Assignment5.R

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Sun May 10 23:44:51 2020

```
#EE435 Wasin Siwasarit  #Assignment5
#Step1: You need to set your current directory
setwd("/Users/wasin_siwasarit/Desktop/EE435")
library(fBasics)
```

```
## Loading required package: timeDate
```

```
## Warning in as.POSIXlt.POSIXct(Sys.time()): unknown timezone 'zone/tz/2019c.
## 1.0/zoneinfo/Asia/Bangkok'
```

```
## Loading required package: timeSeries
```

```
library(timeDate)
library(timeSeries)
library(fGarch)
library(quantmod)
```

```
## Warning: package 'quantmod' was built under R version 3.4.4
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
## Warning: package 'zoo' was built under R version 3.4.4
```

```
##
## Attaching package: 'zoo'
```

```
## The following object is masked from 'package:timeSeries':
##
##     time<-
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: TTR
```

```
##
## Attaching package: 'TTR'
```

```
## The following object is masked from 'package:fBasics':
##
##     volatility
```

```
## Version 0.4-0 included new data defaults. See ?getSymbols.
```

```
# Step2: You need to clear the previous work
cat(rep("\n",50))  #clear R Console
```

```
getSymbols("CAT",from="2006-01-03",to="2017-04-14")
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

We can download the data by using this symbol

```
##
## WARNING: There have been significant changes to Yahoo Finance data.
## Please see the Warning section of '?getSymbols.yahoo' for details.
##
## This message is shown once pe                                  tting
## options("getSymbols.yahoo.war
```

Question1.

Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 13, 2017.You may download the data using quantmod. Let r_t be the log returns, which can be obtained via

rt <- diff(log(as.numeric(CAT[,6])))

(a) Are there any serial correlations in the log return series r_t ? Why?

(b) Are there any ARCH effects in the log return series r_t (the linear depedence of squared returns)? Why?

(c) Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

(d) Build a GARCH(1,1) model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

(e) Write down the fitted model.

```
## [1] "CAT"
```

```
head(CAT)
```

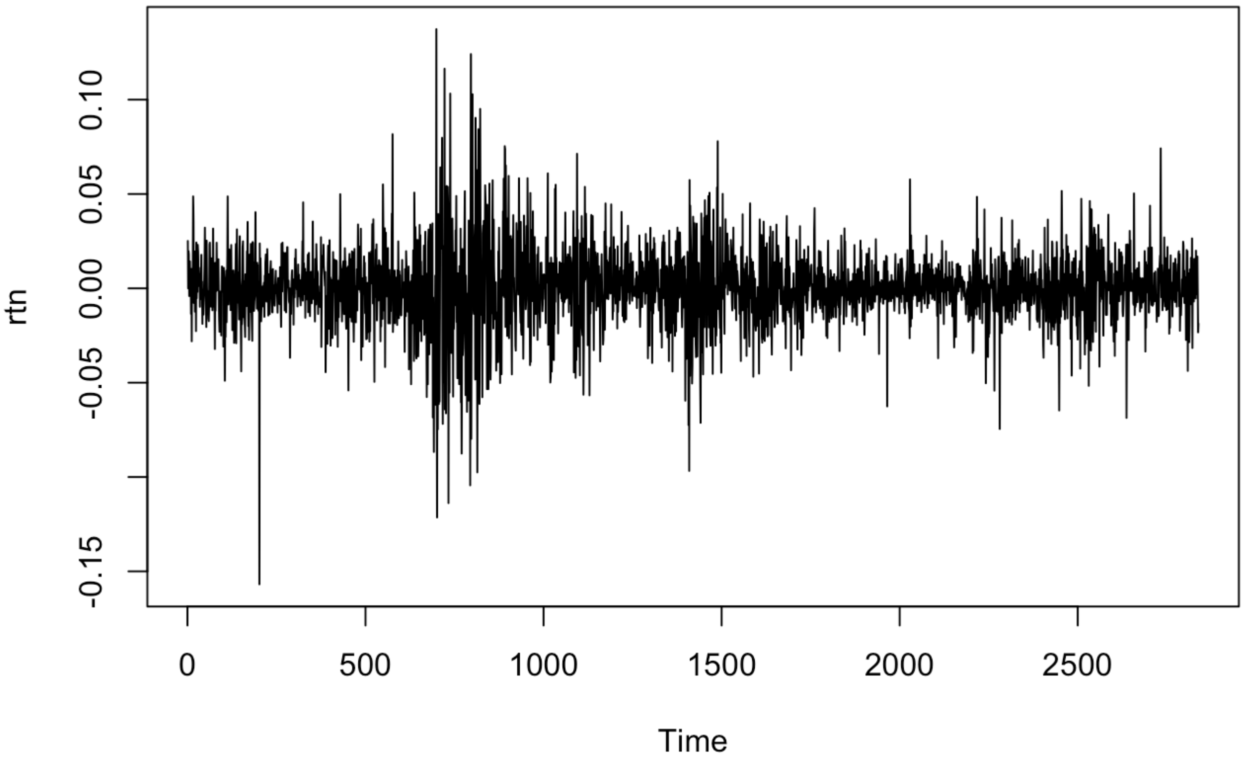
```
##            CAT.Open CAT.High CAT.Low CAT.Close CAT.Volume CAT.Adjusted
## 2006-01-03    57.87    58.11   57.05     57.80    3697500     38.23490
## 2006-01-04    57.95    59.43   57.55     59.27    4577200     39.20730
## 2006-01-05    59.02    59.86   59.00     59.27    4590700     39.20730
## 2006-01-06    59.47    60.76   59.38     60.45    5692300     39.98786
## 2006-01-09    60.45    61.68   60.45     61.55    4408800     40.71552
## 2006-01-10    61.35    61.52   60.64     61.30    3188100     40.55015
```

```
tail(CAT)
```

```
##            CAT.Open CAT.High CAT.Low CAT.Close CAT.Volume CAT.Adjusted
## 2017-04-06    94.42    96.42   94.26     95.82    5421600     87.78250
## 2017-04-07    95.87    96.62   95.42     95.52    4463800     87.50767
## 2017-04-10    96.31    97.89   96.10     97.14    5168300     88.99176
## 2017-04-11    97.30    97.31   95.67     97.10    4250400     88.95512
## 2017-04-12    96.69    96.77   94.66     94.86    4773900     86.90301
## 2017-04-13    94.60    94.86   93.09     93.10    4706000     85.29064
```

```
rtn =diff(log(as.numeric(CAT[,6])))
ts.plot(rtn)
```

Calculate the log-return




```
Box.test(rtn,lag=10,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  rtn
## X-squared = 16.045, df = 10, p-value = 0.09837
```

(a) Are there any serial correlations in the log return series r_t ? Why?

Ans: No, the Ljung-Box statistics give Q(10) = 16.045 with p-value 0.093#

```
t.test(rtn)
```

```
##
##  One Sample t-test
##
## data:  rtn
## t = 0.7338, df = 2838, p-value = 0.4631
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.0004725469  0.0010377572
## sample estimates:
##    mean of x
## 0.0002826052
```

Test $\mu_r = 0$?

$H_0: \mu_r = 0$

$H_1: \mu_r \neq 0$ (mean of $r_t \neq 0$?)

