Question 1 :

ⓐ. $\hat{\beta}_2 = \dfrac{n\sum x_i Y_i - \sum x_i \sum Y_i}{n\sum x_i^2 - [\sum x_i]^2}$

$= \dfrac{18(1254.9) - (388)(50.90)}{18(9620) - 150,544}$

$= \dfrac{2,839}{22,616} = 0.1255$ #

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$= \dfrac{509}{180} - 2.7052 = 0.1226$ #

Estimator of $\hat{\beta}_2$ is 0.1255 and $\hat{\beta}_1$ is 0.1226

$\hat{\beta}_1$ is when $x = 0$, on average y is 0.1226

$\hat{\beta}_2$ is when x rise by 1, on average y will go up by 1.255

ⓑ $TSS = \sum y_i^2 = \sum(Y_i - \bar{Y})^2$

$= 2.5844$

$ESS = TSS - RSS = 2.5844 - 0.5781$

$= 2.0063$

$RSS = \sum \hat{\mu}_i^2 = \sum(Y_i - \hat{Y})^2$

$= 0.5781$

$R^2 = \dfrac{ESS}{TSS} = \dfrac{2.0063}{2.5844} = 0.7763$ #

The R^2 value of 0.7763 mean approximately 77.63% of variation in y explained by x

ⓒ $\hat{Y} = 0.1226 + 0.1255\, X_i$; if $X_i = 30$

$= 0.1226 + 0.1255(30)$

$= 3.8876$

when x is 30, the average Y is approximately 3.8876

ⓓ. $var(\mu) = E(u - E(u))^2$

$= E(u_i)^2$

$= \sigma^2$ or $var(u_i)$

$= \hat{\sigma}^2$

$var(\hat{\beta}_1) = \dfrac{\Sigma x_i^2}{n \Sigma x_i^2} \cdot \sigma^2$

$= \dfrac{9620}{18(211)} \cdot 0.0361$

$= 0.0915$ #

$var(u) = \sigma^2 = \dfrac{\Sigma(y - \hat{y})^2}{n-2}$

$= \dfrac{0.5781}{16}$

$= 0.0361$ #

$var(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{0.0361}{211} \approx 0.00017$ #

$var(u) = 0.0361$

$var(\hat{\beta}_1) = 0.0915$

$var(\hat{\beta}_2) = 0.00017$

ⓔ 90% confidence

$\beta_2 \pm t_{\frac{\alpha}{2}, n-2} \times SE(\hat{\beta}_2)$

$= 0.1255 \pm t_{0.05,16} \times 0.0131$

$= 0.1255 + 1.746 \times 0.0131$

$= (0.1026, 0.1484)$ #

$S.E(\hat{\beta}_2) = \sqrt{var(\hat{\beta}_2)}$

$= 0.0131$

90% confidence mean of y increase by between $(0.1026, 0.1484)$ for each extra x.

ⓕ $\alpha = 0.05$

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

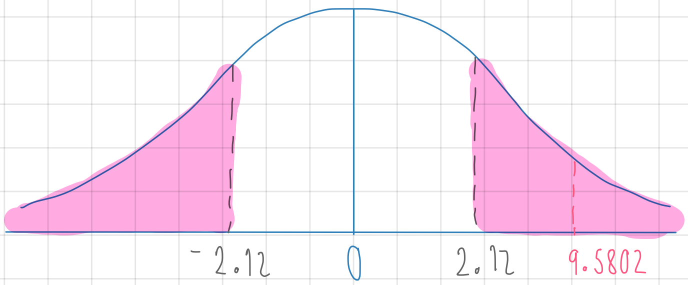
$t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{SE(\hat{\beta}_2)}$

$= \dfrac{0.1255 - 0}{0.0131}$

$= 9.5802$

$t_{\frac{\alpha}{2}, n-2} = t_{0.025,16}$

$= 2.12$



The null hypothesis is rejected. (Reject Ho)

There is enough evidence to say that $\beta_2 \neq 0$.

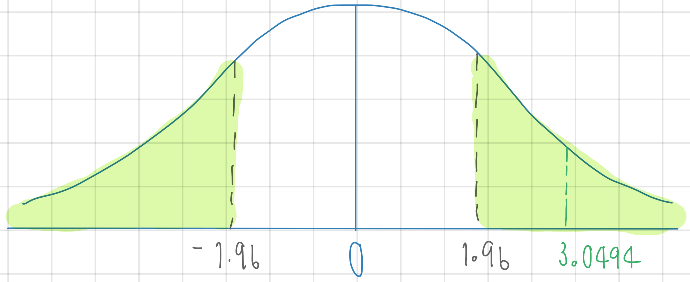
Question 2 :

(a) $\hat{Y} = 0.4279898 + 0.0031338$

$\alpha = 0.05$

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$



$t_{\frac{\alpha}{2}, n-2} = t_{0.025, 27884}$

$= 1.96$

The null hypothesis is rejected. (Reject H_0)

There is enough evidence to say that $\beta_1 \neq 0$.

