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6. Extensions of The Two-Variable Linear Regression Mode

6.1 Functional Form of regression Models

We will consider the following models:

[0] Linear model : $Y = \beta_1 + \beta_2 X$.

[1] The log-linear model (OR LOG-LOG MODEL) : $\ln Y = \beta_1 + \beta_2 \ln X$.

[2] Semilog models (LOG-LIN MODEL) : $\ln Y = \beta_1 + \beta_2 X$.

LIN-LOG MODEL) : $Y = \beta_1 + \beta_2 \ln X$.

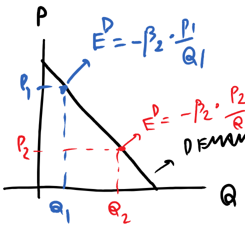
[3] Reciprocal models : $Y = \beta_1 + \beta_2 \left(\frac{1}{X}\right)$.

[4] The logarithmic reciprocal model : $\ln Y = \beta_1 - \beta_2 \left(\frac{1}{X}\right)$.

REVIEW ON LINEAR MODEL : $Y = \beta_1 + \beta_2 X$.

$\hat{\beta}_2$ = IF X CHANGES BY 1 UNIT, Y WILL CHANGE BY $\hat{\beta}_2$ UNIT(S), HOLDING ALL OTHER FACTORS THAT MIGHT AFFECT Y CONSTANT.

(= CETERIS PARIBUS)



$$\text{ELASTITY} = \frac{\frac{\Delta Q}{Q} \times 100}{\frac{\Delta P}{P} \times 100} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \beta_2 \cdot \frac{P}{Q}$$

[NOTE $Q = \beta_1 - \beta_2 P$]

The Log-linear Model (DOUBLE LOG MODEL OR LOG-LOG MODEL)

$\ln Y = \beta_1 + \beta_2 \ln X$ WHICH ORIGINALLY COMES FROM $\underbrace{Y_i = \beta_1 X_i^{\beta_2} e^{u_i}}$

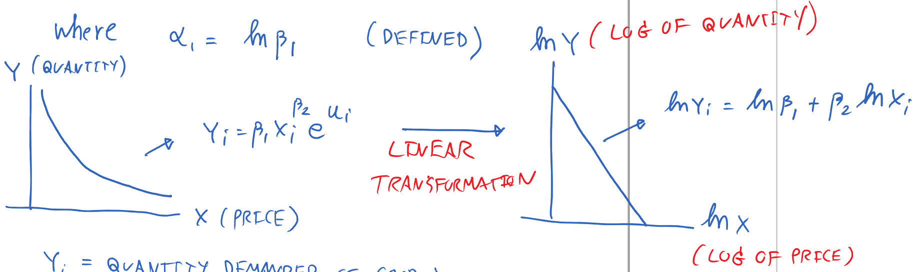
EXPONENTIAL REGRESSION MODEL.

FROM $Y_i = \beta_1 X_i^{\beta_2} e^{u_i}$

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i$$

$$\ln Y_i = \alpha_1 + \beta_2 \ln X_i + u_i$$

where $\alpha_1 = \ln \beta_1$ (DEFINED)



Y_i = QUANTITY DEMANDED OF GOOD X

X_i = PRICE OF GOOD X

FROM $\ln Y = \alpha_1 + \beta_2 \ln X$

$$\frac{d \ln Y}{dX} = \beta_2 \frac{d \ln X}{dX}$$

$$\frac{1}{Y}\frac{dY}{dX} = \beta_2 \frac{1}{X}\cancel{\frac{dX}{dX}}$$

$$\beta_2 = \frac{dY}{dX} \cdot \frac{X}{Y} = \frac{\frac{dY}{Y} \times 100}{\frac{dX}{X} \times 100} = \frac{\text{PERCENTAGE CHANGE IN } Y}{\text{PERCENTAGE CHANGE IN } X}$$

NOTE

$$\boxed{\begin{aligned} \Delta Y &= \text{ABSOLUTE CHANGE IN } Y \\ \frac{\Delta Y}{Y} &= \text{RELATIVE CHANGE IN } Y \\ \% \Delta Y &= \frac{\Delta Y}{Y} \times 100 = \text{PERCENTAGE CHANGE IN } Y \end{aligned}}$$

SO, β_2 = ELASTICITY $\left(= \frac{\% \Delta Y}{\% \Delta X}\right)$

EX: IF $\hat{\beta}_2 = 0.2$ → ALL ELSE EQUAL, IF X CHANGES BY 1% (10%) Y WILL CHANGE BY SLIGHTLY 0.2% (2%).

