_1 X_{1i} + \lambda_2 X_{2i} + \dots + \lambda_k X_{ki} = 0$$

where $\lambda_1, \lambda_2, \dots, \lambda_k$ are constants such that not all of them are zero simultaneously

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The case where the X variables are intercorrelated but not perfectly so, as follows:

$$\lambda_1 X_{1i} + \lambda_2 X_{2i} + \dots + \lambda_k X_{ki} + v_i = 0$$

where v_i is a stochastic error term

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The difference between perfect and less than perfect multicollinearity

$$X_{2i} = \frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \dots - \frac{\lambda_k}{\lambda_2} X_{ki}$$

$$X_{2i} = \frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \dots - \frac{\lambda_k}{\lambda_2} X_{ki} - \frac{1}{\lambda_2} v_i$$

if $\lambda_2 \neq 0$

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Perfect Multicollinearity

$$-5X_{2i} + X_{3i} = 0$$

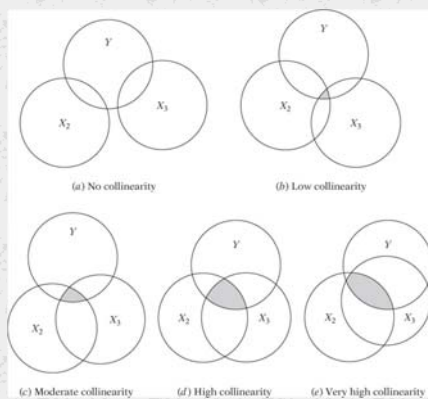
X_2	X_3
10	50
15	75
18	90
24	120
30	150

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NOT PERFECT MULTICOLLINEARITY

X_{2i}	X_{3i}	v_i	$-5X_{2i} + X_{3i} + v_i = 0$
10	52	2	
15	75	0	
18	97	7	
24	129	9	
30	152	2	

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Multicollinearity refers only to linear relationships among the X variables. It does not rule out nonlinear relationships among them.

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + u_i$$

This model is nonlinear, therefore, it does not violate the assumption of no multicollinearity

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If multicollinearity is perfect, the regression coefficients of the X variables are indeterminate and their standard errors are infinite.

$$X_{2i} = \frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \dots - \frac{\lambda_k}{\lambda_2} X_{ki}$$

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If multicollinearity is less than perfect, the regression coefficients, although determinate, possess large standard errors, which means the coefficients cannot be estimated with great precision or accuracy

$$X_{2i} = \frac{\lambda_1}{\lambda_2} X_{1i} - \frac{\lambda_3}{\lambda_2} X_{3i} - \dots - \frac{\lambda_k}{\lambda_2} X_{ki} - \frac{1}{\lambda_2} v_i$$

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SEVERAL SOURCES OF MULTICOLLINEARITY

- The data collection method employed
- Constraints on the model or in the population being sampled
- Model specification
- An overdetermined model

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- Estimation in the presence of perfect multicollinearity
- Estimation in the presence of “High” but “Imperfect” Multicollinearity

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ESTIMATION IN THE PRESENCE OF PERFECT MULTICOLLINEARITY

In the case of perfect multicollinearity the **regression coefficients remain indeterminate** and their **standard errors are infinite**

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$$y_i = Y_i - \bar{Y} \quad x_{2i} = X_{2i} - \bar{X}_2 \quad x_{3i} = X_{3i} - \bar{X}_3$$

$$y_i = \hat{\beta}_2 x_{2i} + \hat{\beta}_3 x_{3i} + \hat{u}_i$$

$$\hat{\beta}_2 = \frac{(\sum y_i x_{2i})(\sum x_{3i}^2) - (\sum y_i x_{3i})(\sum x_{2i} x_{3i})}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i} x_{3i})^2}$$

$$\hat{\beta}_3 = \frac{(\sum y_i x_{3i})(\sum x_{2i}^2) - (\sum y_i x_{2i})(\sum x_{2i} x_{3i})}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i} x_{3i})^2}$$