```
Box.test(rtn^2,lag=10,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  rtn^2
## X-squared = 917.33, df = 10, p-value < 2.2e-16
```

(b) Are there any ARCH effects in the log return series r_t (the linear depedence of squared returns)? Why?

Ans: Yes, the Ljung-Box statistics of r_t^2 give $Q(10) = 917.33$ with p-value close to zero #

```
m1 = garchFit(~garch(1,1),data=rtn,trace=F)
summary(m1)
```

We first estimate the GARCH(1,1) with normal distribution

(c) Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

you can estimate the AR(1)~GARCH(1,1) as well.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rtn, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb0eef0a0>
##  [data = rtn]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 4.9750e-04  4.4051e-06  4.9288e-02  9.3927e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.975e-04   3.075e-04    1.618 0.105726
## omega  4.405e-06   1.254e-06    3.513 0.000443 ***
## alpha1 4.929e-02   8.065e-03    6.112 9.87e-10 ***
## beta1  9.393e-01   1.012e-02   92.773  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7380.076    normalized:  2.599534
##
## Description:
##  Sun May 10 23:44:55 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3250.199  0
##  Shapiro-Wilk Test  R    W      0.9666186 0
##  Ljung-Box Test     R    Q(10)  13.62634  0.1907276
##  Ljung-Box Test     R    Q(15)  15.98849  0.3828173
##  Ljung-Box Test     R    Q(20)  20.59999  0.4210032
##  Ljung-Box Test     R^2  Q(10)  1.004114  0.9998246
##  Ljung-Box Test     R^2  Q(15)  3.702247  0.9985607
##  Ljung-Box Test     R^2  Q(20)  7.015463  0.996634
##  LM Arch Test       R    TR^2   2.746118  0.9970864
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
```

The fitted model is

mean equation:

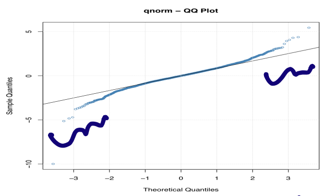
$\hat{r}_t = 4.975\times10^{-4}$
(3.075×10^{-4})

$a_t = {}_t\varepsilon_t$, $\varepsilon_t \overset{iid}{\sim} N(0,1)$

Volatility equation:

$\sigma_t^2 = 4.41\times10^{-6} + 0.049\,a_{t-1}^2 + 0.9396\,{}_{t-1}^{2}$
(1.254×10^{-6}) (0.081) (0.010)

```
## -5.196249 -5.187864 -5.196253 -5.193225
```



fat-tail

```
#plot(m1)

m2 = garchFit(~garch(1,1),data=rtn,cond.dist="std",trace=F)
summary(m2)
```

(d) Build a GARCH(1,1) model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

(e) Write down the fitted model.

Ans: As seen, with the QQ-plot, the model is not adequate as the normality assumption is rejected.

with this result, we then re-estimate the model again with standardized student-t.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb1ae5070>
##  [data = rtn]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9822e-04  4.1699e-06  7.2016e-02  9.2078e-01  5.0886e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.982e-04   2.702e-04    2.214  0.02680 *
## omega  4.170e-06   1.569e-06    2.658  0.00785 **
## alpha1 7.202e-02   1.372e-02    5.248 1.53e-07 ***
## beta1  9.208e-01   1.471e-02   62.586  < 2e-16 ***
## shape  5.089e+00   4.817e-01   10.565  < 2e-16
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.(
##
## Log Likelihood:
##  7509.962    normalized:  2.645284
##
## Description:
##  Sun May 10 23:44:55 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4042.192  0
##  Shapiro-Wilk Test  R    W      0.963916  0
##  Ljung-Box Test     R    Q(10)  14.73913  0.1418661
##  Ljung-Box Test     R    Q(15)  16.77026  0.3327826
##  Ljung-Box Test     R    Q(20)  20.66546  0.4170529
##  Ljung-Box Test     R^2  Q(10)  2.931617  0.9829878
##  Ljung-Box Test     R^2  Q(15)  5.456872  0.9875005
##  Ljung-Box Test     R^2  Q(20)  9.467254  0.9768363
##  LM Arch Test       R    TR^2   4.245483  0.9785882
##
```

e Write down the fitted model:

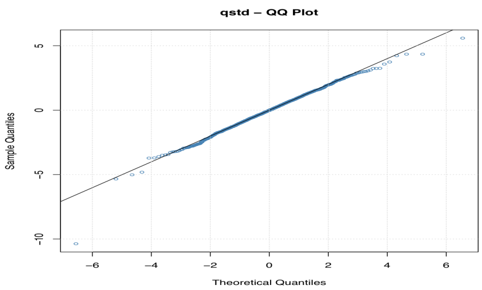
mean equation:

$\hat{r}_t = 5.98 \times 10^{-4}$
(2.70×10^{-4})

$a_t = \sigma_t \varepsilon_t$, $\varepsilon_t \overset{iid}{\sim} t_{5.1}$

$\sigma_t^2 = 4.17 \times 10^{-6} + 0.072 a_{t-1}^2 + 0.926 \sigma_{t-1}^2$
(1.57×10^{-6}) (0.014) (0.015)

We plot the QQ-plot again, the QQ-plot is shown as:



the model seems to be adequate.

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.287046 -5.276565 -5.287052 -5.283265
```

```
pm2 = predict(m2,5)
pm2
```

