EE320 (2/2013)

INTRODUCTORY MATHEMATICAL ECONOMICS

MATHEMATICS AND ECONOMIC RELATIONS

Recap

- Last time, we started talking about the structure of a mathematical economic model, which generally consists of a systems of equations.
- Also, to analyze an economic problem, we need to first determine what are dependent or independent variables to be included in the model.
- So, today's lecture will start with a review of set theory, the concepts of relation and functions, and discuss possible forms of functions that can be used to characterize the *behavioral* equations.

Topics

- Review of set theory
- Relations and functions
  - Ordered pairs and Cartesian products
  - Relations and Functions
  - Inverse Function
- Types of functions
  - Constant functions
  - Polynomial functions
  - Rational functions
  - Non-algebraic functions
  - Functions of two or more independent variables

Set Notation

- A *set* is a collection of distinct objects (e.g. numbers, persons, food items, etc.).
- The objects in a set are called the *elements* of the set.
- Two ways of writing a set:
  - By enumeration:

    A = (7,11,13,17,19)

    S = {apple, blackberry, computer}
  - By description:

    I = {x| x is a positive integer}

    J = {x| 5 < x < 20}

Set Notation (cont'd)

- A *finite* set: A set with a finite number of elements
- An *infinite* set: A set with infinite number of elements

Examples:

B = {3,6,9}

C = {x| 10 < x < 29}

D = {x| x is a rational number}

Questions: Which of the sets above are finite or infinite?

➔ B is a finite set. C and D are infinite sets.

- $\in$ indicates membership in a set:

E.g. $9 \in B$ $9 \notin C$

Relationships between Sets

- Set equality
  - e.g. If A = {2, 5, r, t} and B = {2, r, 5, t}, then A = B.
- Subset: $T \subset S$ iff $x \in T$ implies $x \in S$.
  - e.g. If S = {2, 3, 4, 6} and T = (3, 4}, then $T \subset S$.
  - A proper subset of S is any subset that does not contain *all* the elements of S.
  - Every set is a subset of itself.
- Null set or empty set: { } or Φ.
  - Every set contains the null set.
- Disjoint sets:
  - e.g. Sets A and T are disjoint.

Operations on Sets

- Intersection: $C = A \cap B$

  $$C = \{a \mid a \in A \wedge a \in B\}$$

- Union: $C = A \cup B$

  $$C = \{a \mid a \in A \vee a \in B\}$$

- Complement of a set: $\widetilde{A}$

  $$\widetilde{A} = \{x \mid x \in U \wedge x \notin A\}$$

- Example: Let U = {1,2,3,4,5,6,7}, A = {3,4,5,6}, and B = {5,6,7,8}.

  $$A \cap B = \{5,6\}$$

  $$A \cup B = \{3,4,5,6,7,8\}$$

  $$\widetilde{A} = \{1,2,7\}$$

Laws of Set Operations

- Commutative law:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

- Associative law:

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive law:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Ordered Pairs and Cartesian Products

- Ordered pairs
  - Unordered pair : {a, b} = {b, a} → Ordering does not matter.
  - *Ordered pair*: The ordering of a and b matters.

    (a, b) ≠ (b, a), unless a= b.

  Example: (h, w) = (height in cm., weight in kg.) of students in class

  e.g. (h,w) = (165, 54). Would writing (54, 165) make any senses?

- Given two sets x and y, the *Cartesian product* is the set of all the possible ordered pairs with the first element taken from set x and the second element taken from set y:

$$x \times y = \{(a,b) \mid a \in x \wedge b \in y\}$$

Relations and Functions

- Any subset of the Cartesian product x x y will constitute a *relation* between y and x. Given an x value, one or more y values will be specified by that relation.
- Example: $\{(x, y) \mid y = 2x\}$ and $\{(x, y) \mid y \geq x\}$

Relations and Functions (cont'd)

- A function is a special case of a relation, in which for each x value there exists only *one* corresponding y value.
- Thus, a *function* is a set of ordered pairs with the property that any x value uniquely determines a y value.

$$y = f(x)$$

where x is the *argument* of the function (i.e. independent variable) and y is the *value* of the function (i.e. dependent variable).

