Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1a. $Y_i = \beta_1 + \beta_2 X_i + u_i$

$$\hat{b}_2 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})}$$

$$\hat{b}_2 = \frac{-174.20}{1098.8} = -0.1585$$

$$\hat{b}_1 = \bar{y} - \hat{b}_2 \bar{x}$$

$$\hat{b}_1 = 21.03 - (-0.1585)(12.20) = 22.9637$$

$$\therefore \quad \hat{y} = 22.9637 - 0.1585 X_i$$

If x change by 1 unit, y will change by 0.1585 unit in opposite direction.
If x = 0, y = 22.9637.

b. $$r^2 = 1 - \frac{\Sigma \hat{u}^2}{\Sigma(y-\bar{y})^2}$$

$$r^2 = 1 - \frac{873.14}{882.97} = 0.0111$$

X and y only have a relationship of 1.11%

c. $\hat{y} = 22.9637 - 0.1585 X_i$ $\quad X_i = 5$

$\hat{y} = 22.9637 - 0.1585(5)$

$\hat{y} = 22.1712$

If $X_i = 5$, the value of $\hat{y}$ will be 22.1712

d. $var(u_i) = \sigma^2 = \dfrac{\sum \hat{u}^2}{n-k}$

$var(u_i) = \dfrac{873.14}{30-2} = 31.1836$

$var(b_1) = \sigma^2 \left[\dfrac{1}{n} + \dfrac{\bar{x}^2}{\sum(x-\bar{x})^2} \right]$

$var(b_1) = 31.1836 \left[\dfrac{1}{30} + \dfrac{12.20^2}{1098.8} \right] = 5.2625$

$var(b_2) = \dfrac{\sigma^2}{\sum(x-\bar{x})^2}$

$var(b_2) = \dfrac{31.1836^2}{1098.8^2} = \dfrac{972.4169}{1098.8} = 0.8850$

e. H_0 $b_1 = 0$ at significant level of 0.05
H_1 $b_1 \neq 0$

$$t = \frac{\hat{b}_1 - b_1}{\sqrt{var b_1}} = \frac{22.9637 - 0}{\sqrt{5.2625}} = 10.0103$$

$df = n - k$
$= 30 - 2$
$= 28$

t-table $= 2.04841$ at 0.025

$|t\text{-call}| > |t\text{-table}|$

accept H_1

at 0.05 significant level, b_1 is not equal to 0

H_0 $b_2 = 0$ at significant level of 0.05
H_1 $b_2 \neq 0$

$$t = \frac{\hat{b}_2 - b_2}{\sqrt{Var(b_2)}}$$

$$t = \frac{-0.1585 - 0}{\sqrt{0.8850}} = -0.1685$$

t-table : $df = n - k = 30 - 2 = 28$

t-table = 2.0484

$|t\ cal| < |t\text{-}table|$

accept H_0

coefficient b_2 is not different from zero at 0.05 significant level

f. H_0 $\beta_1 \geq 0$ at significant level $= 0.01$

H_1 $\beta_1 < 0$

$$t = \frac{\hat{b}_1 - \beta_1}{\sqrt{var(\hat{b}_1)}} = \frac{22.96370}{\sqrt{5.2625}} = 10.010$$

t - table $\alpha = 0.01$ $df = n - k = 30 - 2 = 28$

t- table $= -2.4671$

We accept H_0 $\therefore$ b_1 is more then/equal 0 at 0.01 sig. level

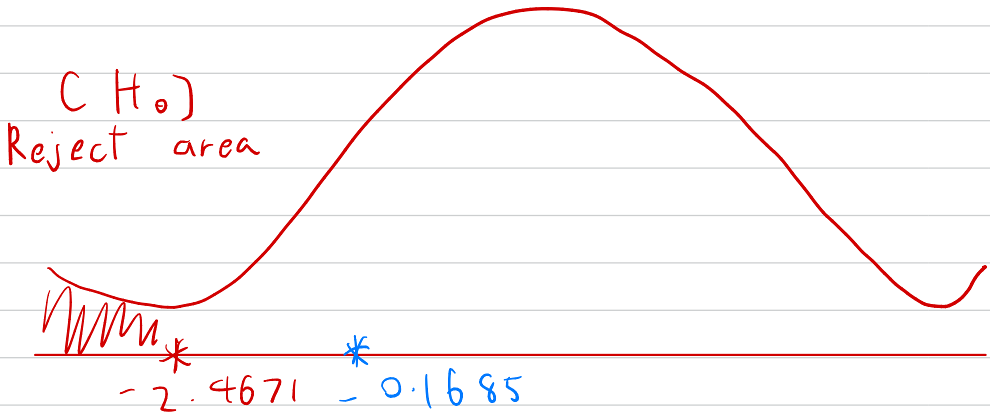
$H_0 \quad B_2 \geq 0$ at significant level $= 0.01$