EX: $\ln Y = \beta_1 + \overset{\ominus}{\beta_2} \ln X_2 + \beta_3 \ln X_3 + \beta_4 \ln X_4$

Y = QUANTITY DEMANDED FOR GOOD X

X_2 = PRICE OF GOOD X

X_3 = PRICE OF ANOTHER GOOD

X_4 = CONSUMERS' INCOME

β_2 = PRICE ELASTICITY OF DEMAND

β_3 = CROSS-PRICE ELASTICITY OF DEMAND

⊕ SUBSTITUTES CASE

⊖ COMPLEMENTARY CASE

β_4 = INCOME ELASTICITY OF DEMAND

⊕ GOOD X IS NORMAL GOOD

⊖ GOOD X IS INFERIOR GOOD

The Semilog Models

The Log-linear Model

$$Q_i = B_1 L_i^{B_2} K_i^{B_3}$$ → COBB-DOUGLAS PRODUCTION FUNCTION.

$$\boxed{\ln Q_i = \ln B_1 + B_2 \ln L_i + B_3 \ln K_i \quad — (2)}$$

DEFINE $\ln B_1 = A$ . . .

$$\ln Q_i = A + B_2 \ln L_i + B_3 \ln K_i \quad —(3)$$ → MATH MODEL

$$\ln Q_i = A + B_2 \ln L_i + B_3 \ln K_i + u_i \quad —(4)$$ ECONOMETRIC MODEL

$$B_2 = \frac{\partial \ln Q}{\partial \ln L} = \frac{\partial Q / Q}{\partial L / L} = \frac{\partial Q}{\partial L} \cdot \frac{L}{Q}$$ = LABOR-OUTPUT ELASTICITY

→ IF L INCREASES BY 1%, Q WILL INCREASE BY B_2 %, CETERIS PARIBUS

$$B_3 = \frac{\partial \ln Q}{\partial \ln K} = \frac{\partial Q / Q}{\partial K / K} = \frac{\partial Q}{\partial K} \cdot \frac{K}{Q}$$ = CAPITAL-OUTPUT ELASTICITY

→ IF K INCREASE BY 1%, Q WILL INCREASE BY B_3 %, CETERIS PARIBUS.

IF $B_2 + B_3 = 1$, THE PRODUCTION FN EXHIBITS "CONSTANT RETURNS TO SCALE (CRS)"

IF $B_2 + B_3 > 1$, " ——— " "INCREASING RETURNS TO SCALE

IF $B_2 + B_2 < 1$, " ——— " "DECREASING RETURNS TO SCALE"

EViews - [Equation: UNTITLED Workfile: CD::Untitled\]

File Edit Object View Proc Quick Options Add-ins Window Help

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Dependent Variable: LNOUTPUT
Method: Least Squares
Date: 10/31/17 Time: 08:54
Sample: 1 51
Included observations: 51

$\beta_2 = 0.468$

$\beta_3 = 0.521279$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.887599	0.396228	9.811517	0.0000
LNLABOR	0.468332	0.098926	4.734169	0.0000
LNCAPITAL	0.521279	0.096887	5.380281	0.0000

R-squared	0.964175	Mean dependent var	16.94139
Adjusted R-squared	0.962683	S.D. dependent var	1.380870
S.E. of regression	0.266752	Akaike info criterion	0.252028
Sum squared resid	3.415518	Schwarz criterion	0.365664
Log likelihood	-3.426703	Hannan-Quinn criter.	0.295452
F-statistic	645.9316	Durbin-Watson stat	1.946388
Prob(F-statistic)	0.000000		

