EE312 Macroeconomics, 2/2013 (Sec. 046402)
Problem Sets 3 : Ch.3 A Closed Economy One-Period Macroeconomic Model
Solution

*** Exam will consist of essay-type questions only.**

1. Suppose the government increases its spending on consumption goods. Use the Closed-Economy, One-Period Macroeconomic Model to determine the effects of the spending increase on aggregate output, consumption, employment and the real wage. Explain the chain of effects among variables correctly. Describe your analysis in words and use diagrams as needed. (If the space provided is not enough, please attach a separate paper.)

Figure 1: Competitive Equilibrium

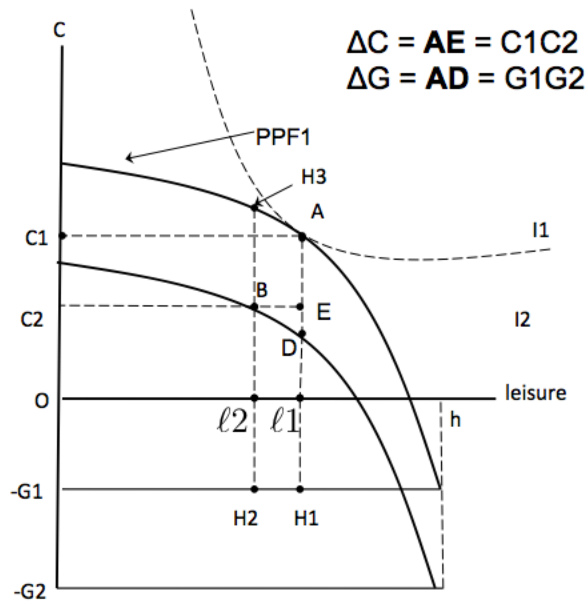
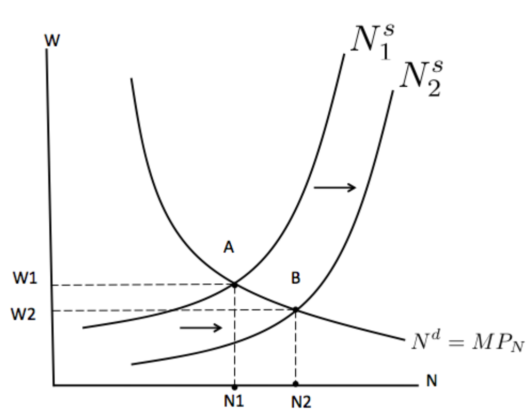


Figure 2: Labor Market Equilibrium



See Figure 1 Competitive Equilibrium.

- Starting with **the initial competitive equilibrium** at point A, the initial competitive equilibrium is at point A where the firm's PPF1 the consumer highest possible indifferece curve I1. At A, the firm maximises profit and the consumer maximizes utility at real wage rate equal to the slope of PPF1 at A (which is equal to the slope of I1 at A). The optimal consumption bundle for consumer is consumption equal to C_1 and leisure equal to ℓ_1. So, the consumer's working time (and labor supply) is $(h - \ell_1)$.
- The government increases spending (G) causing lump-sum tax imposed on the consumer (T) to increase **by the same amount** (Balanced budget constraint: $G = T, \Delta G = \Delta T$).
- **Non-wage income** $(\pi - T)$ and **disposable income** (Y_d) decreases for all ℓ.
- $PPF1$ is illustrated by $C = zF(K, h - \ell) - G_1$. As G increases from G_1 to G_2 , PPF shifts downward by amount of ΔG to PPF2.
- **The new competitive equilibrium** moves from point A to point B. **Consumption drops** from C_1 to C_2 while **leisure decreases** from ℓ_1 to ℓ_2.
- This can be considered as **a pure income effect** which reduces both consumption and leisure since both are normal goods by assumption.
- Less leisure is equivalent to **an increase in working time** $(h - \ell\ )$. The consumer **supplies more labour services** (N^s). [Labour supply shifts to the right.]
  - The real wage(w) drops to induce more labor demand by the firm.
  - The real wage decreases because the slope of PPF2 at point B is less steep than the slope of PPF1 at point A.
  - **Employment rises**.
    See **Figure 2** for the equilibrium in the labor market. N^d remains the same because production function has not been changed. N^s shifts to the right from N_1^s to N_2^s . The consumer is willing to supply more labor for all levels of wage because the consumer's non-wage income is decreased by an increase in tax (T). Real wage decreases and employment rises.
- More labour input in production results in **larger total output** (Y). The original Y can be represented by the distance $H1A$. The new Y can berepresented by the distance $H2H3$. $(\Delta Y = ED\ )$
- The consumer works more, receives a lower real wage and consume less.
- The **decrease in consumption** (the distance C1C2 = AE) **is smaller than the increase in government spending** (the distance G1G2 = AD).
- $Y \uparrow$ but $C \downarrow$. Private consumption is **crowded out** by government purchases.
- This means that when government increases its spending, the firm produces more. The government's share in total output increases while the consumer's share in total output decreases.
- In sum, The consumer's utility decreases as the governmet expenditure increases.
- As the representative consumer pays higher taxes, his or her disposable income falls, and in equilibrium he or she spends less on consumption goods, and work harder to support a larger government.