(f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted GARCH(1,1) model with standardized Student-t innovations.

```
##   meanForecast  meanError standardDeviation
## 1 0.0005982247 0.01557861        0.01557861
## 2 0.0005982247 0.01565614        0.01565614
## 3 0.0005982247 0.01573273        0.01573273
## 4 0.0005982247 0.01580840        0.01580840
## 5 0.0005982247 0.01588318        0.01588318
```

standardized student-t

```
qstd(0.975,nu=5.1)
```

(g) Compute the 95 % 1-step to 5-step interval predictions of the log return series using standardized student-t innovations.

```
## [1] 1.992374
```

find out the t critical value, $\frac{0.05}{2}$ with df=5.1

```
lcl = pm2$meanForecast-1.99*pm2$standardDeviation
ucl = pm2$meanForecast+1.99*pm2$standardDeviation
CI = cbind(lcl,ucl)
CI
```

```
##              lcl        ucl
## [1,] -0.03040321 0.03159965
## [2,] -0.03055749 0.03175394
## [3,] -0.03070991 0.03190636
## [4,] -0.03086050 0.03205695
## [5,] -0.03100929 0.03220574
```

∴ the 95% CI predictions are

```
#Question2
da = read.table("m-kovw-5116.txt",header=T)
dim(da)
```

Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file m-kovw-5116.txt. Obtain the log return series of KO stock.

```
## [1] 780   4
```

(a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

```
ko =log(da$ko+1)
t.test(ko)
```

Change the simple return to be log return.

(a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

```
##
##  One Sample t-test
##
## data:  ko
## t = 4.9853, df = 779, p-value = 7.628e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
## sample estimates:
##  mean of x
## 0.01031992
```

Ans: $H_0: E(r_t) = 0$

$H_1: E(r_t) \neq 0$

since p-value = 7.67e-07 (p-value close to zero) ∴ the expected return is different from zero.

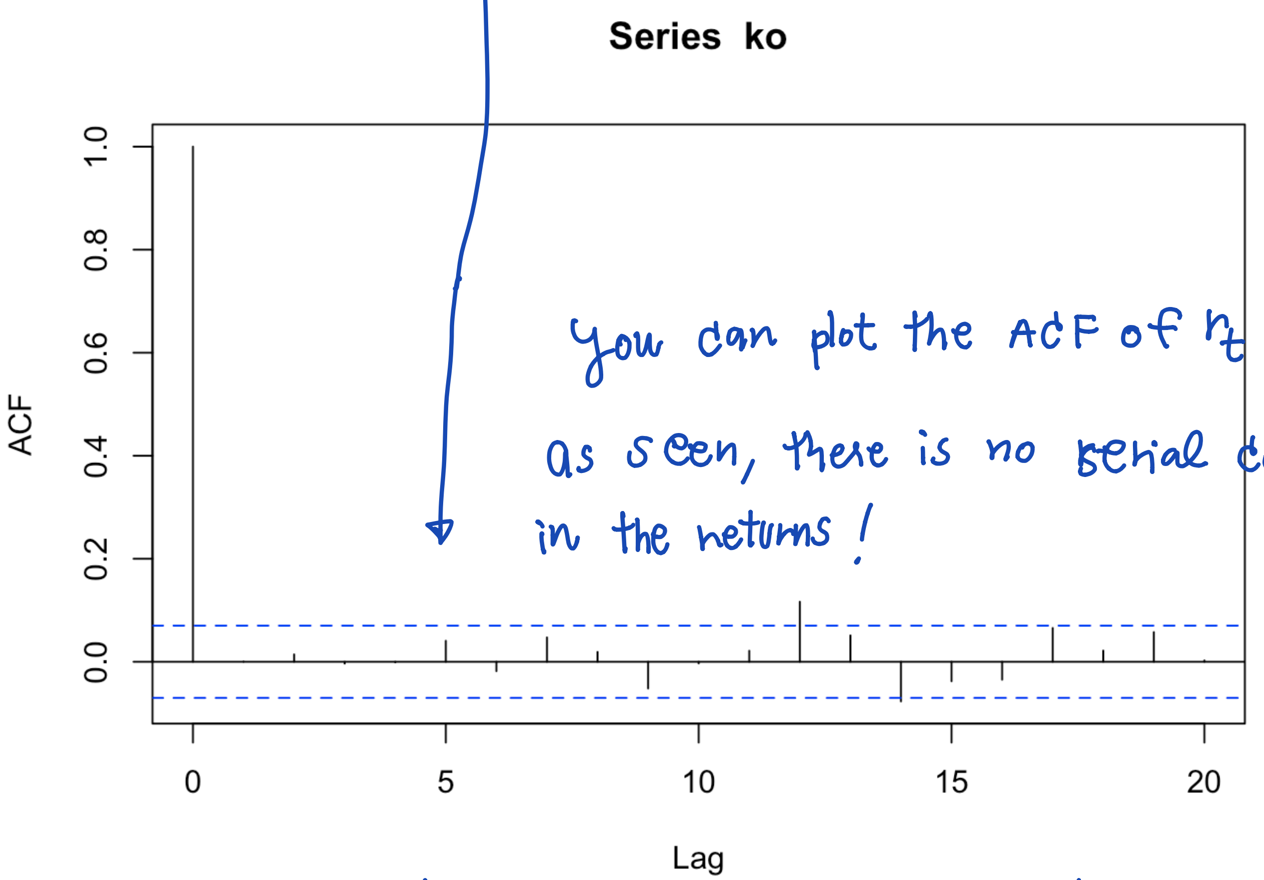
```
Box.test(ko,lag=12,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  ko
## X-squared = 17.06, df = 12, p-value = 0.1474
```

Ans: Since p-value = 0.1474

∴ No, there is no serial correlations in the returns.

```
acf(ko,lag.max = 20)
```



You can plot the ACF of r_t, as seen, there is no serial correlation in the returns!