A	B	C	D	E	F	G	H	I	J	K	L
obs	output	labor	capital	lnoutput	lnlabor	lncapital	lnoutlab	lncaplab	outputstar	capitalstar	laborstar
1	38372840	424471	2689076	17.46286	12.958599	14.804708	4.5042615	1.8461093	-0.1079874	0.0636889	0.1343891
2	1805427	19895	57997	14.406307	9.8982239	10.968146	4.5080838	1.0699228	-0.923066	-0.905514	-0.9410533
3	23736129	206893	2308272	16.98251	12.239957	14.65201	4.7425518	2.4120526	-0.434236	-0.0765868	-0.4439759
4	26981983	304055	1376235	17.11068	12.624964	14.134862	4.4857159	1.5098982	-0.3618868	-0.4199185	-0.1857003
5	217546032	1809756	13554116	19.197922	14.408703	16.422201	4.7892184	2.0134983	3.8857391	4.0660119	3.8167481
6	19462751	180366	1790751	16.784014	12.102743	14.398146	4.6812696	2.2954023	-0.5294886	-0.2672245	-0.5144899
7	28972772	224267	1210229	17.181868	12.320593	14.00632	4.8612742	1.6857276	-0.3175126	-0.4810697	-0.3977924
8	14313157	54455	421064	16.47669	10.90513	12.950541	5.5715599	2.0454102	-0.6442718	-0.7717721	-0.849186
9	159921	2029	7188	11.982435	7.6152983	8.880168	4.367137	1.2648699	-0.959744	-0.9242304	-0.9885446
10	47289846	471211	2761281	17.671806	13.063062	14.831205	4.6087451	1.768144	0.0907705	0.0902868	0.2586331
11	63015125	659379	3540475	17.958885	13.399054	15.079771	4.5598316	1.6807177	0.4412831	0.3773162	0.7588206
12	1809052	17528	146371	14.408314	9.7715549	11.8939	4.6367588	2.122345	-0.9229852	-0.87296	-0.9473452
13	10511786	75414	848220	16.168007	11.230748	13.650895	4.9372597	2.4201472	-0.7290034	-0.6144219	-0.7934729
14	105324866	963156	5870409	18.472561	13.77797	15.585435	4.6945896	1.8074642	1.3843569	1.2355872	1.5663193
15	90120459	835083	5832503	18.316658	13.635286	15.578957	4.6813712	1.9436704	1.0454544	1.2216239	1.2258762
16	39079550	336159	1795976	17.48111	12.72534	14.401059	4.7557702	1.6757196	-0.092235	-0.2652998	-0.1003615
17	22826760	246144	1595118	16.943443	12.413672	14.282458	4.5297723	1.8687862	-0.4545057	-0.3392892	-0.3396391
18	38686340	384484	2503693	17.470997	12.859657	14.733277	4.6113396	1.8736199	-0.1009995	-0.0046001	0.0280958
19	69910555	216149	4726625	18.062727	12.283723	15.368722	5.7790041	3.0849986	0.5949806	0.8142549	-0.4193717
20	7856947	82021	415131	15.876908	11.314731	12.93635	4.5621781	1.6216189	-0.7881791	-0.7739576	-0.7759101
21	21352966	174855	1729116	16.876701	12.071712	14.363121	4.8049889	2.2914085	-0.4873562	-0.2899288	-0.5291392
22	46044292	355701	2706065	17.645115	12.781846	14.811007	4.8632684	2.0291603	0.0630074	0.0699471	-0.0484151
23	92335528	943298	5294356	18.34094	13.757137	15.482152	4.5838022	1.7250143	1.0948278	1.0233883	1.5135329
24	48304274	456553	2833525	17.693031	13.03146	14.857032	4.6615705	1.825572	0.1133819	0.1168991	0.2196693
25	17207903	267806	1212281	16.660879	12.498018	14.008015	4.1628613	1.5099962	-0.5797486	-0.4803138	-0.2820572
26	47340157	439427	2404122	17.672869	12.993227	14.692696	4.6796427	1.6994685	0.0918919	-0.0412788	0.174145
27	2644567	24167	334008	14.788018	10.092743	12.71892	4.6952744	2.6261768	-0.9043618	-0.8038406	-0.9296975
28	14650080	163637	627806	16.499956	12.005405	13.349986	4.4945507	1.3445807	-0.6367618	-0.6956152	-0.5589588
29	7290360	59737	522335	15.802064	10.997706	13.166064	4.8043566	2.1683576	-0.8008082	-0.7344672	-0.8351454
30	9188322	96106	507488	16.033443	11.473207	13.137228	4.5602369	1.6640214	-0.7585031	-0.7399364	-0.7384695