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ASSUME THAT

$$X_{3i} = \lambda X_{2i}$$

$$\bar{X}_{3i} = \lambda \bar{X}_{2i}$$

$$(X_{3i} - \bar{X}_{3i}) = \lambda (X_{2i} - \bar{X}_{2i})$$

$$x_{3i} = \lambda x_{2i}$$

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$\hat{\beta}_2$ AND $\hat{\beta}_3$ ARE INDETERMINATE

$$\hat{\beta}_2 = \frac{\lambda^2 \{ (\sum y_i x_{2i})(\sum x_{3i}^2) - (\sum y_i x_{3i})(\sum x_{2i}^2) \}}{\lambda^2 \{ (\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i} x_{3i})^2 \}} = \frac{0}{0}$$

$$\hat{\beta}_3 = \frac{\lambda \{ (\sum y_i x_{2i})(\sum x_{2i}^2) - (\sum y_i x_{2i})(\sum x_{2i}^2) \}}{\lambda^2 \{ (\sum x_{2i}^2)(\sum x_{2i}^2) - (\sum x_{2i}^2)^2 \}} = \frac{0}{0}$$

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In the case of perfect multicollinearity one cannot get a unique solution for the individual regression coefficients

The variances and standard errors of $\hat{\beta}_2$ and $\hat{\beta}_3$ individually are infinite

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ESTIMATION IN THE PRESENCE OF "HIGH" BUT "IMPERFECT" MULTICOLLINEARITY

$$X_{3i} = \lambda X_{2i} + v_i$$

$$\bar{X}_{3i} = \lambda \bar{X}_{2i}$$

$$X_{3i} - \bar{X}_{3i} = \lambda (X_{2i} - \bar{X}_{2i}) + v_i$$

$$x_{3i} = \lambda x_{2i} + v_i,$$

where $\lambda \neq 0$

$$\sum x_i v_i = 0$$

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$$\hat{\beta}_2 = \frac{\sum (y_i x_{2i}) (\lambda^2 \sum x_{2i}^2 + \sum v_i^2) - (\lambda \sum y_i x_{2i} + \sum y_i v_i) (\lambda \sum x_{2i}^2)}{\sum x_{2i}^2 (\lambda^2 \sum x_{2i}^2 + \sum v_i^2) - (\lambda \sum x_{2i}^2)^2}$$

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THEORETICAL CONSEQUENCES OF MULTICOLLINEARITY

- The OLS estimators are **unbiased**. But unbiasedness is a multisample or repeated sampling property

Keeping the values of the variables X fixed, if one obtains repeated samples and computes the OLS estimators for each of these samples, the average of the sample values will converge to the true population values of the estimators as the number of sample increases

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- Collinearity does not destroy the property of minimum variance
- In the class of all linear unbiased estimators, the OLS estimators have minimum variance- they are **efficient**
- But this does not mean that the variance of an OLS estimator will necessarily be small

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- Multicollinearity is essentially a sample (regression) phenomenon, even if the X variables are not linearly related in the population, they may be so related in the particular sample
- When we postulate the theoretical or population regression function (PRF), we believe that all the X variables included in the model have a separate or independent influence on the dependent variable Y.

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$$Consumption_i = \beta_1 + \beta_2 Income_i + \beta_3 Wealth_i + u_i$$

Two variables may be highly , if not perfectly, correlated: Wealthier people generally tend to have higher incomes.

To assess the individual effects of wealth and income on consumption expenditure we need a sufficient number of sample observations of wealthy individuals with low income, and high income individuals with low wealth

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- OLS estimators are **BLUE** despite multicollinearity

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PRACTICAL CONSEQUENCES OF MULTICOLLINEARITY

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PRACTICAL CONSEQUENCES OF MULTICOLLINEARITY

1. OLS estimators have large variance and covariance
2. The confidence intervals tend to be much wider, leading to the acceptance of the “zero null hypothesis”
3. t ratio of one or more coefficients tends to be statistically insignificant
4. Although the t ratio of one or more coefficients is statistically insignificant, R-Squared can be very high
5. The OLS estimators and their standard errors can be sensitive to small changes in the data

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OLS ESTIMATORS HAVE LARGE VARIANCE AND COVARIANCE

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_{2i}^2 (1 - r_{23}^2)}$$

$$\text{var}(\hat{\beta}_3) = \frac{\sigma^2}{\sum x_{3i}^2 (1 - r_{23}^2)}$$

$$\text{cov}(\hat{\beta}_2, \hat{\beta}_3) = \frac{-r_{23} \sigma^2}{(1 - r_{23}^2) \sqrt{\sum x_{2i}^2 \sum x_{3i}^2}}$$

r_{23} is the coefficient of correlation between X_2 and X_3

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VARIANCE-INFLATING FACTOR (VIF)