∴ the mean equation can be

$r_t = c_0 + a_t$

constant term = mean of r_t

```
at = ko-mean(ko)
Box.test(at^2,lag=12,type="Ljung")
```

$\therefore \hat{a}_t = r_t - c_0$

Check the ARCH effect

```
##
##  Box-Ljung test
##
## data:  at^2
## X-squared = 255.99, df = 12, p-value < 2.2e-16
```

H_0: There is no ARCH effect

H_1: There is ARCH effect

Since p-value is very close to zero, therefore there is ARCH effect with 95% CI

```
m1 = garchFit(~arma(1,0)+garch(1,1),data=ko,trace=F)
summary(m1)
```

(b) Build a AR(1)-GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = ko, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fbbb352b780>
##  [data = ko]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Sun May 10 23:44:55 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
```

The fitted model is

mean equation:

$$\hat{r}_t = \underset{(0.001)}{0.011} - \underset{(0.039)}{0.026}\, r_{t-1}$$

insignificant ! as expected since there is no serial correlat in r_t

$$a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} N(0,1)$$

volatility equation:

$$\sigma_t^2 = 1.81 \times 10^{-4} + 0.095\, a_{t-1}^2 + 0.8496\, \sigma_{t-1}^2$$

① ✓ ② ✓ with the Ljung-box test, both mean equation and volatility equation are adequate

③ but if we plot the qq-plot, the normality assumption is clearly rejected.

with 1, 2, 3, the model is not adequate.

```
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

```
m2 = garchFit(~arma(1,0)+garch(1,1),data=ko,cond.dist="std",trace=F)
summary(m2)
```

(c) Fit a AR(1)-GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = ko, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x7fbbb1919378>
##  [data = ko]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124020  -0.01887601   0.00017395   0.09642928   0.85044150
##       shape
##  7.47877780
```

The fitted model is:

mean eq:

$$\hat{r}_t = \underset{(0.002)}{0.011} - \underset{(0.037)}{0.019}\, r_{t-1}$$

```
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1   -1.888e-02   3.691e-02   -0.511  0.60904
## omega  1.739e-04   6.596e-05    2.637  0.00836 **
## alpha1 9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1  8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape  7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} t_{7.479}$$

Volatility eq:

$$\sigma_t^2 = \underset{(0.6596\times e^{-05})}{1.739\times e^{-04}} + \underset{(0.023)}{0.096}\, a_{t-1}^2 + \underset{(0.033)}{0.8506}\, \sigma_{t-1}^2$$

```
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021
```

1 OK

2 OK

```
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816988
##  LM Arch Test       R    TR^2   10.48089  0.5738501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```

③ With the QQ-plot, the model seems to be adequate.

```
m3 = garchFit(~garch(1,1),data=ko,trace=F)
summary(m3)
```

(d) Build a GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

↳ we can conclude that the model is not adequate as the normality assumption is rejected (qqplot)

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = ko, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb06dc208>
##  [data = ko]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
```

```
## -2.990752 -2.966858 -2.990804 -2.981562
```

```
m4 = garchFit(~garch(1,1),data=ko,cond.dist="std",trace=F)
summary(m4)
```

Again, the model is adequate.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = ko, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb21b05c0>
##  [data = ko]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
```

(f) Compare the model (b)-(e) which model you select.

Lo

Compare m_1, m_2, m_3, m_4

We select m_4 model

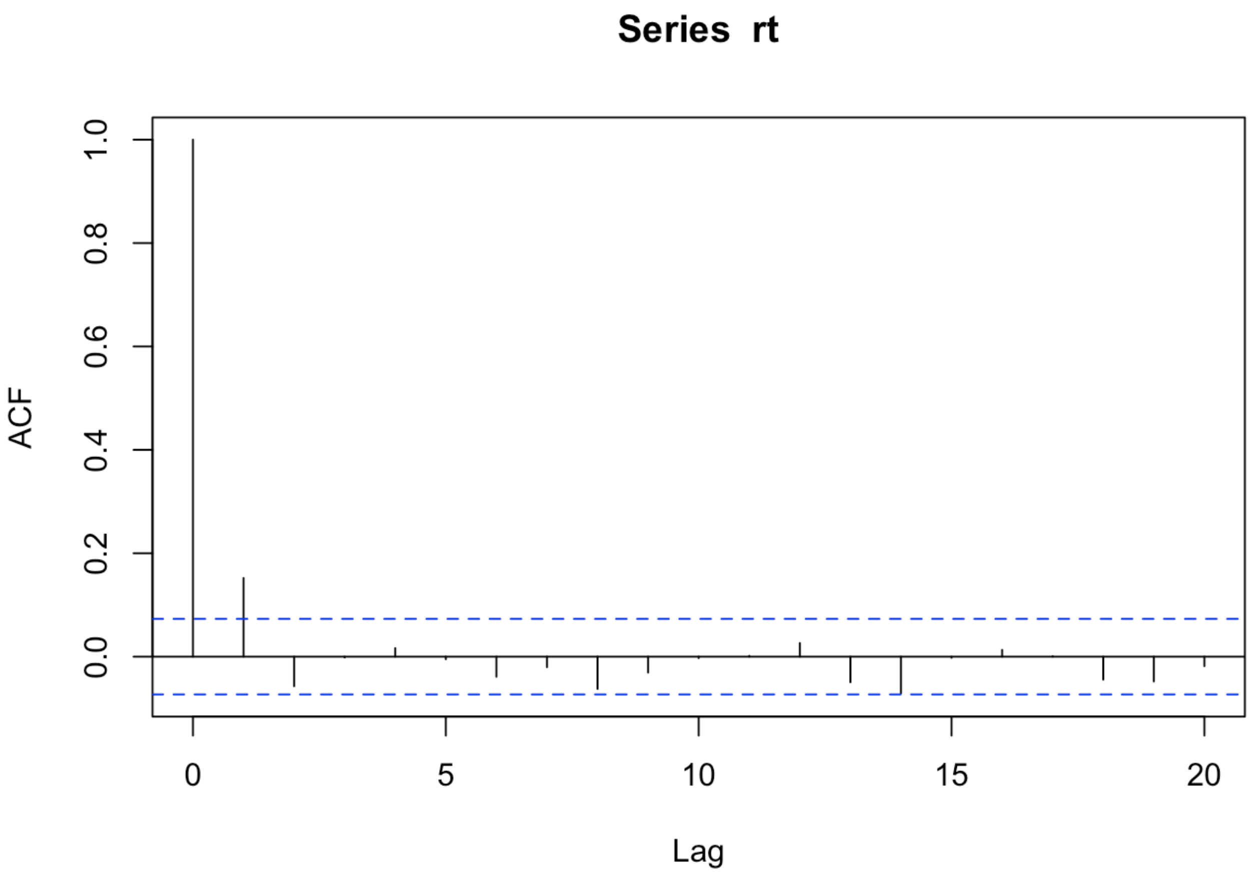
with the smallest BIC

and model is adequate.

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

You can check the code for Q_4 by yourself.