$G\uparrow$	$\rightarrow$	$(\pi - T)\downarrow$	$\rightarrow$	$Y_d\downarrow$	$\rightarrow$	$\ell\downarrow$	$\rightarrow (h-\ell)\uparrow$
			$\swarrow$				$\downarrow$
		$C\downarrow$					$N^S\uparrow$
			$\nwarrow$				
				$Y\uparrow$			(shift right)
			$\nearrow$				$\downarrow$
		$zF(K, N^d)$	$\leftarrow$	$N^d\uparrow$ (move along)		$\leftarrow$	$w^*\downarrow$

$$|\Delta C| < |\Delta G|$$
$$\Delta C = \Delta Y - \Delta G$$

2. Use the closed-economy, one period macroeconomic model to determine the effects of a decrease in total factor productivity on the aggregate output, consumption, employment and real wage. Explain the chain of effects among variables correctly. Describe your analysis in words and use diagrams as needed. (If the space provided is not enough, please attach a separate sheet.)

Figure 3: Competitive Equilibrium

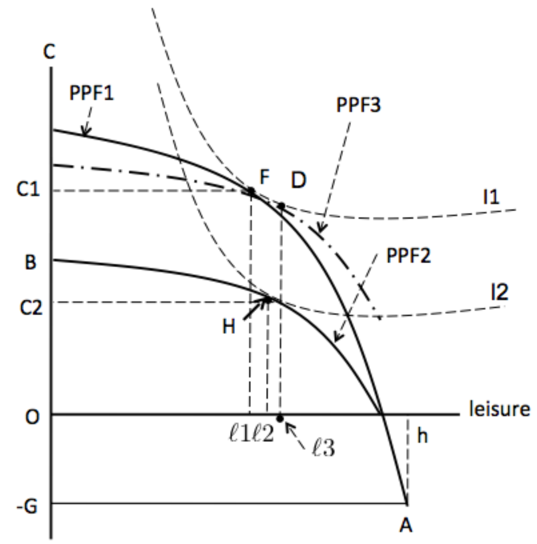
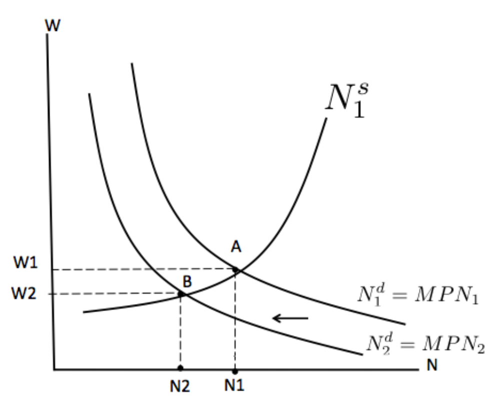


Figure 4: Labor Market Equilibrium



See figure 3. Let assume **a stronger substitution effect** so that labour supply has a postitive slope.

- Starting with **the initial competitive equilibrium** at point F, the initial competitive equilibrium is at point F where the firm's PPF1 the consumer highest possible indifferece curve I1. At A, the firm maximises profit and the consumer maximizes utility at real wage rate equal to the slope of PPF1 at F (which is equal to the slope of I1 at F). The optimal consumption bundle for consumer is consumption equal to C_1 and leisure equal to ℓ_1. So, the consumer's working time (and labor supply) is $(h - \ell_1)$.
- A decrease in total factor productivity causes a decrease in MPN. The **PPF rotates downwards** from PPF1 to PPF2.
- The new PPF is less steeper (flatter) than the original one.

