8. The investor's degree of risk aversion is characterized by the slope of his or her indifference curve. Indifference curves show, at any level of expected return and risk, the required risk premium for taking on one additional percentage point of standard deviation. More risk-averse investors have steeper indifference curves; that is, they require a greater risk premium for taking on more risk.
9. The optimal position, y^*, in the risky asset, is proportional to the risk premium and inversely proportional to the variance and degree of risk aversion:

$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2}$$

Graphically, this portfolio represents the point at which the indifference curve is tangent to the CAL.

10. A passive investment strategy disregards security analysis, targeting instead the risk-free asset and a broad portfolio of risky assets such as the S&P 500 stock portfolio. If in 2009 investors took the mean historical return and standard deviation of the S&P 500 as proxies for its expected return and standard deviation, then the values of outstanding assets would imply a degree of risk aversion of about $A = 2.15$ for the average investor. This is in line with other studies, which estimate typical risk aversion in the range of 2.0 through 4.0.

Related Web sites for this chapter are available at **www.mheducation.asia/olc/bodie**

Key Terms

risk premium	risk neutral	risk-free asset
fair game	risk lover	capital allocation line
risk averse	mean-variance (M-V) criterion	reward-to-volatility ratio
utility	indifference curve	passive strategy
certainty equivalent rate	complete portfolio	capital market line

Problem Sets

Basic

1. Which of the following choices best completes the following statement? Explain. An investor with a higher degree of risk aversion, compared to one with a lower degree, will prefer investment portfolios
   a. with higher risk premiums.
   b. that are riskier (with higher standard deviations).
   c. with lower Sharpe ratios.
   d. with higher Sharpe ratios.
   e. None of the above is true.
2. Which of the following statements are true? Explain.
   a. A lower allocation to the risky portfolio reduces the Sharpe (reward-to-volatility) ratio.
   b. The higher the borrowing rate, the lower the Sharpe ratios of levered portfolios.
   c. With a fixed risk-free rate, doubling the expected return and standard deviation of the risky portfolio will double the Sharpe ratio.
   d. Holding constant the risk premium of the risky portfolio, a higher risk-free rate will increase the Sharpe ratio of investments with a positive allocation to the risky asset.
3. What do you think would happen to the expected return on stocks if investors perceived higher volatility in the equity market? Relate your answer to Equation 6.7.

Intermediate

4. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either $70,000 or $200,000 with equal probabilities of .5. The alternative risk-free investment in T-bills pays 6% per year.

a. If you require a risk premium of 8%, how much will you be willing to pay for the portfolio?

b. Suppose that the portfolio can be purchased for the amount you found in (*a*). What will be the expected rate of return on the portfolio?

c. Now suppose that you require a risk premium of 12%. What is the price that you will be willing to pay?

d. Comparing your answers to (*a*) and (*c*), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?

5. Consider a portfolio that offers an expected rate of return of 12% and a standard deviation of 18%. T-bills offer a risk-free 7% rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to bills?

6. Draw the indifference curve in the expected return–standard deviation plane corresponding to a utility level of .05 for an investor with a risk aversion coefficient of 3. (*Hint:* Choose several possible standard deviations, ranging from 0 to .25, and find the expected rates of return providing a utility level of .05. Then plot the expected return–standard deviation points so derived.)

7. Now draw the indifference curve corresponding to a utility level of .05 for an investor with risk aversion coefficient $A = 4$. Comparing your answer to Problem 6, what do you conclude?

8. Draw an indifference curve for a risk-neutral investor providing utility level .05.

9. What must be true about the sign of the risk aversion coefficient, A, for a risk lover? Draw the indifference curve for a utility level of .05 for a risk lover.

For Problems 10 through 12: Consider historical data showing that the average annual rate of return on the S&P 500 portfolio over the past 80 years has averaged roughly 8% more than the Treasury bill return and that the S&P 500 standard deviation has been about 20% per year. Assume these values are representative of investors' expectations for future performance and that the current T-bill rate is 5%.

10. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

w_{bills}	w_{index}
0	1.0
0.2	0.8
0.4	0.6
0.6	0.4
0.8	0.2
1.0	0

11. Calculate the utility levels of each portfolio of Problem 10 for an investor with $A = 2$. What do you conclude?

12. Repeat Problem 11 for an investor with $A = 3$. What do you conclude?

Use these inputs for Problems 13 through 19: You manage a risky portfolio with expected rate of return of 18% and standard deviation of 28%. The T-bill rate is 8%.

13. Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund. What is the expected value and standard deviation of the rate of return on his portfolio?

14. Suppose that your risky portfolio includes the following investments in the given proportions:

Stock A	25%
Stock B	32%
Stock C	43%

What are the investment proportions of your client's overall portfolio, including the position in T-bills?

15. What is the reward-to-volatility ratio (S) of your risky portfolio? Your client's?
16. Draw the CAL of your portfolio on an expected return–standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.
17. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16%.
    a. What is the proportion y?
    b. What are your client's investment proportions in your three stocks and the T-bill fund?
    c. What is the standard deviation of the rate of return on your client's portfolio?
18. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%.
    a. What is the investment proportion, y?
    b. What is the expected rate of return on the complete portfolio?
19. Your client's degree of risk aversion is $A = 3.5$.
    a. What proportion, y, of the total investment should be invested in your fund?
    b. What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?
20. Look at the data in Table 6.7 on the average risk premium of the S&P 500 over T-bills, and the standard deviation of that risk premium. Suppose that the S&P 500 is your risky portfolio.
    a. If your risk-aversion coefficient is $A = 4$ and you believe that the entire 1926–2009 period is representative of future expected performance, what fraction of your portfolio should be allocated to T-bills and what fraction to equity?
    b. What if you believe that the 1968–1988 period is representative?
    c. What do you conclude upon comparing your answers to (a) and (b)?
21. Consider the following information about a risky portfolio that you manage, and a risk-free asset: $E(r_P) = 11\%$, $\sigma_P = 15\%$, $r_f = 5\%$.
    a. Your client wants to invest a proportion of her total investment budget in your risky fund to provide an expected rate of return on her overall or complete portfolio equal to 8%. What proportion should she invest in the risky portfolio, P, and what proportion in the risk-free asset?
    b. What will be the standard deviation of the rate of return on her portfolio?
    c. Another client wants the highest return possible subject to the constraint that you limit his standard deviation to be no more than 12%. Which client is more risk averse?
22. Investment Management Inc. (IMI) uses the capital market line to make asset allocation recommendations. IMI derives the following forecasts:
    - Expected return on the market portfolio: 12%.
    - Standard deviation on the market portfolio: 20%.
    - Risk-free rate: 5%.

    Samuel Johnson seeks IMI's advice for a portfolio asset allocation. Johnson informs IMI that he wants the standard deviation of the portfolio to equal half of the standard deviation for the market portfolio. Using the capital market line, what expected return can IMI provide subject to Johnson's risk constraint?

For Problems 23 through 26: Suppose that the borrowing rate that your client faces is 9%. Assume that the S&P 500 index has an expected return of 13% and standard deviation of 25%, that $r_f = 5\%$, and that your fund has the parameters given in Problem 21.

23. Draw a diagram of your client's CML, accounting for the higher borrowing rate. Superimpose on it two sets of indifference curves, one for a client who will choose to borrow, and one who will invest in both the index fund and a money market fund.
24. What is the range of risk aversion for which a client will neither borrow nor lend, that is, for which $y = 1$?
25. Solve Problems 23 and 24 for a client who uses your fund rather than an index fund.
26. What is the largest percentage fee that a client who currently is lending ($y < 1$) will be willing to pay to invest in your fund? What about a client who is borrowing ($y > 1$)?
    d. What is the Jensen measure for the Miranda Fund?

Challenge

For Challenge Problems 27, 28 and 29: You estimate that a passive portfolio, that is, one invested in a risky portfolio that mimics the S&P 500 stock index, yields an expected rate of return of 13% with a standard deviation of 25%. You manage an active portfolio with expected return 18% and standard deviation 28%. The risk-free rate is 8%.