```
#Question4
# You can download the data set from the folder "EE435" if your current folder
is named "EE435"
da=read.table("m-deciles.txt",header =T )
#Select the column you would like to estimate, in this example, it is the colum
n with the name "CAP9RET"
simple_return = da$CAP9RET
# Transform the simple return to be log return
rt =log(1+simple_return)
acf(rt,lag.max = 20)
```




```
t.test(rt)
```

```
##
##  One Sample t-test
##
## data:  rt
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054
```

```
Box.test(rt,lag=12,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  rt
## X-squared = 24.768, df = 12, p-value = 0.01596
```

```
at = rt-mean(rt)
Box.test(at^2,lag=10,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  at^2
## X-squared = 20.53, df = 10, p-value = 0.02462
```

```
m1 = garchFit(~arma(1,0)+garch(1,0),data=rt,trace=F)
summary(m1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x7fbbb0ccfab0>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      mu      ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1    0.1470670   0.0424301    3.466 0.000528 ***
## omega  0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
```

```
## -3.207301 -3.181861 -3.207363 -3.197480
```

```
m2 = garchFit(~arma(1,0)+garch(1,0),data=rt,cond.dist="std",trace=F)
summary(m2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x7fbbb0901b88>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu        ar1      omega     alpha1      shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115897    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
##
```

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

```
m3 = garchFit(~garch(1,0),data=rt,trace=F)
summary(m3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = rt, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7fbbb353cb90>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

```
m4 = garchFit(~garch(1,1),data=rt,cond.dist="std",trace=F)
summary(m4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb30e6000>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 0.01253321  0.00013013  0.11435407  0.83665662  6.46000315
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.253e-02   1.538e-03    8.150 4.44e-16 ***
## omega  1.301e-04   5.817e-05    2.237  0.02528 *
## alpha1 1.144e-01   3.494e-02    3.273  0.00107 **
## beta1  8.367e-01   4.591e-02   18.225  < 2e-16 ***
## shape  6.460e+00   1.315e+00    4.913 8.97e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1201.367    normalized:  1.668565
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1171.213  0
##  Shapiro-Wilk Test  R    W      0.9457868 1.420853e-15
##  Ljung-Box Test     R    Q(10)  16.52228  0.08562605
##  Ljung-Box Test     R    Q(15)  20.6328   0.1489773
##  Ljung-Box Test     R    Q(20)  25.48915  0.183354
##  Ljung-Box Test     R^2  Q(10)  2.235555  0.9941952
##  Ljung-Box Test     R^2  Q(15)  4.909806  0.9928629
##  Ljung-Box Test     R^2  Q(20)  5.757538  0.9991875
##  LM Arch Test       R    TR^2   2.506995  0.9981363
##
```

Assignment 6 : Question 3

```
## Information Criterion Statistics:
##        AIC       BIC       SIC      HQIC
## -3.323241 -3.291441 -3.323337 -3.
```

Question3.

Consider the monthly returns of McDonalds stock from August 1966 to December 2014. The data are available from CRSP and in the file m-mcd3dx6614.txt. Obtain the log return series of MCD stock.

3.1 Is the expected MCD log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

```
source("Igarch.R")
da=read.table("m-mcd3dx6614.txt",header=T)
head(da)
```

```
##   PERMNO     date       mcd    vwretd    ewretd    sprtrn
## 1  43449 19660831 -0.203390 -0.075388 -0.100714 -0.077751
## 2  43449 19660930 -0.106383 -0.006958 -0.018161 -0.007004
## 3  43449 19661031 -0.047619  0.042294 -0.014385  0.047544
## 4  43449 19661130  0.018750  0.017516  0.040725  0.003117
## 5  43449 19661230  0.386503  0.006210  0.017306 -0.001492
## 6  43449 19670131  0.212389  0.085530  0.172104  0.078178
```

```
mcd=log(da$mcd+1)
t.test(mcd)
```

$H_0: E[r_t] = 0$

$H_1: E[r_t] \neq 0$

with the p-value 1.59×10^{-5}, indicating that the mean is not zero.

```
##
##  One Sample t-test
##
## data:  mcd
## t = 4.3532, df = 580, p-value = 1.586e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.007509528 0.019856397
## sample estimates:
##  mean of x
## 0.01368296
```

```
Box.test(mcd,lag=12,type='Ljung')
```

The Ljung-Box statistics of the returns show $Q(12) = 12.69$ with p-value = 0.39

∴ there is no serial correlation in the returns.

```
##
##  Box-Ljung test
##
## data:  mcd
## X-squared = 12.693, df = 12, p-value = 0.3918
```

```
at=mcd-mean(mcd)
Box.test(at^2,lag=12,type='Ljung')
```

The Ljung-Box statistics of the squared residuals show $Q(12) = 157.99$, which is significant.

∴ There is the ARCH effects in the returns.

```
##
##  Box-Ljung test
##
## data:  at^2
## X-squared = 157.99, df = 12, p-value < 2.2e-16
```

```
n1=garchFit(~garch(1,1),data=mcd,trace=F)
summary(n1)
```

3.2 Build a GARCH model with Gaussian innovations for the log return series. Per- form model checking and write down the fitted model.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = mcd, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb1c13f58>
##  [data = mcd]
##
## Conditional Distribution:
##  norm
##
```

the fitted model is

$\hat{r}_t = \underset{(0.003)}{0.012}$

$a_t = \sigma_t \varepsilon_t \,, \quad \varepsilon_t \overset{iid}{\sim} N(0,1)$

```
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.2156e-02  4.9595e-05  8.9200e-02  9.0242e-01
##
## Std. Errors:
##  based on Hessian
##
```

$\sigma_t^2 = \underset{(4.699\times10^{-5})}{4.96\times10^{-5}} + \underset{(0.026)}{0.089}\, a_{t-1}^2 + \underset{(0.028)}{0.9026}\, \sigma_{t-1}^2$

