HW

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1) Find cournot equilibrium when there are 3 firms in the market

$P = a - bQ$, $Q = q_1 + q_2 + q_3$, $c_1 = c_2 = c_3 = c$

What is equilibrium price? p^*

What are firm's profit? $\pi_1 = \pi_2 = \pi_3 = ?$

- Firm 1: $\pi = pq - c_1$

$$= [a - b(q_1 + q_2 + q_3)]q_1 - c_1$$

$$= aq_1 - q_1^2 b - q_1 q_2 b - q_1 q_3 b - c_1$$

$$\frac{\partial \pi}{\partial q_1} = a - 2q_1 b - q_2 b - q_3 b$$

$$2q_1 b = a - q_2 b - q_3 b$$

$$q_1 = \frac{a - q_2 b - q_3 b}{2b}$$

$$q_1 = \frac{a - \left(\frac{a - q_3 b}{3b}\right) b - q_3 b}{2b}$$

$$q_1 = \frac{3a - a + q_3 b - 3q_3 b}{6b}$$

$$= \frac{2a - 2q_3 b}{6b} = \frac{a - q_3 b}{3b} = \frac{a - \left(\frac{a}{4b}\right) b}{3b}$$

$$= \frac{4a - a}{12b} = \frac{a}{4b} \quad \#$$

- Firm 2: $\pi = p \cdot q_2 - c_2$

$$= [a - b(q_1 + q_2 + q_3)]q_2 - c_2$$

$$= aq_2 - q_1 q_2 b - q_2^2 b - q_2 q_3 b - c_2$$

$$\frac{\partial \pi}{\partial q_2} = a - q_1 b - 2q_2 b - q_3 b$$

$$0 = a - q_1 b - 2q_2 b - q_3 b$$

$$2q_2 b = a - q_1 b - q_3 b$$

$$2q_2 b = a - \left(\frac{a - q_2 b - q_3 b}{2b}\right) b - q_3 b$$

$$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$$

$$4q_2 b = a + q_2 b - q_3 b$$

$$3q_2 b = a - q_3 b$$

$$q_2 = \frac{a - q_3 b}{3b} = \frac{a - \left(\frac{a}{4b}\right) b}{3b} = \frac{4a - a}{12b}$$

$$= \frac{3a}{12b} = \frac{a}{4b} \quad \#$$

- Firm 3: $\pi = pq_3 - c_3 = (a - b(q_1 + q_2 + q_3))q_3 - c_3 = aq_3 - bq_1 q_3 - b q_2 q_3 - bq_3^2 - c_3$

$$\frac{\partial \pi}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3$$

$$2bq_3 = a - bq_1 - bq_2$$

$$q_3 = \frac{a - bq_1 - bq_2}{2b} = \frac{a - b\left(\frac{a - q_3 b}{3b}\right) - b\left(\frac{a - q_3 b}{3b}\right)}{2b} = \frac{3a - a + q_3 b - a + q_3 b}{6b}$$

$$6bq_3 = a + 2q_3 b$$

$$4bq_3 = a$$

$$q_3 = \frac{a}{4b} \quad \#$$

- Equilibrium price: $p = a - bQ$

$$= a - b(q_1 + q_2 + q_3)$$

$$= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right) = a - b\left(\frac{3a}{4b}\right) = \frac{4a - 3a}{4} = \frac{a}{4}$$

$$\begin{cases} \pi_1 = pq - c_1 = \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - c_1 = \frac{a^2}{16b} - c_1 \\ \pi_2 = pq - c_1 = \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - c_2 = \frac{a^2}{16b} - c_2 \\ \pi_3 = pq - c_1 = \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - c_3 = \frac{a^2}{16b} - c_3 \end{cases}$$

2) If there are N firms

$$\begin{cases} q_i^* = f(N) \text{ (production of each individual firm)} \\ p_i = f(N) \text{ (price of the mkt)} \\ \pi_i = f(N) \end{cases}$$

$p = a - bQ$; $Q = q_n$

$C_N = C_N = C$

Assume that $q_1 + q_2 + q_3 + \dots q_n = A$

$$p = a - b(q_1 + q_2 + \dots q_n)$$

$$p = a - bq_1 - bq_2 - \dots bq_n$$

$$\pi_1 = (a - bq_1 - bq_2 \dots - bq_n)q_1 - c_1$$

$$\vdots$$

$$\pi_n = (a - bq_1 - bq_2 - \dots - bq_n)q_n - c_n$$

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_1 - bq_2 - \dots - bq_n = 0$$

$$\frac{a}{2b} - 0.5(q_2 + q_3 + \dots q_n) = q_1$$

$$\frac{a}{2b} - 0.5(q_1 + q_2 + \dots q_n) = q_n$$

$$q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots q_n)$$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A$$

$$q_2 = \frac{a}{b} - A$$

$$\vdots$$

$$q_n = \frac{a}{b} - A$$

$$q_1 + q_2 + \dots q_n \rightarrow A = n\left(\frac{a}{b} - A\right) = nq$$

$$= n\frac{a}{b} - nA$$

$$(n+1)A = \frac{na}{b}$$

$$A = nq \rightarrow A = \frac{na}{(n+1)b}$$

$$q_i = \frac{a}{(n+1)b}$$

$$P = a - b(A) = a - b\left(\frac{na}{(n+1)b}\right)$$

$$P = a - \frac{n}{n+1}(a) = \frac{a(n+1) - na}{n+1} = \frac{na + a - na}{n+1}$$

$$P = \frac{a}{n+1}$$

$$\pi_i = p q_i - c_i = \frac{a}{n+1} \times \frac{a}{(n+1)b} - c_i$$

$$\pi_i = \frac{a^2}{(n+1)b} - c_i$$

3) from question 2,

what happen if , $N \to \infty$?

, $N \to 1$?

if $n \to \infty$ $\quad q_i = \frac{a}{(n+1)b} \to 0$

$$A = n q_i \to \infty$$

$$p = \frac{a}{n+1} \to 0$$

$$\pi = \frac{a^2}{(n+1)^2 b} - c_i \to -c_i$$

if $n \to 1$ $\quad q_i = \frac{a}{2b} \qquad Q = \frac{a}{2b} < Q = \frac{na}{(n+1)b}$

$A = nq_i = Q$ ✱ $n = 1$ firm is monopoly

$P = \frac{a}{n+1} = \frac{a}{2}$ ✱ $P_m = \frac{a}{2} > P = \frac{a}{n+1}$

$$\pi = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_1 = \pi_m > \pi_i = \frac{a^2}{(n+1)^2 b} - c_1$$