27. Draw the CML and your funds' CAL on an expected return–standard deviation diagram.
    a. What is the slope of the CML?
    b. Characterize in one short paragraph the advantage of your fund over the passive fund.
28. Your client ponders whether to switch the 70% that is invested in your fund to the passive portfolio.
    a. Explain to your client the disadvantage of the switch.
    b. Show him the maximum fee you could charge (as a percentage of the investment in your fund, deducted at the end of the year) that would leave him at least as well off investing in your fund as in the passive one. (*Hint:* The fee will lower the slope of his CAL by reducing the expected return net of the fee.)
29. Consider again the client in Problem 19 with $A = 3.5$.
    a. If he chose to invest in the passive portfolio, what proportion, y, would he select?
    b. Is the fee (percentage of the investment in your fund, deducted at the end of the year) that you can charge to make the client indifferent between your fund and the passive strategy affected by his capital allocation decision (i.e., his choice of y)?

CFA® PROBLEMS

Use the following data in answering CFA Problems 1–3:

Utility Formula Data

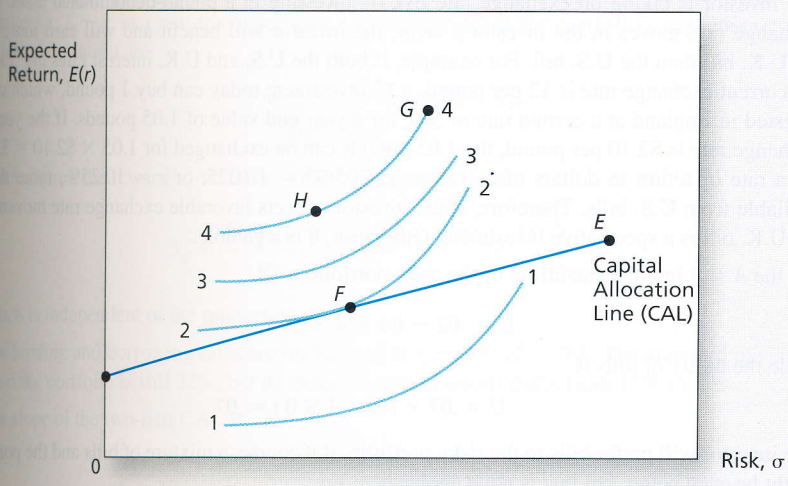
Investment	Expected Return, $E(r)$	Standard Deviation, σ
1	.12	.30
2	.15	.50
3	.21	.16
4	.24	.21

$U = E(r) - \frac{1}{2} A\sigma^2$, where $A = 4$

1. On the basis of the utility formula above, which investment would you select if you were risk averse with $A = 4$?
2. On the basis of the utility formula above, which investment would you select if you were risk neutral?
3. The variable (A) in the utility formula represents the:
   a. investor's return requirement.
   b. investor's aversion to risk.

c. certainty equivalent rate of the portfolio.
d. preference for one unit of return per four units of risk.

Use the following graph to answer CFA Problems 4 and 5.



4. Which indifference curve represents the greatest level of utility that can be achieved by the investor?
5. Which point designates the optimal portfolio of risky assets?
6. Given $100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills on the basis of the following table?

Action	Probability	Expected Return
Invest in equities	.6	$50,000
	.4	−$30,000
Invest in risk-free T-bills	1.0	$ 5,000

7. The change from a straight to a kinked capital allocation line is a result of the:
   a. Reward-to-volatility ratio increasing.
   b. Borrowing rate exceeding the lending rate.
   c. Investor's risk tolerance decreasing.
   d. Increase in the portfolio proportion of the risk-free asset.
8. You manage an equity fund with an expected risk premium of 10% and an expected standard deviation of 14%. The rate on Treasury bills is 6%. Your client chooses to invest $60,000 of her portfolio in your equity fund and $40,000 in a T-bill money market fund. What is the expected return and standard deviation of return on your client's portfolio?
9. What is the reward-to-volatility ratio for the *equity fund* in CFA Problem 8?

E-Investments Exercises

Risk Aversion

There is a difference between an investor's *willingness* to take risk and his or her *ability* to take risk. Take the quizzes offered at the Web sites below and compare the results. If they are significantly different, which one would you use to determine an investment strategy?

http://mutualfunds.about.com/library/personalitytests/blrisktolerance.htm
http://mutualfunds.about.com/library/personalitytests/blriskcapacity.htm