```
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.216e-02   2.534e-03    4.797 1.61e-06 ***
## omega  4.960e-05   4.699e-05    1.056 0.291177
## alpha1 8.920e-02   2.561e-02    3.483 0.000496 ***
## beta1  9.024e-01   2.785e-02   32.406  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  728.4167    normalized:  1.253729
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  34.04279  4.052308e-08
##  Shapiro-Wilk Test  R    W      0.9880677 0.0001116285
##  Ljung-Box Test     R    Q(10)  8.955288  0.5363521
##  Ljung-Box Test     R    Q(15)  10.16644  0.809142
##  Ljung-Box Test     R    Q(20)  13.90125  0.8354673
##  Ljung-Box Test     R^2  Q(10)  5.007499  0.8906765
##  Ljung-Box Test     R^2  Q(15)  6.432461  0.9715326
##  Ljung-Box Test     R^2  Q(20)  9.453338  0.9770369
##  LM Arch Test       R    TR^2   4.794391  0.9644961
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
```

However, QQ-plot for the standardized residuals for GARCH (1,1) with normal distribution shows that the model is not adequate.

3.3 Fit an IGARCH(1,1) model for the MCD log returns. Write down the fitted model.

```
## -2.493689 -2.463639 -2.493783 -2.481975
```

```
n2=Igarch(mcd,include.mean=T)
```

```
## Estimates:  0.01159262 0.9260433
## Maximized log-likehood:  -727.3589
##
## Coefficient(s):
##         Estimate  Std. Error  t value   Pr(>|t|)
## mu    0.01159262  0.00250469  4.62836 3.6856e-06 ***
## beta 0.92604332  0.01571166 58.93987 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The fitted model for IGARCH(1,1) is:

$\hat{r}_t = 0.012$
(0.003)

$a_t = \sigma_t \cdot \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} N(0,1)$

$\sigma_t^2 = (1-0.926)a_{t-1}^2 + 0.926\sigma_{t-1}^2$
$(\;-\;) \quad (0.0157)$

```
n3=garchFit(~garch(1,1),data=mcd,trace=F,cond.dist="sstd")
summary(n3)
```

3.4 Fit a GARCH model with skew-Student-t innovations to the log return series. Perform model checking and write down the fitted model.

3.5 Based on the fitted model, is the monthly log returns of MCD stock skewed? Why?

```
##
## Title:
##  GARCH Modellin
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = mcd, cond.dist = "sstd",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb12af8d0>
##  [data = mcd]
##
## Conditional Distribution:
##  sstd
##
## Coefficient(s):
##         mu       omega      alpha1       beta1        skew       shape
## 1.2030e-02  7.4863e-05  9.7026e-02  8.9262e-01  8.3331e-01  1.0000e+01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.203e-02   2.562e-03    4.696 2.66e-06 ***
## omega  7.486e-05   7.091e-05    1.056 0.291114
## alpha1 9.703e-02   3.194e-02    3.037 0.002386 **
## beta1  8.926e-01   3.601e-02   24.790  < 2e-16 ***
## skew   8.333e-01   5.069e-02   16.441  < 2e-16 ***
## shape  1.000e+01   3.005e+00    3.328 0.000875 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  736.3457    normalized:  1.267376
##
## Description:
##  Sun May 10 23:44:56 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  35.86292  1.631041e-08
##  Shapiro-Wilk Test  R    W      0.9877696 8.812423e-05
##  Ljung-Box Test     R    Q(10)  9.098441  0.5227874
##  Ljung-Box Test     R    Q(15)  10.44539  0.7908282
##  Ljung-Box Test     R    Q(20)  14.30697  0.8145971
##  Ljung-Box Test     R^2  Q(10)  5.265189  0.8727737
##  Ljung-Box Test     R^2  Q(15)  6.738114  0.9645136
##  Ljung-Box Test     R^2  Q(20)  9.89765   0.9699901
##  LM Arch Test       R    TR^2   5.237744  0.9495629
```

Using this command for the skew-student t

The fitted model is:

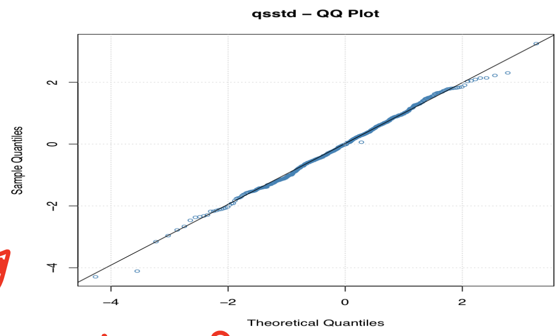
$\hat{r}_t = 0.012$
(0.003)

$a_t = \sigma_t \varepsilon_t$, $\varepsilon_t \sim t^*_{10}(0.833)$

$\sigma_t^2 = 7.49\times10^{-5} + 0.097 a_{t-1}^2 + 0.8936 \sigma_{t-1}^2$
(7.091×10^{-5}) (0.032) (0.036)

✓ ✓

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC       HQI
## -2.514099 -2.469024 -2.514309 -2.49652
```




```
#plot(n3)
tt=(.8333-1)/.0507
tt
```

```
## [1] -3.287968
```

QQ-plot for the standardized residuals for GARCH(1,1) model with skew Student-t innovations fitted to the monthly log returns of McDonald's stock

```
source("garchM.R")
n4=garchM(mcd)
```

```
## Maximized log-likehood:  729.8678
##
## Coefficient(s):
##         Estimate  Std. Error  t value  Pr(>|t|)
## mu    4.00381e-03 4.09801e-03  0.97701 0.32856266
## gamma 1.66190e+00 8.37012e-01  1.98551 0.04708765 *
## omega 4.84952e-05 4.93846e-05  0.98199 0.32610394
## alpha 8.91997e-02 2.43178e-02  3.66808 0.00024438 ***
## beta  9.02421e-01 2.69120e-02 33.53228 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Ans: Model checking indicates the model is adequate. The skew parameter is highly significant! As a matter of fact, QQ-plot of student t innovations indicates that skew distribution is needed.

```
n5=garchFit(~garch(1,1),data=mcd,trace=F,leverage=T)
summary(n5)
```

3.5 Based on the fitted model, is the monthly log returns of MCD stock skewed? Why?

3.6 Fit a GARCM-M model to the monthly log returns. Write down the model? Is the risk premium statistically significant? Why?