6.1 Functional Form of regression Models LOG-LIN MODEL 103

The Semilog Models

LIN-LOG MODEL

LOG-LIN MODEL OR GROWTH MODEL

BASIC FINANCE : $Y_t = Y_0 (1+r)^t$

IF WE TAKE ln BOTH SIDES:

$$\ln Y_t = \underbrace{\ln Y_0}_{B_1} + t \underbrace{\ln(1+r)}_{B_2}$$

0.03

$$\therefore \ln Y_t = B_1 + B_2 t$$

THE PRF: $\ln Y_t = B_1 + B_2 t + u_t$

$$\frac{d \ln Y_t}{dt} = B_2$$

$$\frac{\frac{dY_t}{Y_t}}{dt} =$$

$B_1 = 7.87$

IF WE TAKE ANTILOG OF B_1, YOU WILL OBTAIN Y_0

ANTILOG (7.87...) = 2632.27

ESTIMATED VALUE OF REAL GDP AT THE BEGINNING PERIOD (1960).

$$\frac{\frac{dY_t}{Y_t} \times 100}{dt} = B_2 \times 100$$

PERCENTAGE CHANGE IN Y_t OR GROWTH RATE OF Y_t !!!

IF Y_t = REAL GDP

$\frac{dY_t}{Y_t} \times 100$ = REAL GDP GROWTH (%)

EXAMPLE GDP FOR U.S.A FOR 1960 - 2007.

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LNRGDP
Method: Least Squares
Date: 10/31/17 Time: 10:49
Sample: 1960 2007
Included observations: 48

$\widehat{\ln(RGDP)} = \overset{B_1}{7.875662} + \overset{B_2}{0.031490}\, t$

⟹ GDP HAS BEEN RISING AT THE RATE OF 3.15 PERCENT PER YEAR

INSTANTENEOUS GROWTH RATE (AT A POINT IN TIME)

$= B_2 \cdot 100 = 0.031490 \cdot 100$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C $\hat{B}_1$	7.875662	0.009759	807.0064	0.0000
TIME B_2	0.031490	0.000347	90.81649	0.0000

R-squared	0.994454	Mean dependent var	8.647156
Adjusted R-squared	0.994333	S.D. dependent var	0.442081
S.E. of regression	0.033280	Akaike info criterion	-3.926967
Sum squared resid	0.050947	Schwarz criterion	-3.849001
Log likelihood	96.24722	Hannan-Quinn criter.	-3.897504
F-statistic	8247.634	Durbin-Watson stat	0.347739
Prob(F-statistic)	0.000000		

HOW TO COMPUTE "COMPOUND GROWTH RATE"? (GROWTH RATE OVER A PERIOD OF TIME)

RECALL THAT $B_2 = \ln(1+r)$

WHERE r = COMPOUND GROWTH RATE

r = ANTILOG (B_2) − 1

$r = 0.031991$

$\hat{B}_1 = 7.87562$

$B_1 = \ln Y_0$ AS WE DEFINED

IF YOU TAKE ANTILOG OF B_1, YOU WILL GET ESTIMATED GDP 1960

β_1 = ln Y_0 AS WE DEFINED

IF YOU TAKE ANTILOG OF β_1, YOU WILL GET ESTIMATED GDP$_{1960}$

$r = 0.031991$

THE BEGINNING VALUE OF REAL GDP

$$\text{ANTILOG}(\hat{\beta}_1) = \text{ANTILOG}(7.875662)$$

$$= 2632.27$$

COMPOUND GROWTH RATE IS ABOUT 3.2% SLIGHTLY GREATER THAN THE INSTANTANEOUS GROWTH RATE OF 3.15%

NOTE ACTUAL RGDP$_{1960}$ = 2501.8 BILLION USD.

