

# 1 Comparative statics of equilibrium

- Nicholson, Ch. 12, pp. 422-424
- Supply and Demand function of parameter  $\alpha$  :
  - $Y_i^S(p_i, w, r, \alpha)$
  - $X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$

- How does  $\alpha$  affect  $p^*$  and  $Y^*$ ?
- Comparative statics with respect to  $\alpha$
- Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$$

- Can write equilibrium as implicit function:

$$Y_i^S(p_i, w, r, \alpha) - X_i^D(\mathbf{p}, \mathbf{M}, \alpha) = 0$$

- What is  $dp^*/d\alpha$ ?

- Implicit function theorem:

$$\frac{\partial p^*}{\partial \alpha} = - \frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- What is sign of denominator?

- Sign of  $\partial p^*/\partial \alpha$  is negative of sign of numerator

- Examples:

1. *Fad*. Good becomes more fashionable:  $\frac{\partial X^D}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

2. *Recession in Europe*. Negative demand shock for US firms:  $\frac{\partial X^D}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$

3. *Oil shock*. Import prices increase:  $\frac{\partial Y^S}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

4. *Computerization*. Improvement in technology.  $\frac{\partial Y^S}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$

## 2 Elasticities

- Nicholson, Ch.1, pp. 28-29
- How do we interpret magnitudes of  $\partial p^* / \partial \alpha$ ?
- Result depends on units of measure.
- Can we write  $\partial p^* / \partial \alpha$  in a unit-free way?
- Yes! Use **elasticities**.
- Elasticity of  $x$  with respect to parameter  $p$  is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{p}{x}$$

- Interpretation: Percent response in  $x$  to percent change in  $p$  :

$$\begin{aligned}\varepsilon_{x,p} &= \frac{\partial x}{\partial p} \frac{p}{x} = \lim_{dp \rightarrow 0} \frac{x(p+dp) - x(p)}{dp} \frac{p}{x} = \\ &= \lim_{dp \rightarrow 0} \frac{dx/x}{dp/p}\end{aligned}$$

where  $dx \equiv x(p+dp) - x(p)$ .

- Now, show

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

- Notice: This makes sense only for  $x > 0$  and  $p > 0$

- Proof. Consider function

$$x = f(p)$$

- Rewrite as

$$\ln(x) = \ln f(p) = \ln f(e^{\ln(p)})$$

- Define  $\hat{x} = \ln(x)$  and  $\hat{p} = \ln(p)$

- This implies

$$\hat{x} = \ln f(e^{\hat{p}})$$

- Get

$$\begin{aligned} \frac{\partial \hat{x}}{\partial \hat{p}} &= \frac{\partial \ln x}{\partial \ln p} = \\ &= \frac{1}{f(e^{\hat{p}})} \frac{\partial f(e^{\hat{p}})}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{p}{x} \end{aligned}$$

- Example with Cobb-Douglas utility function

- $U(x, y) = x^\alpha y^{1-\alpha}$  implies solutions

$$x^* = \alpha \frac{M}{p_x}, y^* = (1 - \alpha) \frac{M}{p_y}$$

- Elasticity of demand with respect to own price  $\varepsilon_{x,p_x}$ :

$$\varepsilon_{x,p_x} = \frac{\partial x^*}{\partial p_x} \frac{p_x}{x^*} = -\frac{\alpha M}{(p_x)^2} \frac{p_x}{\alpha \frac{M}{p_x}} = -1$$

- Elasticity of demand with respect to other price  $\varepsilon_{x,p_y} = 0$

- Go back to problem above:

$$\frac{\partial p^*}{\partial \alpha} = - \frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- Use elasticities to rewrite response of  $p$  to change in  $\alpha$  :

$$\frac{\partial p^*}{\partial \alpha} \frac{\alpha}{p} = - \frac{\left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \frac{\alpha}{Y}}{\left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{Y}}$$

or (using fact that  $X^{D*} = Y^{S*}$ )

$$\varepsilon_{p,\alpha} = - \frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- We are likely to know elasticities from empirical studies

### 3 Response to taxes

- Nicholson, Ch. 12, pp. 442-446
- Per-unit tax  $t$
- Write price  $p_i$  as price including tax
- Supply:  $Y_i^S(p_i - t, w, r)$
- Demand:  $X_i^D(\mathbf{p}, \mathbf{M})$

$$Y_i^S(p_i - t, w, r) - X_i^D(\mathbf{p}, \mathbf{M}) = 0$$

- What is  $dp^*/dt$ ?

- Comparative statics:

$$\begin{aligned}
 \frac{\partial p^*}{\partial t} &= -\frac{-\frac{\partial Y^S}{\partial p}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} = \\
 &= \frac{-\frac{\partial Y^S}{\partial p} \frac{p}{X}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right) \frac{p}{X}} = \\
 &= \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
 \end{aligned}$$

- How about price received by suppliers  $p^* - t$ ?

$$\begin{aligned}
 \frac{\partial (p^* - t)}{\partial t} &= \frac{\frac{\partial Y^S}{\partial p}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} - 1 = \\
 &= \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
 \end{aligned}$$

- *Inflexible Supply.* (Capacity is fixed) Supply curve vertical ( $\varepsilon_{S,p} = 0$ )

- Producers bear burden of tax

- *Flexible Supply.* (Constant Returns to Scale) Supply curve horizontal ( $\varepsilon_{S,p} \rightarrow \infty$ )

- Consumers bear burden of tax

- *Inflexible demand.* Demand curve vertical ( $\varepsilon_{D,p} = 0$ )?

- Consumers bear burden
- General lesson: Least elastic side bears larger part of burden
- What happens with a subsidy ( $t < 0$ )?
- What happens to quantity sold?
- Use demand curve:

$$\frac{\partial X^{D*}}{\partial t} = \frac{\partial X^{D*}}{\partial p^*} \frac{\partial p^*}{\partial t}$$

and use expression for  $\partial p^* / \partial t$  above