VIF shows how the variance of an estimator is inflated by the presence of multicollinearity

$$VIF = \frac{1}{(1 - r_{23}^2)}$$

$$r_{23}^2 \rightarrow 1, VIF \rightarrow \infty$$

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$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_{2i}^2} VIF$$

$$\text{var}(\hat{\beta}_3) = \frac{\sigma^2}{\sum x_{3i}^2} VIF$$

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EXAMPLE

TABLE 10.1
The Effect of Increasing r_{23} on $\text{var}(\hat{\beta}_2)$ and $\text{cov}(\hat{\beta}_2, \hat{\beta}_3)$

Value of r_{23} (1)	VIF (2)	$\text{var}(\hat{\beta}_2)$ (3) $\frac{\sigma^2}{\sum x_{2i}^2} = A$	$\text{var}(\hat{\beta}_2) (r_{23} \neq 0)$ $\text{var}(\hat{\beta}_2) (r_{23} = 0)$ (4)	$\text{cov}(\hat{\beta}_2, \hat{\beta}_3)$ (5)
0.00	1.00	$\frac{\sigma^2}{\sum x_{2i}^2} = A$	—	0
0.50	1.33	$1.33 \times A$	1.33	$0.67 \times B$
0.70	1.96	$1.96 \times A$	1.96	$1.37 \times B$
0.80	2.78	$2.78 \times A$	2.78	$2.22 \times B$
0.90	5.76	$5.76 \times A$	5.76	$4.73 \times B$
0.95	10.26	$10.26 \times A$	10.26	$9.74 \times B$
0.97	16.92	$16.92 \times A$	16.92	$16.41 \times B$
0.99	50.25	$50.25 \times A$	50.25	$49.75 \times B$
0.995	100.00	$100.00 \times A$	100.00	$99.50 \times B$
0.999	500.00	$500.00 \times A$	500.00	$499.50 \times B$

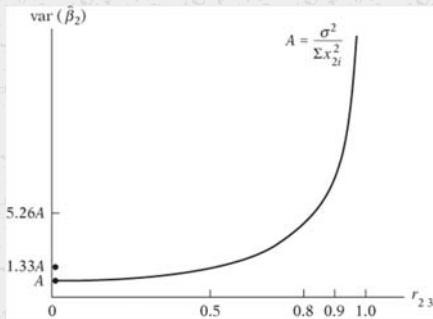
$$\text{Note: } A = \frac{\sigma^2}{\sum x_{2i}^2}$$

$$B = \frac{-\sigma^2}{\sqrt{\sum x_{2i}^2 \sum x_{3i}^2}}$$

$$\times = \text{times}$$

*To find out the effect of increasing r_{23} on $\text{var}(\hat{\beta}_1)$, note that $A = \sigma^2 / \sum x_{1i}^2$ when $r_{23} = 0$, but the variance and covariance magnifying factors remain the same.

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$$\text{var}(\hat{\beta}_j) = \frac{\sigma^2}{\sum x_j^2} \text{VIF}$$

$\text{var}(\hat{\beta}_j)$ is large or small will depend on the three ingredients:

- (1) σ^2
- (2) VIF
- (3) $\sum x_j^2$

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TABLE 10.2
The Effect of
Increasing
Collinearity on the
95% Confidence
Interval for
 β_2 : $\hat{\beta}_2 \pm 1.96 \text{ se}(\hat{\beta}_2)$

Value of r_{23}	95% Confidence Interval for β_2
0.00	$\hat{\beta}_2 \pm 1.96 \sqrt{\frac{\sigma^2}{\sum x_{2i}^2}}$
0.50	$\hat{\beta}_2 \pm 1.96 \sqrt{(1.33)} \sqrt{\frac{\sigma^2}{\sum x_{2i}^2}}$
0.95	$\hat{\beta}_2 \pm 1.96 \sqrt{(10.26)} \sqrt{\frac{\sigma^2}{\sum x_{2i}^2}}$
0.995	$\hat{\beta}_2 \pm 1.96 \sqrt{(100)} \sqrt{\frac{\sigma^2}{\sum x_{2i}^2}}$
0.999	$\hat{\beta}_2 \pm 1.96 \sqrt{(500)} \sqrt{\frac{\sigma^2}{\sum x_{2i}^2}}$

Note: We are using the normal distribution because σ^2 is assumed for convenience to be known. Hence the use of 1.96, the 95% confidence factor for the normal distribution. The standard errors corresponding to the various r_{23} values are obtained from Table 10.1.