$H_1 \quad B_2 < 0$

$$t = \frac{\hat{b}_2 - B_2}{\sqrt{var(b_2)}} = \frac{-0.1585 - 0}{\sqrt{0.8850}} = -0.1685$$

t-table $\alpha = 0.01 \quad df = n - k = 30 - 2 = 28$

t-table $= -2.4671$



∴ We accept H_0, b_2 is more than / equal to 0 at 0.01 sig. level

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \qquad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

2a. The sign in front of $\hat{B}_2$ makes economical sense. This is because as car aged by n year (year = x), the value of the car should be decreasing. In this case, as time increase by 1 year, price decrease by 85024.

b. $x = 5$, $\hat{y} = 7836 - 2512 = 5{,}324$

confident = 95%

$$\text{Con} = \hat{y} \pm t_{\alpha/2},\ S\sqrt{\frac{(x-\bar{x})^2}{(n-1)S_x^2}}$$

$t_{\alpha/2}$ $\quad \alpha = 1 - 0.95 = 0.05$

$\alpha/2 = 0.05/2 = 0.025$

$df = n - k = 11 - 2 = 9$

at t-distribution 0.025, df = 9, $t = 2.2616$

$$S_x^2 = \frac{\Sigma[x-\bar{x}]^2}{n-1} = \frac{78.73}{11-1} = 7.873$$

$$\text{confident interval} = \underset{(\hat{y})}{5324} \pm \underset{(t)}{2.26} \times \underset{(\text{error})}{411.8}\sqrt{\frac{[\underset{(x)}{5} - \underset{(\bar{x})}{7.45}]^2}{\underset{(n-1)}{[11-1]} \cdot \underset{(S_x^2)}{7.873}}}$$

$$= 5324 \pm 2.26 \times 411.8\sqrt{0.0762}$$

$= 5324 \pm 256.9048$

$= 5580.9048, 5067.0952$

$\therefore$ confident interval $= 5067.0952$ to 5580.9048

c. $\hat{b}_2 = \dfrac{\Sigma [x - \bar{x}][y - \bar{y}]}{\Sigma [x - \bar{x}]^2}$ $\hat{b}_1 = \bar{y} - \hat{b}_2 \bar{x}$

New: $\hat{b}_2 = \dfrac{\Sigma [10x - 10\bar{x}][y - \bar{y}]}{\Sigma [10x - 10\bar{x}]^2}$

$\hat{b}_2 = \dfrac{\cancel{10} \Sigma [x - \bar{x}][y - \bar{y}]}{10^{\cancel{2}} \Sigma [x - \bar{x}]^2}$

$\hat{b}_2^{N} = \frac{1}{10} \cdot \hat{b}_2^{old}$

$\hat{b}_2^{N} = \frac{1}{10} [-502.4] = -50.24$

$$\hat{b}_1^N = \bar{y} - (\cancel{1}/_{10} \cdot \hat{b}_2^0)(\cancel{1}0\,\bar{x})$$

$$\hat{b}_1^N = \bar{y} - \hat{b}_2^0\,\bar{x}$$

$$\hat{b}_1^N = = \hat{b}_1^0$$

$$\hat{b}_1^N = 7836$$

d. $\varepsilon = \frac{dY}{dx} \cdot \frac{x}{Y}$

$$\frac{dy}{dx} = -502.4$$

$$\hat{y} = 7,836 - 502.4(10)$$

$$\hat{y} = 2,812$$

$$\varepsilon = \frac{dy}{dx} \cdot \frac{\bar{x}}{\bar{y}} = -502.4 \cdot \frac{10}{2812}$$

$$= -1.7866$$