CHAPTER 3

Static and Comparative Static Equilibrium Analysis

Topics: Static and Comparative Static Equilibrium Analysis, PART1

Outline:

- Linear models in economics
- Simultaneous system of equations
- A partial-equilibrium market model: A model of price determination in an isolated market
- Excise Tax and Market Equilibrium
- What is elasticity ? How can we derive elasticity?
- Tax incidence and Elasticity

"Equilibrium" means

An equilibrium is a group of selected interelated variables so adjusted to one another that no inherent tendency to change is in the model.

All variables in the model must simultaneously be in a state of rest due to the balancing of the internal forces of the model, while the external factors are assumed fixed

"Static" means

Statics are equilibrium

Comparative statics = Comparing equilibrium

Nongoal type of equilibrium:

The equilibrium that is not a result of any particular objective, but is derived from an impersonal process of interaction.

e.g. underemployment equilibrium level of national income

Linear economic model vs. Nonlinear economic model

$Q^D = Q^S$	$Q^D = Q^S$
$Q^D = a - bP$	$Q^D = a - bP^2$
$Q^S = -c + dP$	$Q^S = -c + dP^2$

"Simultaneous system of equations" means

system of equations in which each equation is considered to be related

The solution to system of equations must make every equations true simultaneously.

3.1

A partial-equilibrium market model: A model of price determination in an isolated market

Suppose we are interested in one commodity, energy drink. Since only one commodity is being considered, the economic model for this market is comprised of**:**

1. Q^d Quantity demanded
2. Q^s Quantity supplied
3. P Price

After having chosen the variables, we next make certain assumptions regarding the working of the market. First, we must specify an equilibrium condition- something indispensable in an equilibrium model. The standard assumption is that equilibrium occurs in the market if and only if the excess demand is zero, that is, if and only if the market is cleared. The Clearing Market Condition can be written as:

$$Q^d = Q^s \qquad \textbf{(1)}$$

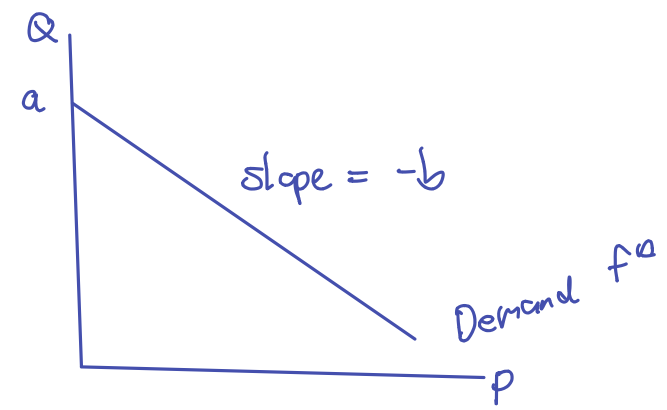
Equation (1.) is conditional equation of the market. The model requires that we specify behavioral equations to explain how exactly the demand and supply are each determined.

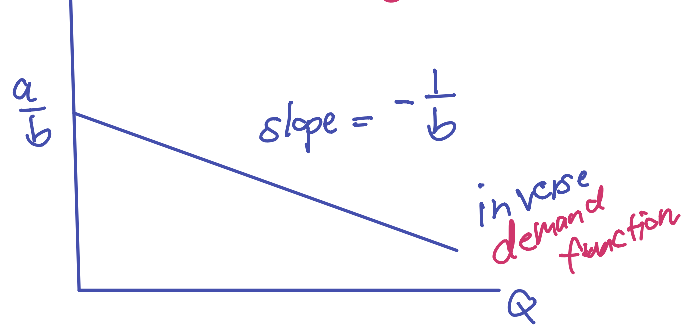
Demand for energy drink

We assume that Q_d is a decreasing linear function of P.