- Alternative names of a function: a *mapping* or *transformation*:

$$f: x \longrightarrow y$$

  - Domain: The set of all permissible values that x can take.
  - Range: The set of all images (y value into which an x value is mapped)

Example: Domain and Range

- The total cost C of a firm is a function of its daily output Q: $C = 300 + 4Q$.
- Suppose that there is a capacity limit of 200 units of output per day.
- What are the domain and range of the cost function?

Domain = $\{Q \mid 0 \le Q \le 200\}$

Range:

If $Q=0$, $C= 300$.

If $Q=200$, $C = 300 + (4 \times 200) = 1100$

➔ Range = $\{C \mid 300 \le C \le 1100\}$

Inverse Function

- Let y = f(x) be an invertible function, and f maps X to Y.

  ➔ The inverse function $f^{-1}(x) = g(y)$ maps Y to X.

- Example: Given $y = f(x) = 3x + 2$, what is f^{-1} ?

  ➔ The inverse function: $x = g(y) = (y-2)/3$

- Example:

  Suppose a demand function is given by $Q_d = 100 – 0.2P$. Derive the inverse demand function.

  ➔ $P = 500 - 5Q_d$

Constant Function

- A constant function is a function whose range consists of only one element:

  $y = f(x) = y_0$, where y_0 is a constant.

- *Example*: In a cost function of a firm, the total cost (TC) includes only a fixed cost (F), say $50. Thus, the cost function can be written as: TC = F = 50.

- Graph

Polynomial Functions

- General form of a polynomial function of a variable x:

  $$y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

  where $a_0 \ldots a_m$ are the coefficients and the value of n is the degree of the polynomial function.

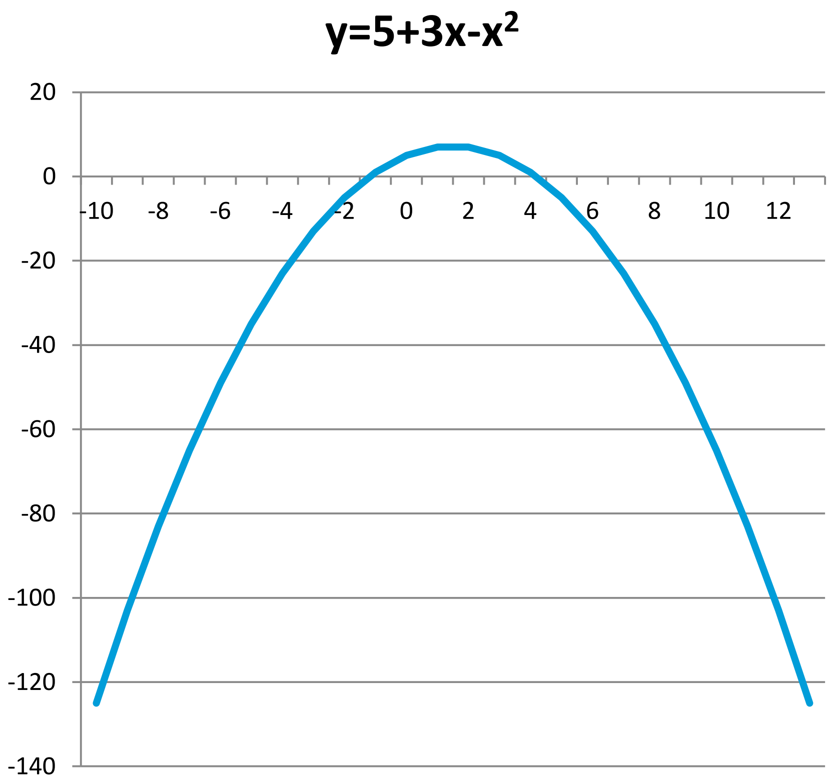
- Subclass of polynomial functions:
  - n=0: $y = a_0$ [constant function]
  - n=1: $y = a_0 + a_1x$ [linear function]
  - n=2: $y = a_0 + a_1x + a_2x^2$ [quadratic function]
  - n=3: $y = a_0 + a_1x + a_2x^2 + a_3x^3$ [cubic function]