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WIDER CONFIDENCE INTERVALS

Because of the large standard errors, the confidence intervals for the relevant population parameters tend to be larger

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“INSIGNIFICANT” T RATIOS

In cases of high collinearity the estimated standard errors increase dramatically, thereby making the t values smaller

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A HIGH R^2 BUT FEW SIGNIFICANT T RATIOS

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$$

In cases of high collinearity, it is possible to find, the partial slope coefficients are individually statistically insignificant on the basis of the t test

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SENSITIVITY OF OLS ESTIMATORS AND THEIR STANDARD ERRORS TO SMALL CHANGES IN DATA

As long as multicollinearity is not perfect, estimation of the regression coefficients is possible but the estimates and their standard errors become very sensitive to even the slightest change in the data

EXAMPLE

TABLE 10.3 Hypothetical Data on Y, X₂, and X₃

Y	X ₂	X ₃
1	2	4
2	0	2
3	4	12
4	6	0
5	8	16

$$\hat{Y}_i = 1.1939 + 0.4463X_{2i} + 0.0030X_{3i}$$
$$(0.7737) \quad (0.1848) \quad (0.0851)$$
$$t = (1.5431) \quad (2.4151) \quad (0.0358)$$
$$R^2 = 0.8101$$
$$r_{23} = 0.5523$$
$$\text{cov}(\hat{\beta}_2, \hat{\beta}_3) = -0.00868$$
$$df = 5 - 3 = 2$$

Source	SS	df	MS			
Model	8.18121951	2	4.09060976	Number of obs =	5	
Residual	1.89878849	2	.949398244	F(2, 2) =	4.27	
Total	10	4	2.5	Prob > F =	0.1899	
				R-squared =	0.8181	
				Adj R-squared =	0.6282	
				Root MSE =	.97437	

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x2	.4463415	.1848184	2.42	0.137	-.3488336 1.241517
x3	.0030488	.0850659	0.34	0.975	-.3629692 .3690578
_cons	1.193982	.7736789	1.54	0.263	-2.134969 4.522774

TABLE 10.4 Hypothetical Data on Y, X₂, and X₃

Y	X ₂	X ₃
1	2	4
2	0	2
3	4	0
4	6	12
5	8	16

$$\hat{Y}_i = 1.2108 + 0.4014X_{2i} + 0.0270X_{3i}$$
$$(0.7480) \quad (0.2721) \quad (0.1252)$$
$$t = (1.6187) \quad (1.4752) \quad (0.2158)$$
$$R^2 = 0.8143$$
$$r_{23} = 0.8285$$
$$\text{cov}(\hat{\beta}_2, \hat{\beta}_3) = -0.0282$$
$$df = 5 - 3 = 2$$

Source	SS	df	MS			
Model	8.14324324	2	4.07162162	Number of obs =	5	
Residual	1.85675676	2	.928378378	F(2, 2) =	4.39	
Total	10	4	2.5	Prob > F =	0.1857	
				R-squared =	0.8143	
				Adj R-squared =	0.6286	
				Root MSE =	.96352	

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x2	.4013514	.272065	1.48	0.278	-.7692498 1.571953
x3	.027827	.1252281	0.22	0.849	-.5117858 .5658399
_cons	1.210811	.7480215	1.62	0.247	-2.697666 4.429288

DETECTION OF MULTICOLLINEARITY

Kmenta (1986)

1. Multicollinearity is a question of degree and not of kind. The meaning distinction is not between the presence and the absence of multicollinearity, but between its various degrees
2. Since multicollinearity refers to the condition of the explanatory variables that are assumed to be nonstochastic, it is a feature of the sample and not of the population

Therefore, we do not “test for multicollinearity” but can, if we wish, measure its degree in any particular sample

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DETECTION OF MULTICOLLINEARITY

- High R-Squared but few significant t-ratios
- High pair-wise correlations among regressors
- Examination of partial correlations
- Auxiliary regressions
- VIF
- Scatter plot

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HIGH R-SQUARED BUT FEW SIGNIFICANT T-RATIOS

- If R-Squared is high, say, in excess of 0.8, the F-test in most cases will reject the hypothesis that the partial slope coefficients are simultaneously equal to zero, but the individual t tests will show that none or very few of the partial slope coefficients are statistically different from zero.