THE DIFFERENCE IS DUE TO "COMPOUNDING"

8.11.17 LIN-LOG MODEL

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i \rightarrow \text{ECONOMETRIC MODEL}$$

$$Y = \beta_1 + \beta_2 \ln X \rightarrow \text{MATHEMATICAL MODEL}$$

$$\frac{dY}{dX} = \beta_2 \frac{d \ln X}{dX}$$

$$\frac{dY}{dX} = \beta_2 \frac{1}{X} \frac{\cancel{dX}}{\cancel{dX}}$$

$$\beta_2 = X \cdot \frac{dY}{dX} = \frac{dY}{\frac{dX}{X}} = \frac{\text{ABSOLUTE CHANGE IN } Y}{\text{RELATIVE CHANGE IN } X} \rightarrow \frac{X_{NEW} - X_{OLD}}{X_{OLD}}$$

$$\boxed{\frac{\beta_2}{100} = \frac{dY}{\frac{dX}{X} \cdot 100} \quad \begin{array}{l} \rightarrow \text{ABSOLUTE CHANGE IN } Y \\ \rightarrow \text{PERCENTAGE CHANGE IN } X \end{array}}$$

It is used to tackle w/ the question:

$$\boxed{\text{If } X \text{ changes by } 1\%, Y \text{ will change by } \underline{?} \text{ unit(s)}}$$

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The Reciprocal Models

$$Y_i = B_1 + B_2\left(\frac{1}{X_i}\right) + u_i$$

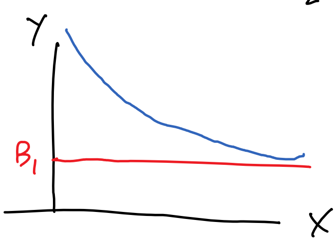
$$\boxed{Y_i = B_1 + B_2\left(\frac{1}{X_i}\right)}$$

$$\frac{dy}{dx} = \frac{d\left(B_2 X^{-1}\right)}{dx} = B_2(-1)X^{-1-1} = B_2(-1)X^{-2} = -B_2 \cdot \frac{1}{X^2} \quad \text{OR} \quad = -\frac{B_2}{X^2}$$

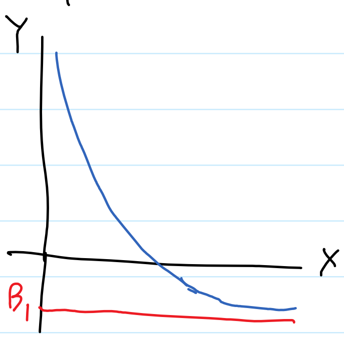
SO $$\boxed{\frac{dy}{dx} = -B_2 \cdot \left[\frac{1}{X^2}\right]}$$

→ SAYS THAT IF X INCREASES INDEFINITELY, $B_2 \frac{1}{X}$ WILL APPROACH TO ZERO, AND THEN Y WILL APPROACH TO ITS ASYMTOTE VALUE OF B_1.

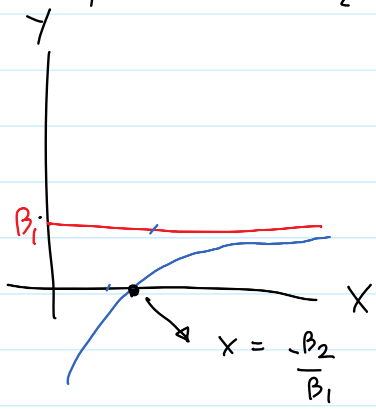
IF $B_1 > 0$ AND $B_2 > 0$, $\frac{dy}{dx}$ OR SLOPE WILL BE NEGATIVE.



IF $B_1 < 0$ AND $B_2 > 0$



IF $B_1 > 0$ AND $B_2 < 0$, SLOPE WILL BE POSITIVE.



$X = \frac{-B_2}{B_1}$ WHEN $Y = 0$

$$\boxed{\text{NOTE } \frac{dy}{dx} = \frac{-B_2}{X^2}}$$

$$\boxed{Y = B_1 + B_2 \frac{1}{X}}$$

IF $B_1 < 0$ AND $B_2 < 0$, SLOPE WILL BE POSITIVE.