

EE325 Section 1 HW 2 Due Thursday February 20<sup>th</sup> (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a.  $X_i$  is total microeconomics exam point (total points are 100) and  $Y_i$  is GPA of each student.

Table 1.a

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{Y}_i = 3.2125$$

$$\bar{X}_i = 77.625$$

1.1 Now consider the two-variable  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Use OLS to find the estimator of  $\beta_0$  and  $\beta_1$ . (Note: NIID = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2} \quad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= \frac{-0.4 \cdot -14.6 \quad 0.2 \cdot -5.6 \quad -0.1 \cdot 0.7 \quad 0.7 \cdot 3.4 \quad 0.4 \cdot 9.4 \quad -0.1 \cdot -2.1 \quad -0.5 \cdot -2.1 \quad 0.5 \cdot 12.4}{(2.8-3.2)(63-77.6) + (3.4-3.2)(72-77.6) + (3-3.2)(78-77.6) + (3.5-3.2)(81-77.6) + (3.6-3.2)(87-77.6) + (3.0-3.2)(75-77.6) + (2.7-3.2)(75-77.6) + (3.7-3.2)(90-77.6)}$$

$$(2.8-3.2)^2 + (3.4-3.2)^2 + (3-3.2)^2 + (3.5-3.2)^2 + (3.6-3.2)^2 + (3.0-3.2)^2 + (2.7-3.2)^2 + (3.7-3.2)^2$$

$$\hat{\beta}_1 = \frac{17.44}{511.84} = 0.03$$

$$\hat{\beta}_0 = 3.2 - 0.03(77.6)$$

$$\hat{\beta}_0 = 0.87$$

$\hat{\beta}_1 \neq \beta_1$

1.2 For each observation  $i$ , find  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\hat{Y}_1 = (0.87) + (0.03)(63) = 2.76$$

$$\hat{u}_1 = 0.44$$

$$\hat{Y}_2 = (0.87) + (0.03)(72) = 2.93$$

$$\hat{u}_2 = 0.47$$

$$\hat{Y}_3 = (0.87) + (0.03)(78) = 3.21$$

$$\hat{u}_3 = -0.01$$

$$\hat{Y}_4 = (0.87) + (0.03)(81) = 3.3$$

$$\hat{u}_4 = -0.1$$

$$\hat{Y}_5 = (0.87) + (0.03)(87) = 3.48$$

$$\hat{u}_5 = -0.18$$

$$\hat{Y}_6 = (0.87) + (0.03)(75) = 3.12$$

$$\hat{u}_6 = 0.04$$

$$\hat{Y}_7 = (0.87) + (0.03)(75) = 3.12$$

$$\hat{u}_7 = 0.09$$

$$\hat{Y}_8 = (0.87) + (0.03)(90) = 3.57$$

$$\hat{u}_8 = -0.32$$

$$\text{So, } \sum_{i=0}^N \hat{u}_i = 0.01 \approx 0 \neq 0$$

$$SSR = \sum (\hat{y}_i - \bar{y})^2$$

$$SST = \sum_i (x_i - \bar{x})^2$$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$Var(\hat{u}_i) = \sigma^2 = \frac{SSR}{n-2} = \frac{1.02875}{8-2}$$

/

2. Data is listed in the table

$X_i$	$Y_i$	$Y_i - \bar{Y}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
10	0	-9.1	-10	100
12	2	-7.1	-8	64
14	5	-4.1	-6	36
16	6	-3.1	-4	16
18	7	-2.1	-2	4
22	10	0.9	2	4
24	10	0.9	4	16
26	15	5.9	6	36
28	16	6.9	8	64
30	20	10.9	10	100

$$\bar{x} = 20$$

$$\bar{y} = 9.1$$

2.1 From the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Find estimators of  $\beta_0$  and  $\beta_1$  from the OLS method and interpret the meaning.

$$\beta_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

$$\beta_1 = \frac{(-9.1)(-10) + (-7.1)(-8) + (-4.1)(-6) + (-3.1)(-4) + (-2.1)(-2) + (0.9)(2) + (0.9)(4) + (5.9)(6) + (6.9)(8) + (10.9)(10)}{100 + 64 + 36 + 16 + 4 + 4 + 16 + 36 + 64 + 100}$$

$$\beta_1 = \frac{394}{480} = 0.89545455$$

$$y = -8.8 + 0.8955 X_i$$

$$\beta_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

2

$$= 9.1 - 0.8955(20) = -8.91$$

$$\hat{Y} = \beta_0 + \beta_1 X_i$$

2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\hat{Y}_1 = -8.84 + 0.8955(10) = 0.155$$