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EXAMPLE

Source	SS	df	MS	Number of obs = 18		
Model	8565.55487	2	4282.77794	F(2, 7) =	92.48	
Residual	324.445926	7	46.348418	Prob > F =	0.0000	
Total	8890	9	987.777778	R-squared =	0.9035	
				Adj R-squared =	0.9531	
				Root MSE =	6.808	

	y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x2		.9415373	.8228983	1.14	0.299	-1.004388	2.887383
x3		-.0424345	.0886845	-0.53	0.615	-.2331757	.1483967
_cons		24.77473	6.7525	3.67	0.008	6.897699	48.74186

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HIGH PAIR-WISE CORRELATIONS AMONG REGRESSORS

- The pair-wise or zero-order correlation coefficient between two regressors is high, say, in excess of 0.8

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EXAMPLE

Correlation between income (x2) and wealth (x3)

	x2	x3
x2	1.0000	
x3	0.9990	1.0000

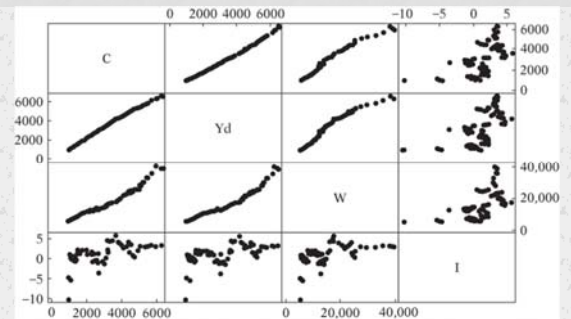
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VARIANCE INFLATION FACTOR

- **As a rule of thumb**, if the VIF of a variable exceeds 10, which will happen if R_j^2 exceeds 0.90, that variable is said to be highly collinear

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SCATTER PLOT



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C – Consumption

Y_d – Real disposable personal income

W – Real wealth

I – Real Interest Rate

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REMEDIAL MEASURES

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REMEDIAL MEASURES

- Do Nothing
- Rule-Thumb Procedures
 1. A priori information
 2. Combining cross-sectional and time series data
 3. Dropping a variable (s) and specification bias
 4. Adding or new data
 5. Transformation of variables

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A PRIORI INFORMATION

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

$$Y = \text{consumption}, X_2 = \text{income}, X_3 = \text{wealth}$$

$$\beta_3 = 0.10\beta_2$$

$$Y_i = \beta_1 + \beta_2 X_{2i} + 0.10\beta_2 X_{3i} + u_i$$

$$= \beta_1 + \beta_2 X_i + u_i$$

$$X_i = X_{2i} + 0.10X_{3i}$$

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COMBINING CROSS-SECTIONAL AND TIME SERIES DATA

A variant of the extraneous or a priori information technique is the combination of cross-sectional and time series data known as pooling the data

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DROPPING A VARIABLE (S) AND SPECIFICATION BIAS

But in dropping a variable from the model we may be committing a **specification bias** or **specification error**.

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ADDING OR NEW DATA

As the sample size increases, $\sum_i (X_{ji} - \bar{X}_j)^2$ will generally increase. Therefore, for any given r_{23} , the variance of $\hat{\beta}_2$ will decrease, thus decreasing the standard error, which will enable us to estimate more precisely

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TRANSFORMATION OF VARIABLES

- First difference form
- Ratio transformation

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- First difference form

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

$$Y_{t-1} = \beta_1 + \beta_2 X_{2,t-1} + \beta_3 X_{3,t-1} + u_{t-1}$$

$$Y_t - Y_{t-1} = \beta_2 (X_{2t} - X_{2,t-1}) + \beta_3 (X_{3t} - X_{3,t-1}) + v_t$$

where $v_t = u_t - u_{t-1}$

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First difference form may not satisfy one of the assumptions of the CLRM – the disturbances are serially uncorrelated (We will see in Autocorrelation chapter)

First differencing – may not appropriate in cross-sectional data where there no logical ordering of the observations

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- Ratio transformation

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

$$\frac{Y_t}{X_{3t}} = \beta_1 \left(\frac{1}{X_{3t}} \right) + \beta_2 \left(\frac{X_{2t}}{X_{3t}} \right) + \beta_3 + \left(\frac{u_t}{X_{3t}} \right)$$

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- Ratio transformation, the error term $\left(\frac{u_t}{X_{3t}} \right)$ will be **heteroscedastic**, if the original error term is homoscedastic

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Multicollinearity may not pose a serious problem
- When R-squared is high and the regression coefficients are individually significant as revealed by the higher t values