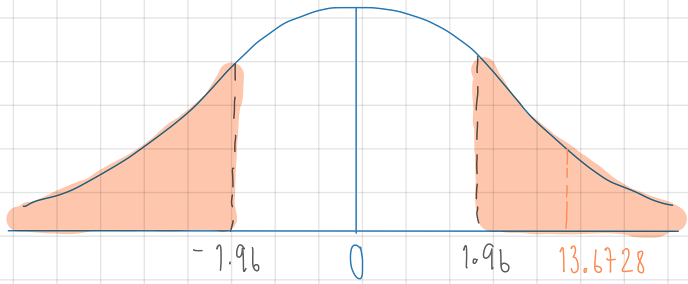
$H_0 = \beta_2 = 0$

$H_1 = \beta_2 \neq 0$

$t = \dfrac{\hat{\beta}_2 - \beta_2}{SE(\hat{\beta}_2)}$

$= \dfrac{0.0031338 - 0}{0.0002292}$

$= 13.6728$



$t_{\frac{\alpha}{2}, n-2} = t_{0.025, 27884}$

$= 1.96$

The null hypothesis is rejected. (Reject H_0)

There is enough evidence to say that $\beta_2 \neq 0$.

(b) # $\hat{\beta}_2$ is about 0.003 suggesting that age of a person goes up by 1 year, on average, number of time person visit hospital go up about 0.003 times.

Yes, if the age increase, the person has to visit hospital more frequency.

(c) $\ln outp_i = \beta_1 + \beta_2 age_i + u_i$

$\ln \hat{Y} = 0.4279898 + 0.0031338X$

$\beta_2 = 0.0031338$

It means that an increase in age of 1 year, on average, lead on about 0.31% of number of time that person visit hospital increase.

ⓓ $\hat{Y} = 0.4279898 + 0.0031338X$

$w_1 = 1$, $w_2 = \frac{1}{10}$

$\hat{\beta}_1^{+} = w_1 \hat{\beta}_1$

$= 1 \cdot 0.4279898$

$= 0.4279898$ #

$\hat{\beta}_2^{+} = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2$

$= 10 \cdot 0.0031338$

$= 0.031338$ #

$SE(\hat{\beta}_1^{+}) = w_1 \cdot SE(\hat{\beta}_1)$

$= 1 \cdot 0.140339$

$= 0.140339$ #

$SE(\hat{\beta}_2^{+}) = \left(\frac{w_1}{w_2}\right) SE(\hat{\beta}_2)$

$= 10 \cdot 0.0002292$

$= 0.002292$ #

Confidence interval at 95%

$\beta_1^{+} = \hat{\beta}_1^{+} \pm t_{\frac{\alpha}{2}} \cdot SE(\hat{\beta}_1^{+})$

$= 0.4279898 \pm t_{0.025,\,27886} \cdot 0.140339$

$= (0.1529, 0.70305)$ #

$\beta_2^{+} = \hat{\beta}_2^{+} \pm t_{\frac{\alpha}{2}} \cdot SE(\hat{\beta}_2^{+})$

$= 0.031338 \pm 1.96(0.002292)$

$= (0.0268, 0.0358)$ #

ⓔ $\hat{Y} = 0.4279898 + 0.0031338X$

$= 0.4279898 + 0.0031338(50)$

$= 0.5847$

$var(\hat{Y}_0) = 0.00002$

$SE(\hat{Y}_0) = 0.0045$

$t_{\frac{\alpha}{2}, n-2} = t_{0.005,\,27884}$

$= 2.576$

$E\left(\frac{Y_0}{X_0}\right) = \hat{Y} \pm t_{\frac{\alpha}{2}, n-2} \cdot SE(\hat{Y}_0)$

$= 0.5849 \pm 2.576(0.0045)$

$= (0.5731, 0.5964)$ #

At $\alpha = 0.01$, the confidence interval of mean prediction at age of 50 years old is between 0.5731 and 0.5964

Question 3 : According to both mean prediction and individual prediction variance formula, when we move x further away from $\bar{x}$, both variances increase, therefore the confidence interval for these is wider than while we move x closer to $\bar{x}$.