

MA 332 Linear Algebra
Exercise 2

1. Let
$$A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 0 \\ 0 & 3 & 2 \end{bmatrix}$$

- a) Factor A as $A=LU$, where L is lower triangular and U is upper triangular.
- b) Find a basis for the column space of A
- c) What is the rank of A ?

2. Decide whether or not the following vectors are linearly independent, by solving $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Decide also if they span \mathbb{R}^4 , by trying to solve $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = (0,0,0,1)$.

3. By locating the pivots, find a basis for the column space of

$$U = \begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Express each column that is not in the basis as a combination of the basic columns. Find also a matrix A with this echelon form U , but a different column space.

4. Show that the set H of all points in \mathbb{R}^2 of the form $(3s, 2+5s)$ is not a vector space, by showing that it is not closed under multiplication. (Find a specific vector \mathbf{u} , in H and a scalar c such that $c\mathbf{u}$ is not in H .)

5. Determine if $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is in $\text{Nul } A$, where $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$

6. Let $A = \begin{bmatrix} -6 & 12 \\ -3 & 6 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ determine if \mathbf{w} is in $\text{Col } A$. Is \mathbf{w} in $\text{Nul } A$?

7. Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$. Determine if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis in \mathbb{R}^3 . Is $\{\mathbf{v}_1, \mathbf{v}_2\}$ a basis for \mathbb{R}^2 ?

8. Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, $v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$. Find the basis for the subspace W spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.