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EXAMPLE

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CONSUMPTION EXPENDITURE IN RELATION TO INCOME AND WEALTH

TABLE 10.5
Hypothetical Data
on Consumption
Expenditure Y ,
Income X_2 , and
Wealth X_3

Y , \$	X_2 , \$	X_3 , \$
70	80	810
65	100	1009
90	120	1273
95	140	1425
110	160	1633
115	180	1876
120	200	2052
140	220	2201
155	240	2435
150	260	2686

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Source	SS	df	MS	Number of obs =	10
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Total	8890	9	987.777778	R-squared =	0.9635
				Adj R-squared =	0.9531
				Root MSE =	6.888

	y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x2		.9415373	.8228983	1.14	0.298	-1.984388 2.867383
x3		-.0424345	.0896645	-0.53	0.615	-.2331757 .1483867
_cons		24.77473	6.7525	3.67	0.008	8.807689 40.74186

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

$$\hat{Y}_i = 24.7747 + 0.9415X_{2i} - 0.0424X_{3i}$$

$$(6.7525) \quad (0.8229) \quad (0.0807)$$

$$t = (3.6690) \quad (1.1442) \quad (-0.5261)$$

$$R^2 = 0.9635 \quad \bar{R}^2 = 0.9531 \quad df = 10 - 3 = 7$$

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

Regression shows that income and wealth together explain about 96 % of the variation in consumption expenditure, and yet **neither of the slope coefficients is individually statistically significant**. Moreover, not only is the wealth variable statistically insignificant but also it has the wrong sign

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

TABLE 10.6
ANOVA Table for
the Consumption-
Income-Wealth
Example

Source of Variation	SS	df	MSS
Due to regression	8,565.5541	2	4,282.7770
Due to residual	324.4459	7	46.3494

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

$$H_0 = \beta_2 = \beta_3 = 0$$

$$F = \frac{4282.7770}{46.3494} = 92.4019$$

Reject the null hypothesis
(92.4019 > Critical F-value)

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

This example shows dramatically what multicollinearity does. The fact that the F test is significant but the t values of X_2 and X_3 are individually insignificant means that the two variables are so highly correlated that it is impossible to isolate the individual impact of either income and wealth on consumption

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

$$\hat{X}_{3i} = 7.5454 + 10.1909X_{2i}$$

Source	SS	df	MS	Number of obs = 10		
Model	3427202.73	1	3427202.73	F(1, 8) =	3849.02	
Residual	7123.27273	8	890.409091	Prob > F =	0.0000	
Total	3434326	9	381591.778	R-squared =	0.9979	
				Adj R-squared =	0.9977	
				Root MSE =	29.84	
	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x3	10.19091	.1842623	62.04	0.000	9.81212	10.5697
x2_cons	7.545455	29.47581	0.26	0.804	-60.42589	75.5168

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

$$\hat{Y}_i = 24.4545 + 0.5091X_{2i}$$

Source	SS	df	MS	Number of obs =	19
Model	8552.72727	1	8552.72727	F(1, 8) =	292.87
Residual	337.272727	8	42.1590909	Prob > F	= 0.0000
Total	8890	9	987.777778	R-squared	= 0.9621
				Adj R-squared	= 0.9573
				Root MSE	= 6.493

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x2	.5090909	.0357428	14.24	0.000	.4266678 .591514
_cons	24.45455	6.413817	3.81	0.005	9.664256 39.24483

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

$$\hat{Y}_i = 24.411 + 0.0498X_{3i}$$

Source	SS	df	MS	Number of obs =	19
Model	8504.87666	1	8504.87666	F(1, 8) =	176.67
Residual	385.123344	8	48.1404181	Prob > F	= 0.0000
Total	8890	9	987.777778	R-squared	= 0.9567
				Adj R-squared	= 0.9513
				Root MSE	= 6.9383

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x3	.0497638	.003744	13.29	0.000	.0411301 .0583974
_cons	24.41104	6.874097	3.55	0.007	8.559349 40.26274

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

Regressions show very clearly that in situations of extreme multicollinearity dropping the highly collinear variable will often make the other X variable statistically significant.
This result would suggest that a way out of extreme collinearity is to drop the collinearity variable.

EE 325 (Ajarn Kaewkwan
Tangtipongkul)

SOURCE

Gujarati, D.N. (2009) Basic Econometrics. 5th ed. Singapore, McGraw-Hill.

EE 325 (Ajarn Kaewkwan
Tangtipongkul)