- **The new competitive equilibrium** moves from point F to point H. Consumption drops from C_1 to C_2 while leisure increases from ℓ_1 to ℓ_2.
- This means that wage decreases for all N , ℓ
- **substitution effect and income effect :** $w \downarrow$
  substitution effect : $\ell \uparrow$ and $C \downarrow$
  leisure is less costly.
  income effect : $\ell \downarrow$ and $C \downarrow$
  lower wage implies lower income.
- Consumption declines for sure.
- As we assume **a stronger substitution effect** , leisure increases. (See note*)
- **To separate income effect and substitution effect**, we draw an imaginary PPF (PPF3) which has the same slope as the new PPF (PPF2).
  - Point D is the point where the imaginary PPF (PPF3) touches the initial IC (I1).
  - FD is therefore the substitution effect (the effect due only to the relative price change, controlling for the change in real income). Leisure inceases from $\ell 1$ to $\ell 3$.
  - DH is then the income effect (the effect due to only the income change, controlling for the change in the relative price). Leisure decreases from $\ell 3$ to $\ell 2$.
  - Since Substitution Effect (SE) $>$ Income Effect (IE), $\ell 1 \ell 3 > \ell 3 \ell 2$. Leisure increases from from ℓ_1 to ℓ_2.
- More leisure is equivalent to **an decrease in working time** $(h - \ell\ )$. The consumer **supplies less labour services** (N^s). **Employment decreases.**
- The original real wage is the slope of PPF1 at point F.
- The new real wage is the slope of PPF2 at point H.
- **Real wage decreases. (or the effect on real wage is ambiguous**.)
- **See figure 4** for the equilibrium in the labor market. N^d shifts to the left (From N_1^d to N_2^d) because the decrease in z causing MPN to decrease for all N. Real wage decreases..
- Less labour input in production results in **smaller total output** $(Y \downarrow)$.
- The consumer works less, consumes less, and receives a lower real wage. (or an ambiguous effect on real wage)
- The consumer utility is decreased.

$z, K \downarrow \ \rightarrow \ MP_N \downarrow \ \rightarrow \ N^d \downarrow \ \rightarrow \ w \downarrow$

$\swarrow \qquad \searrow$

Substitution Effect $\ell \uparrow C \downarrow$ | Income Effect $\ell \downarrow C \downarrow$

$\searrow \qquad \swarrow$

$C \downarrow$

N^Sdepends

$SE > IE \ : C \downarrow, \ell \uparrow$

$SE = IE \ : C \downarrow, \overline{\ell}$

$SE < IE \ : C \downarrow, \ell \downarrow$

$\rightarrow \ Y \downarrow$

$\downarrow$

$C \downarrow$

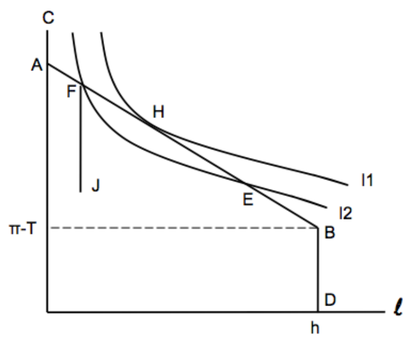
* Note: A student may analyze all the three possible cases; a stronger income effect, an equal effect and a stronger income effect. Please make sure you understand all cases especially the stronger substitution effect case. Later, we will always assume a stronger substitution effect.

* Note: In my opinion, the graph for labour market is a little tricky. We analyze a general equilibrium where the consumer is the owner of the firm. When TFP(total factor productivity) changes, firm's profit changes. This must affect labour supply somehow. When we analyze the model by using graphical illustration, the effect on the labour market is unclear. However, the mathematical appendix of the textbook shows us how the model is derived. When it becomes sequence of equations, all the solutions are clear. It can be proved that real wage decreases when TFP decreases (real wage increases when TFP increases).

1. Consider a simple one-period, closed-economy model where the representative consumer has utility function $U(C) = C^{1/2}\ell^{1/2}$ and has h available hours to divide between work and leisure. The representative firm has technology given by $Y = zK^{2/3}N^{1/3}$. There is a government that sets its expenditure level at a value $G > 0$.

   (a) Define the consumer budget constraint.
   ... $C + w\ell = wh + (\pi - T)$....or ...$C = w(h - \ell) + (\pi - T)$...

   (b) Define the consumer's utility maximization condition. Solve for labour supply function $N^s(w)$.



The consumer's utility is maximized when the slope of IC is equal to the slope of the budget line.

$$\text{The slope of IC} = \text{The slope of the budget line}$$

$$MRS_{C,\ell} = \text{...w...}$$

$$\frac{MU_\ell}{MU_C} = ...w....$$

$$MU_C = \frac{\partial U}{\partial C} = \frac{1}{2}C^{-1/2}\ell^{1/2}$$

$$MU_\ell = \frac{\partial U}{\partial \ell} = ...\frac{1}{2}C^{1/2}\ell^{-1/2}.....$$

$$\text{The slope of IC} = \text{The slope of the budget line}$$

$$\frac{MU_\ell}{MU_C} = w$$

$$\frac{\frac{1}{2}C^{1/2}\ell^{-1/2}.............}{\frac{1}{2}C^{-1/2}\ell^{1/2}} = w$$

$$\left(\frac{C}{\ell}\right)^{..1.....} = w$$

$$C = ...w\ell.....$$

Substitute $C = w\ell$ into the equation for the budget line

$$C + w\ell = wh + (\pi - T)$$

$$...w\ell... + w\ell = wh + (\pi - T)$$

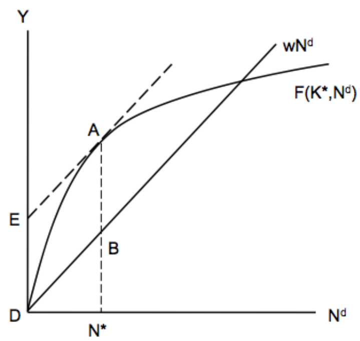
$$\ell = \frac{wh + (\pi - T)}{...2w......}$$