Linear Function: $y = a_0 + a_1 x$

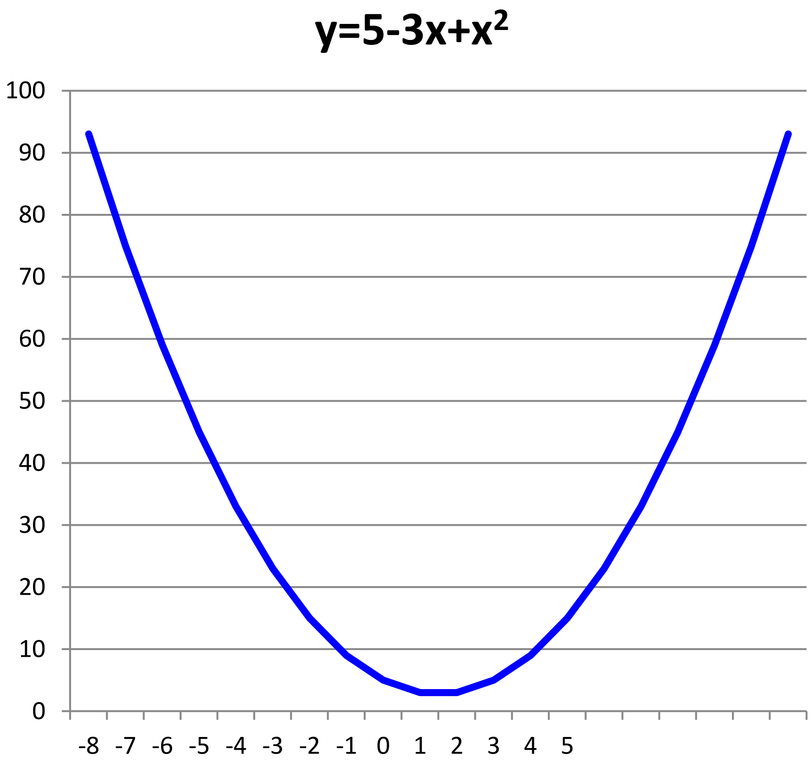
- Example: $TC = FC + VC = a + bQ$

Quadratic Function: $y = a_0 + a_1x + a_2x^2$

- Case of $a_2 < 0$

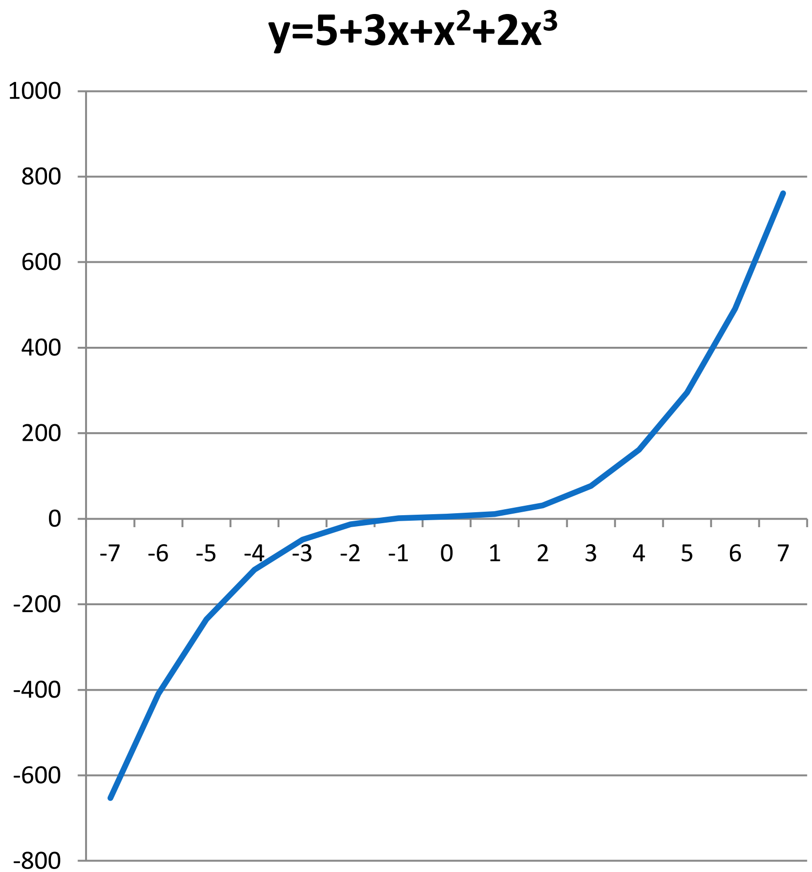


- Case of $a_2 > 0$

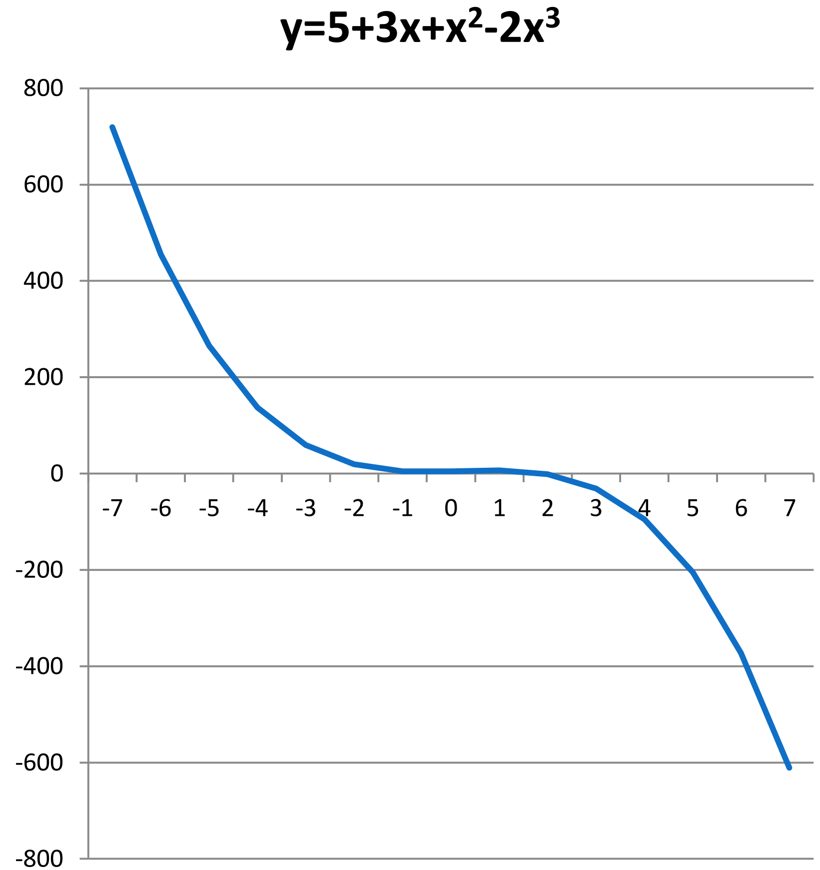


Cubic Function: $y = a_0 + a_1x + a_2x^2 + a_3x^3$