Ans: The fitted GARCH-M model is:

$$\hat{r}_t = \underset{(4.098\times10^{-3})}{0.004} + \underset{(0.837)}{1.6626}\,\sigma_t^2$$

$$a_t = \sigma_t \varepsilon_t , \quad \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = 4.85\times10^{-5} + 0.089\, a_{t-1}^2 + 0.9026\, \sigma_{t-1}^2$$

(4.938×10^{-5}) (0.024) (0.269)

The risk premium is significant at 5% because the t-ratio has a p-value of 0.047 which is less than $\alpha = 0.05$.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = mcd, leverage = T, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb3604908>
##  [data = mcd]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      gamma1       beta1
## 1.1633e-02  7.7137e-05  9.6154e-02  1.1872e-01  8.8874e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.163e-02   2.569e-03    4.528 5.97e-06 ***
## omega  7.714e-05   6.460e-05    1.194 0.232430
## alpha1 9.615e-02   2.726e-02    3.528 0.000419 ***
## gamma1 1.187e-01   8.987e-02    1.321 0.186478
## beta1  8.887e-01   3.252e-02   27.327  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  729.3407    normalized:  1.25533
##
## Description:
##  Sun May 10 23:44:58 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  29.76947  3.432749e-07
##  Shapiro-Wilk Test  R    W      0.9892262 0.0002866107
##  Ljung-Box Test     R    Q(10)  9.133716  0.5194623
##  Ljung-Box Test     R    Q(15)  10.37262  0.7956695
##  Ljung-Box Test     R    Q(20)  14.18361  0.8210657
##  Ljung-Box Test     R^2  Q(10)  4.353095  0.9300158
##  Ljung-Box Test     R^2  Q(15)  5.664229  0.9848529
##  Ljung-Box Test     R^2  Q(20)  9.055389  0.9822568
##  LM Arch Test       R    TR^2   4.450539  0.9738684
##
## Information Criterion Statistics:
```

3.7 Fit a TGARCH(1,1) model to the monthly log returns. Write down the fitted model. Is the leverage effect statistically significant? Why?

For TGARCH(1,1), the fitted model is:

mean eq:

$$\hat{r}_t = \underset{(0.0026)}{0.0116}$$

$$a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

Volatility eq:

$$\sigma_t^2 = \underset{(6.46\times10^{-5})}{7.714\times10^{-5}} + (\underset{(0.027)}{0.096} + \underset{(0.119)}{0.119} I_{t-1}) a_{t-1}^2 + \underset{(0.033)}{0.8896}\,\sigma_{t-1}^2$$

Indicator function

$I_{t-1} = 1$ if $a_{t-1} < 0$

$I_{t-1} = 0$ if $a_{t-1} > 0$

The leverage effect is not significant at 5%.

```
##        AIC       BIC       SIC      HQIC
## -2.493428 -2.455865 -2.493574 -2.478784
```

```
n7=garchFit(~garch(1,1),data=mcd,trace=F,leverage=T,cond.dist="sstd")
summary(n7)
```

∴ It doesn't have the leverage effect.

3.7 Fit a TGARCH(1,1) model to the monthly log returns. Write down the fitted model. Is the leverage effect statistically significant? Why?

We can reestimate again TGARCH(1,1) with skew student-t innovation.

The conclusion does not change if we use this skew student-t innovation.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = mcd, cond.dist = "sstd",
##     leverage = T, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb1b50038>
##  [data = mcd]
##
## Conditional Distribution:
##  sstd
##
## Coefficient(s):
##         mu       omega      alpha1      gamma1       beta1        skew
## 1.1881e-02  8.9897e-05  1.0184e-01  5.1258e-02  8.8512e-01  8.3312e-01
##      shape
## 1.0000e+01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.188e-02   2.584e-03    4.598 4.27e-06 ***
## omega  8.990e-05   8.392e-05    1.071 0.284077
## alpha1 1.018e-01   3.398e-02    2.998 0.002722 **
## gamma1 5.126e-02   9.954e-02    0.515 0.606592
## beta1  8.851e-01   4.034e-02   21.944  < 2e-16 ***
## skew   8.331e-01   5.068e-02   16.440  < 2e-16 ***
## shape  1.000e+01   2.988e+00    3.346 0.000819 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  736.4794    normalized:  1.267607
##
## Description:
##  Sun May 10 23:44:58 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  33.53332  5.227963e-08
##  Shapiro-Wilk Test  R    W      0.9883878 0.0001443072
##  Ljung-Box Test     R    Q(10)  9.192195  0.513966
##  Ljung-Box Test     R    Q(15)  10.55198  0.7836595
##  Ljung-Box Test     R    Q(20)  14.46827  0.8059826
##  Ljung-Box Test     R^2  Q(10)  4.871001  0.8996269
```

```
##  Ljung-Box Test     R^2  Q(15)  6.279247  0.9746656
##  Ljung-Box Test     R^2  Q(20)  9.639969  0.9742404
##  LM Arch Test       R    TR^2   4.992423  0.9582316
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.511117 -2.458529 -2.511402 -2.490616
```

Q4

```
vw=log(da$vwretd+1)
t.test(vw)
```

Question4.

Consider the monthly returns of the value-weighted index, including dividends from 1966 to 20014. The simple returns are in the file m-mcd3dx6614.txt (column with heading vwretd). Transform the simple returns to log returns.

4.1 Find an adequate model for the monthly log return series. Perform model checking to justify your model.

4.2 Obtain 1-step to 5-step ahead predictions of the log return and its volatility at the forecast origin December 2014.

4.3 Fit a GJR model (may use the APARCH command) to the monthly log return series. Write down the model. Is the leverage effect statistically significant? Why?

```
##
##  One Sample t-test
##
## data:  vw
## t = 4.2134, df = 580, p-value = 2.917e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.004274232 0.011738564
## sample estimates:
##   mean of x
## 0.008006398
```