From leisure function $\ell(w) = \dfrac{wh + (\pi - T)}{2w}$. We can find the labour supply function.

$$\begin{aligned}
N^s(w) &= \text{....}h\text{....} - \ell(w) \\
&= \text{...}h\text{......} - \frac{wh + (\pi - T)}{2w} \\
&= \frac{-(\pi - T) + ..wh..}{2w} \\
&= \frac{..wh..}{2w} - \frac{\pi - T}{2w} \\
&= \frac{\ldots h..}{2} - \frac{\pi - T}{2w}
\end{aligned}$$

This is the labour supply function. $N^s(w) = \dfrac{h}{2} - \dfrac{\pi - T}{2w}$. From the labour supply function, $w \uparrow \Rightarrow N^s$...↑..... (↑ or ↓).

(c) Define the firm's profit maximization condition. Solve for labour demand function.



Profit Maximization

- Y = revenue;
- MP_N = marginal revenue;
- wN^d = variable cost;
- w = marginal cost;
- Profit $= Y - wN^d$;
- Max profit = AB where $MP_N = w$.

The firm's profit is maximized when the slope of TR $\left(zF(K, N^d)\right)$ is equal to the slope of TC $\left(wN^d\right)$ or MR = MC.

$Y = zK^{2/3}N^{1/3} = zK^{2/3}N^{d1/3}$

$$\begin{aligned}
\text{Slope of Y} &= \text{Slope of } wN^d \\
\frac{\partial Y}{\partial N^d} &= \frac{\partial \left(wN^d\right)}{\partial N^d} \\
\frac{1}{3}zK^{2/3}N^{d-2/3\cdots\cdots} &= ..w.... \\
N^d &= \text{.....} \left(\frac{3w}{zK^{2/3}}\right)^{-3/2} \text{......}
\end{aligned}$$

$N^d = \left(\dfrac{3w}{zK^{2/3}}\right)^{-3/2} = \left(\dfrac{z^{3/2}K}{(3w)^{3/2}}\right)$. This is labor demand function. $w \uparrow \Rightarrow N^d$..↓.... (↑ or ↓).

(d) Find the competitive equilibrium values given $h = 16$, $z = 1$, $K = 8$, $G = 0$.

$$\begin{aligned}
Y &= zK^{2/3}N^{1/3} \\
&= (..1..)(..8....)^{2/3}N^{1/3}
\end{aligned}$$

$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2$.

$$Y = 4N^{1/3}. \tag{1}$$

Marginal Product of Capital.

$$\begin{aligned} MP_N &= \frac{\partial Y}{\partial N} \\ &= (\frac{\dots 1 \dots}{\dots 3 \dots})4N^{(1/3 - ..1..)} \\ &= \left(\frac{\dots 4 \dots}{\dots 3 \dots}\right) N^{\dots -2/3 \dots} \end{aligned}$$

$$MP_N = \frac{4}{3}N^{-2/3} = w. \tag{2}$$

From consumer's optimization solution, $C = w\ell$.

$$C = w\ell$$

Substitute $MP_N = \frac{4}{3}N^{-2/3} = w$

$$C = \dots\dots\dots \frac{4}{3}N^{-2/3}\ell \dots\dots\dots\dots\dots\dots$$

$$C = \frac{4}{3}N^{-2/3}\ell \tag{3}$$

As G=0, $Y = C + G$. Then,

$$C = Y$$

From (1),

$$C = 4N^{1/3} \tag{4}$$

Equation (3) = equation (4).
Substitute $\ell = h - N$ into $\ell = 3N$. Let $h = 16$. Solve for N.

$$N^* = \dots\frac{4}{3} \times \frac{1}{4} \times (16 - N^*); .4N^* = 16, \dots N^* = 4\dots\dots\dots$$

Substitute $N = 4$ into $\ell = 16 - N$ and get
$\ell^* = \dots 12 \dots\dots\dots$
From equation (4), substitute $N = 4$ and get C*.
$C^* = \dots 4(4)^{1/3} \dots\dots\dots$
From equation (2), substitute $N = 4$ and get w*.
$w^* = \dots\frac{4}{3}(4)^{-2/3} = \frac{1}{3}(4)^{1/3} \dots\dots\dots\dots\dots$
From equation (1), substitute $N = 4$ and get Y*.
$Y^* = \dots\dots 4(4)^{1/3}. = C \dots\dots\dots$