- Case of $a_3 > 0$



- Case of $a_3 < 0$



Rational Functions

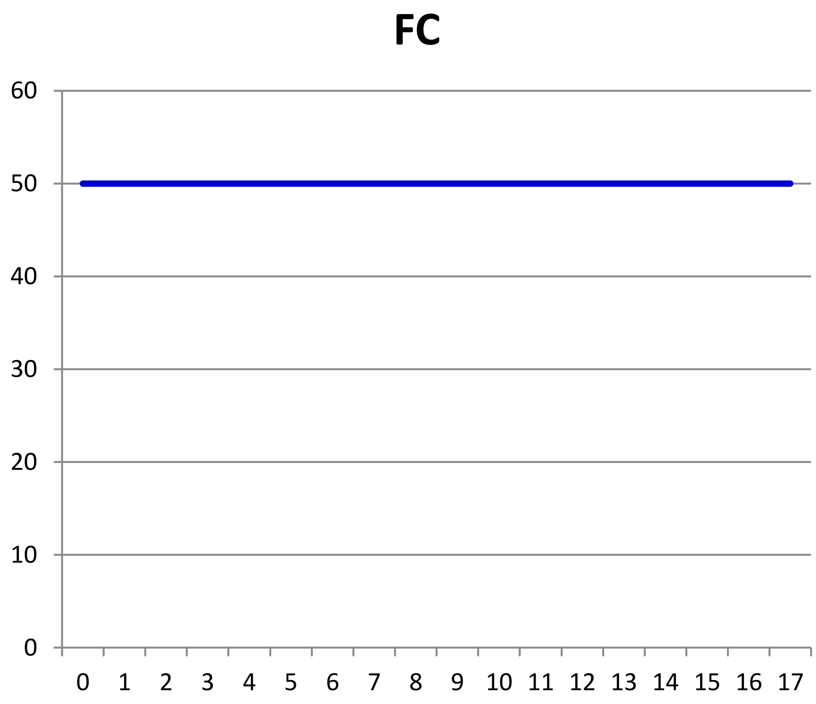
- A *rational function* is a function expressed as a ratio of two polynomials in the variable x: $y = f(x)/g(x)$, where $g(x) \neq 0$.
- Special case: A rectangular hyperbola:

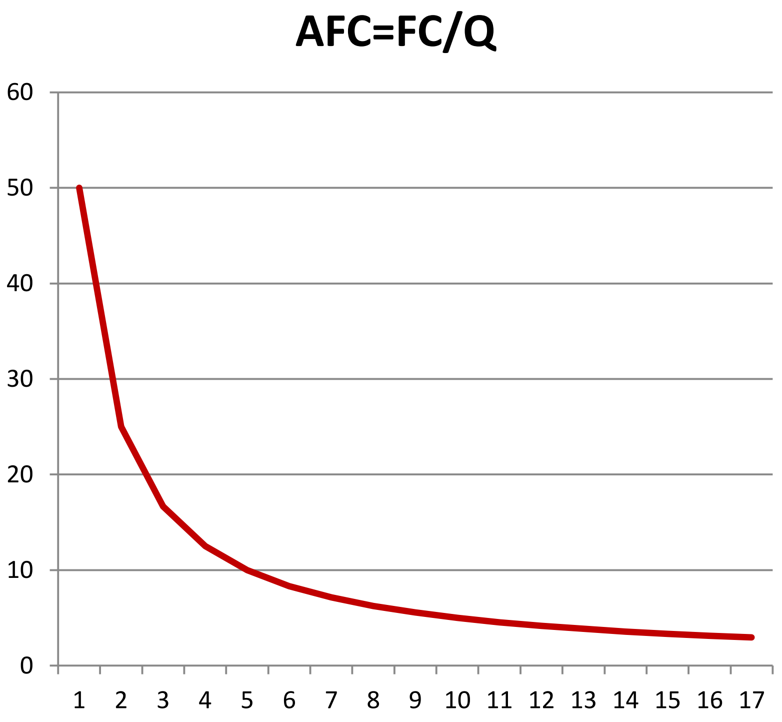
$$y = a/x \quad \text{or} \quad xy = a.$$

- Applications:
  - A demand curve $Q = f(P)$ with constant total expenditure PQ, where the price elasticity is unitary.
  - An average fixed cost (AFC) curve: $AFC = f(Q) = c$, where c is a constant. Here, total fixed cost is a constant because $TFC = cQ$.