$$\hat{Y}_2 = -8.84 + 0.8955(12) = 1.946$$

$$\hat{Y}_3 = -8.84 + 0.8955(14) = 3.737$$

$$\hat{Y}_4 = -8.84 + 0.8955(16) = 5.528$$

$$\hat{Y}_5 = -8.84 + 0.8955(18) = 7.319$$

$$\hat{Y}_6 = -8.84 + 0.8955(20) = 9.110$$

$$\hat{Y}_7 = -8.84 + 0.8955(22) = 10.902$$

$$\hat{Y}_8 = -8.84 + 0.8955(24) = 12.693$$

$$\hat{Y}_9 = -8.84 + 0.8955(28) = 16.274$$

$$\hat{Y}_{10} = -8.84 + 0.8955(30) = 18.065$$

$$\hat{u}_1 = -8.945 \quad \hat{u}_6 = 1.801$$

$$\hat{u}_2 = -7.154 \quad \hat{u}_7 = 3.592$$

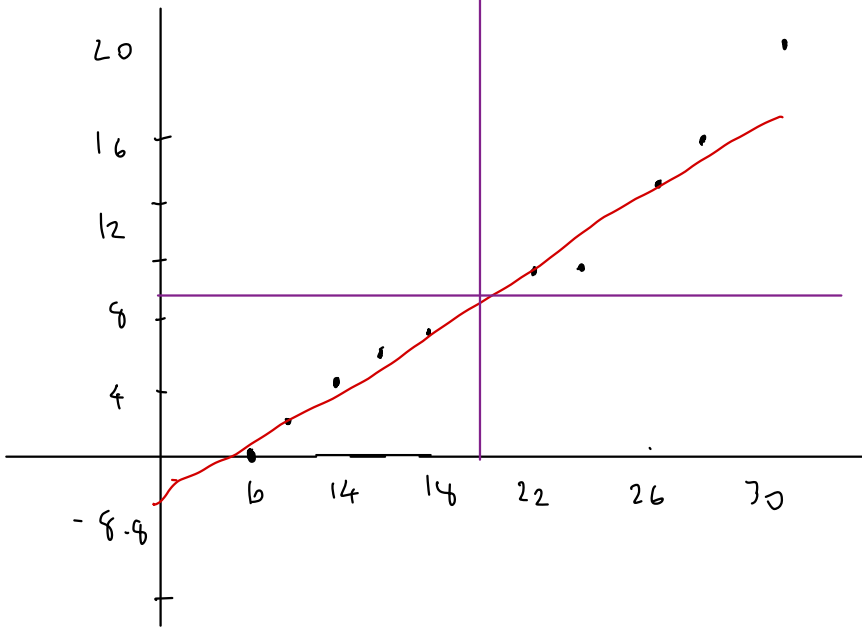
$$\hat{u}_3 = -5.363 \quad \hat{u}_8 = 5.383$$

$$\hat{u}_4 = -3.572 \quad \hat{u}_9 = 7.174$$

$$\hat{u}_5 = -1.781 \quad \hat{u}_{10} = 8.965$$

$$\sum \hat{u}_i = 0 \quad \#$$

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ?



$$\beta_0 + \beta_1 X_i$$

$$Y = -8.8 + 0.8955(20) = 9.1$$

Pass  $\#$

2.4 If  $X_i = 16$ , what is the predicted Y?

$$9.11$$

2.5 Find  $\text{var}(\hat{u}_i)$ ,  $\text{var}(\hat{\beta}_0)$ ,  $\text{var}(\hat{\beta}_1)$

$$SSR = \sum (Y - \hat{Y})^2 = 366.9$$

$$SST = \sum (X - \bar{X})^2 = 440$$

$$\text{var}(\hat{u}_i) = \frac{SSR}{n-2} = \frac{366.9}{10-2} = 45.8625$$

$$\text{var}(\hat{\beta}_0) = \frac{\sigma^2}{SST} \sum X_i^2 = \frac{(45.8625)(440)}{10(440)} = 46.2794$$

$$\text{var}(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{45.8625}{440} = 0.104233$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where  $u_i \sim NIID(0, \sigma^2)$ . Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$Y_i = \beta_1 X_i + u_i$$

$$\text{argmin} \sum_i (Y_i - \tilde{\beta}_1 X_i)^2$$

$$\text{F.O.C w.r.t. } \tilde{\beta}_1$$

$$\frac{\partial \sum_i (Y_i - \tilde{\beta}_1 X_i)^2}{\partial \tilde{\beta}_1} = 0$$

$$\sum_i 2(Y_i - \tilde{\beta}_1 X_i)(-X_i) = 0$$

$$\sum_i X_i (Y_i - \tilde{\beta}_1 X_i) = 0 \quad (-2)$$

$$\sum_i X_i Y_i - \sum_i \tilde{\beta}_1 X_i^2 = 0$$

$$\sum_i X_i Y_i = \sum_i \tilde{\beta}_1 X_i^2$$

$$\hat{\beta}_1 = \frac{\sum_i X_i Y_i}{\sum_i X_i^2}$$

Substitute  $Y_i = \beta_1 X_i + u_i$

$$\hat{\beta}_1 = \frac{\sum_i X_i (\beta_1 X_i + u_i)}{\sum_i X_i^2}$$

$$\hat{\beta}_1 = \frac{\sum_i \beta_1 X_i^2}{\sum_i X_i^2} + \frac{\sum_i u_i X_i}{\sum_i X_i^2}$$

$$E(\hat{\beta}_1) = E\left(\beta_1\right) + E\left(\frac{\sum u_i X_i}{\sum X_i^2}\right) \text{ by SLR4 } E(u_i | X_i) = 0$$

$$E(\hat{\beta}_1) = \beta_1$$