g2 is the optimal model, you can study the codes as provided.

```
Box.test(vw,lag=12,type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  vw
## X-squared = 10.874, df = 12, p-value = 0.5398
```

```
g1=garchFit(~garch(1,1),data=vw,trace=F,cond.dist="std")
summary(g1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = vw, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb1c0e348>
##  [data = vw]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##        mu      omega     alpha1      beta1      shape
## 0.0108521  0.0001307  0.1157731  0.8248038  6.8746049
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.085e-02   1.626e-03    6.676 2.46e-11 ***
## omega  1.307e-04   6.935e-05    1.884  0.05950 .
## alpha1 1.158e-01   3.615e-02    3.202  0.00136 **
## beta1  8.248e-01   5.163e-02   15.975  < 2e-16 ***
## shape  6.875e+00   1.665e+00    4.129 3.64e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1003.914    normalized:  1.727908
##
## Description:
##  Sun May 10 23:44:58 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  346.1475  0
##  Shapiro-Wilk Test  R    W      0.9591998 1.311288e-11
##  Ljung-Box Test     R    Q(10)  6.752436  0.7485915
##  Ljung-Box Test     R    Q(15)  9.888828  0.826674
##  Ljung-Box Test     R    Q(20)  12.55099  0.8958081
##  Ljung-Box Test     R^2  Q(10)  3.919834  0.9508909
##  Ljung-Box Test     R^2  Q(15)  5.915419  0.9811337
##  Ljung-Box Test     R^2  Q(20)  7.017659  0.9966267
##  LM Arch Test       R    TR^2   4.181749  0.9799288
##
```

```
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.438604 -3.401041 -3.438750 -3.423960
```

```
#plot(g1)
g2=garchFit(~garch(1,1),data=vw,trace=F,cond.dist="sstd")
summary(g2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = vw, cond.dist = "sstd",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fbbb0e41910>
##  [data = vw]
##
## Conditional Distribution:
##  sstd
##
## Coefficient(s):
##         mu       omega      alpha1       beta1        skew       shape
## 0.00897757  0.00010541  0.10940382  0.84211308  0.74324957  7.53820168
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     8.978e-03   1.659e-03    5.410 6.29e-08 ***
## omega  1.054e-04   5.392e-05    1.955 0.050581 .
## alpha1 1.094e-01   3.080e-02    3.552 0.000382 ***
## beta1  8.421e-01   4.151e-02   20.288  < 2e-16 ***
## skew   7.432e-01   4.863e-02   15.284  < 2e-16 ***
## shape  7.538e+00   2.052e+00    3.673 0.000239 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1014.613    normalized:  1.746322
##
## Description:
##  Sun May 10 23:44:58 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  323.2159  0
##  Shapiro-Wilk Test  R    W      0.9598587 1.713623e-11
##  Ljung-Box Test     R    Q(10)  6.792263  0.7449003
##  Ljung-Box Test     R    Q(15)  9.944184  0.8232361
##  Ljung-Box Test     R    Q(20)  12.43423  0.90032
##  Ljung-Box Test     R^2  Q(10)  4.03123   0.9459271
##  Ljung-Box Test     R^2  Q(15)  5.812654  0.9827257
##  Ljung-Box Test     R^2  Q(20)  6.90882   0.9969735
##  LM Arch Test       R    TR^2   4.29014   0.9776136
```

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.471990 -3.426916 -3.472201 -3.454419
```

```
predict(g2,5)
```

4.2

4.2 Obtain 1-step to 5-step ahead predictions of the log return and its volatility at the forecast origin December 2014.

```
##   meanForecast  meanError standardDeviation
## 1  0.008977569 0.03178158        0.03178158
## 2  0.008977569 0.03265735        0.03265735
## 3  0.008977569 0.03346940        0.03346940
## 4  0.008977569 0.03422418        0.03422418
## 5  0.008977569 0.03492724        0.03492724
```

```
g3=garchFit(~aparch(1,1),data=vw,trace=F,cond.dist="sstd",delta=2,include.delta
=F)
summary(g3)
```

4.3 Fit a GJR model (may use the APARCH command) to the monthly log return series. Write down the model. Is the leverage effect statistically significant? Why?

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~aparch(1, 1), data = vw, delta = 2, cond.dist = "sstd",
##     include.delta = F, trace = F)
##
## Mean and Variance Equation:
##  data ~ aparch(1, 1)
## <environment: 0x7fbbb4641dc0>
##  [data = vw]
##
## Conditional Distribution:
##  sstd
##
## Coefficient(s):
##         mu       omega      alpha1      gamma1       beta1        skew
## 0.00811271  0.00019259  0.05061437  0.98074307  0.80157030  0.73763144
##      shape
## 7.64060867
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     8.113e-03   1.685e-03    4.815 1.47e-06 ***
## omega  1.926e-04   7.264e-05    2.651 0.008019 **
## alpha1 5.061e-02   4.377e-02    1.156 0.247574
## gamma1 9.807e-01   8.091e-01    1.212 0.225449
## beta1  8.016e-01   5.035e-02   15.919  < 2e-16 ***
## skew   7.376e-01   4.719e-02   15.630  < 2e-16 ***
## shape  7.641e+00   2.075e+00    3.683 0.000231 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1021.46    normalized:  1.758107
##
## Description:
##  Sun May 10 23:44:58 2020 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  625.9286  0
##  Shapiro-Wilk Test  R    W      0.9513571 6.618149e-13
##  Ljung-Box Test     R    Q(10)  7.147308  0.7114689
##  Ljung-Box Test     R    Q(15)  9.736043  0.8360068
##  Ljung-Box Test     R    Q(20)  13.2887   0.8646566
##  Ljung-Box Test     R^2  Q(10)  3.283148  0.9739538
```

The GJR GARCH:

mean eq:

$$\hat{r}_t = \underset{(0.0017)}{0.00811}$$

$$a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim t^*_{(0.738)}\ 7.614$$

volatility eq:

$$\sigma_t^2 = \underset{(7.26\times10^{-5})}{1.926\times10^{-4}} + \underset{(0.044)}{0.051}\left(|a_{t-1}| - \underset{(0.801)}{0.981}\, a_{t-1}\right)^2 + \underset{(0.050)}{0.802}\,\sigma_{t-1}^2$$

4.3 Fit a GJR model (may use the APARCH command) to the monthly log return series. Write down the model. Is the leverage effect statistically significant? Why?

```
##  Ljung-Box Test      R^2  Q(15)  6.13536   0.9773832
##  Ljung-Box Test      R^2  Q(20)  7.559482  0.9944042
##  LM Arch Test        R    TR^2   3.501375  0.9908501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.492118 -3.439531 -3.492404 -3.471617
```

Is the leverage effect statistically significant?

Ans: No, the leverage parameter is not significant at 5%. It has a p-value of 0.23 #