Example: Rectangular hyperbola

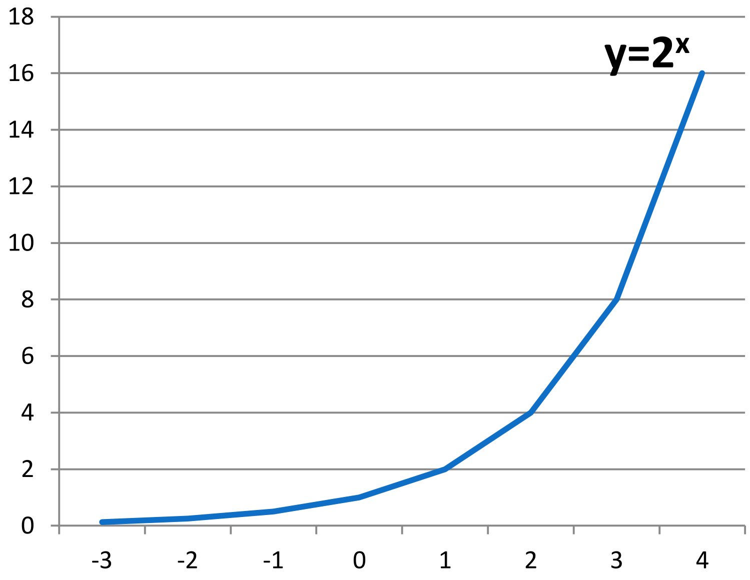
- Suppose FC = 50.

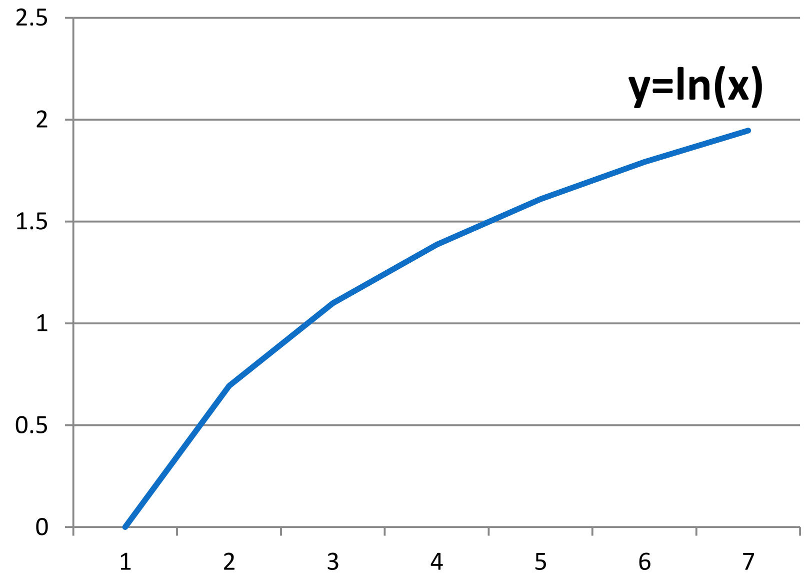




Nonalgebraic Function

- A *nonalgebraic* function is a function that *cannot* be expressed in terms of polynomials and/or roots of polynomials.
  - Exponential function: $y = b^x$
  - Logarithm function: $y = \log_b x$
- Graphs





Example: Exponential Function

Functions of Multiple Independent Variables

- Given two independent variables x and y, a function of uniquely determined (dependent) variable z is:

$$z = g(x, y),$$

where the domain of the function will be some subset of the points in the xy plane.

- The association between the three variables are summarized by the ordered triple (x, y, z).
- Examples:
  - Production function: $Q = Q(K, L)$
  - Utility function: $U = U(x_1, x_2)$

Example: Production function in a 3-dimension space