$Q^d = a - bP \qquad b > 0$

$P^d = \frac{a}{b} - \frac{1}{b} Q$ (2)



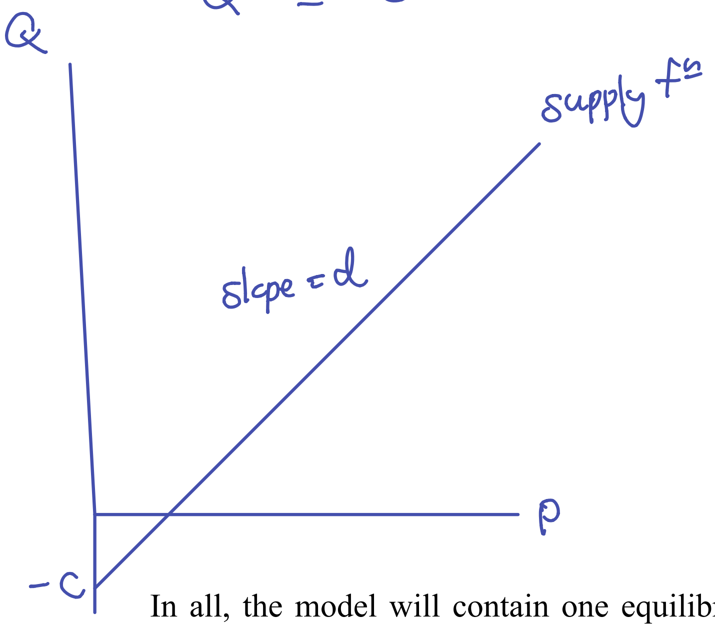


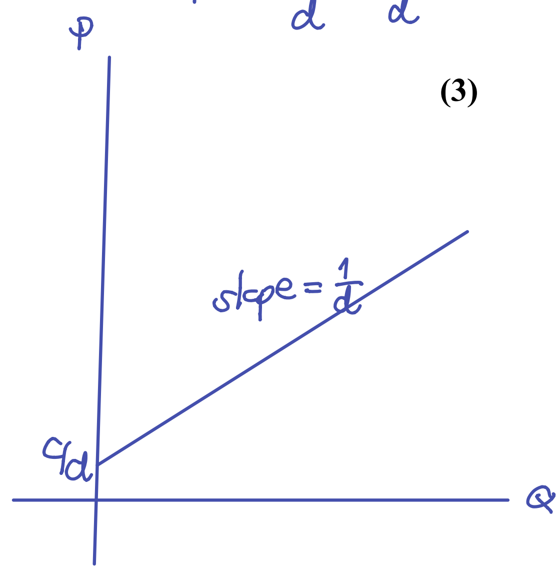
Supply for energy drink

We assume that Q_S is an increasing linear function of P.

$Q^S = -c + dP$

$P = \frac{c}{d} + \frac{1}{d} Q$ (3)





In all, the model will contain one equilibrium condition plus two behavioral equations which govern the demand and supply sides of the market.

market model

$$Q_d = Q_s \Rightarrow \text{ 1 Conditional equation}$$

$$\left.\begin{aligned} Q_d &= a - bP \quad (a, b > 0) \\ Q_s &= -c + dP \end{aligned}\right\} \text{ 2 Behavioral equations} \qquad (4)$$

!!

The next step is to obtain the solution values of the three endogenous variables, Q^d, Q^S, P.

The solution values are those values that satisfy the three equations in (4) simultaneously.

We usually denote the solution value of an endogenous variable with an asterisk. Thus, the solution values of Q_d, Q_s, P are denoted by Q^{d*}, Q^{s*}, P^*.

Since $\boxed{Q_d^* = Q_s^*,}$ they can be replaced by a single symbol Q^*.

Hence, an equilibrium solution of the model may simply be denoted by an ordered pair (Q^*, P^*).

$P^* = \frac{a+c}{b+d}$, $Q^* = \frac{ad-bc}{b+d}$

$Q^* = f(a, b, c, d)$

$P^* = g(a, b, c, d)$

Solution by Elimination of Variables:

$Q^D = Q^S$

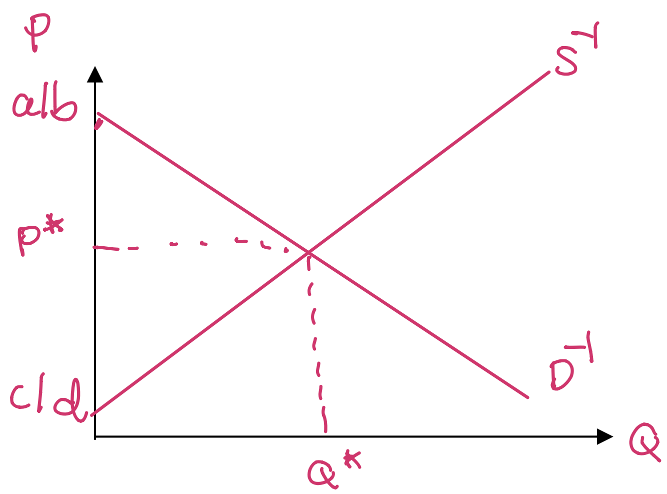
$a - bP = -c + dP$

$P^* = \frac{a+c}{b+d}$

$Q^* = Q^{D*} = a - b\left(\frac{a+c}{b+d}\right)$

$Q^* = \frac{ad-bc}{b+d}$

need to have endo = f(exo, parameter) for solution

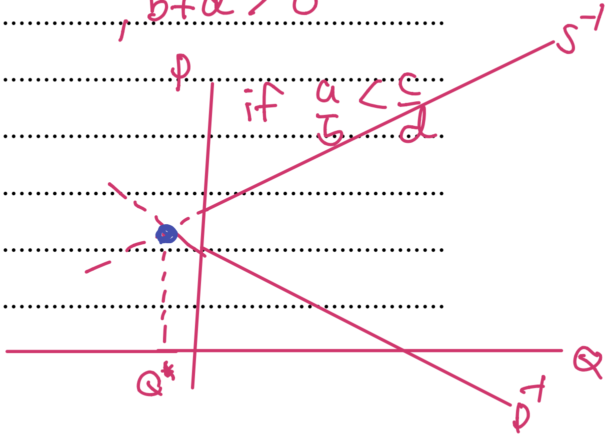


Additional restriction for an economically meaningful solution: $Q^* > 0$

$$\frac{ad-bc}{b+d} > 0 \quad , \; b+d > 0$$

$$ad - bc > 0$$

$$\frac{a}{b} > \frac{c}{d}$$



Note: Compare between partial equilibrium and general equilibrium model

Partial

$$Q_x^d = Q_x^s$$
$$Q_x^d = a - bP_x + cP_y$$
$$Q_x^s = d + eP_x$$

endo : Q_x^d, Q_x^s, P_x

exo : P_y

para : a, b, c, d, e

General

$$Q_X^d = Q_X^s$$
$$Q_X^d = a - bP_X + cP_Y$$
$$Q_X^s = d + eP_X$$
$$Q_Y^d = Q_Y^s$$
$$Q_Y^d = f - gP_Y + hP_X$$
$$Q_Y^s = j + kP_Y$$

$\left.\begin{matrix} P_X, P_Y \\ Q_X^s, Q_X^D, Q_Y^D, Q_Y^s \end{matrix}\right\}$ endo

para $\Rightarrow a, b, c, d, e, f, g, h, j, k$

3.2

Excise Tax and Market Equilibrium

We will use a partial-equilibrium market model from previous section in an analysis of the impact of excise tax.

Definition of an excise tax:

Excise taxes are narrowly based taxes on consumption, levied on specific goods, services, and activities. They can be either a per unit tax (such as the per gallon tax on gasoline) or a percentage of price (such as the airline ticket tax). Generally, excise taxes are collected from producers or wholesalers, and are embedded in the price paid by final consumers.

(a.) **<u>Specific Tax</u>** or per unit tax or unit tax: is a tax that is defined as a fixed amount for each unit of a good or service sold, such as cents per kilogram.

t baht per unit

(b.) **<u>Ad Valorem Tax</u>** is a tax whose amount is based on the value of a transaction or of property. It is a charge based on a fixed percentage of the product value. It is typically imposed at the time of a transaction, as in the case of a sale or value-added tax (VAT).

t % of price

Tax can be collected from buyers or producers, depending on convenience and efficacy of tax collection. Either way, market equilibrium will change. We can compare *market equilibrium before tax to market equilibrium after tax*, and analyze how the change is. This is called "Comparative Static Analysis".

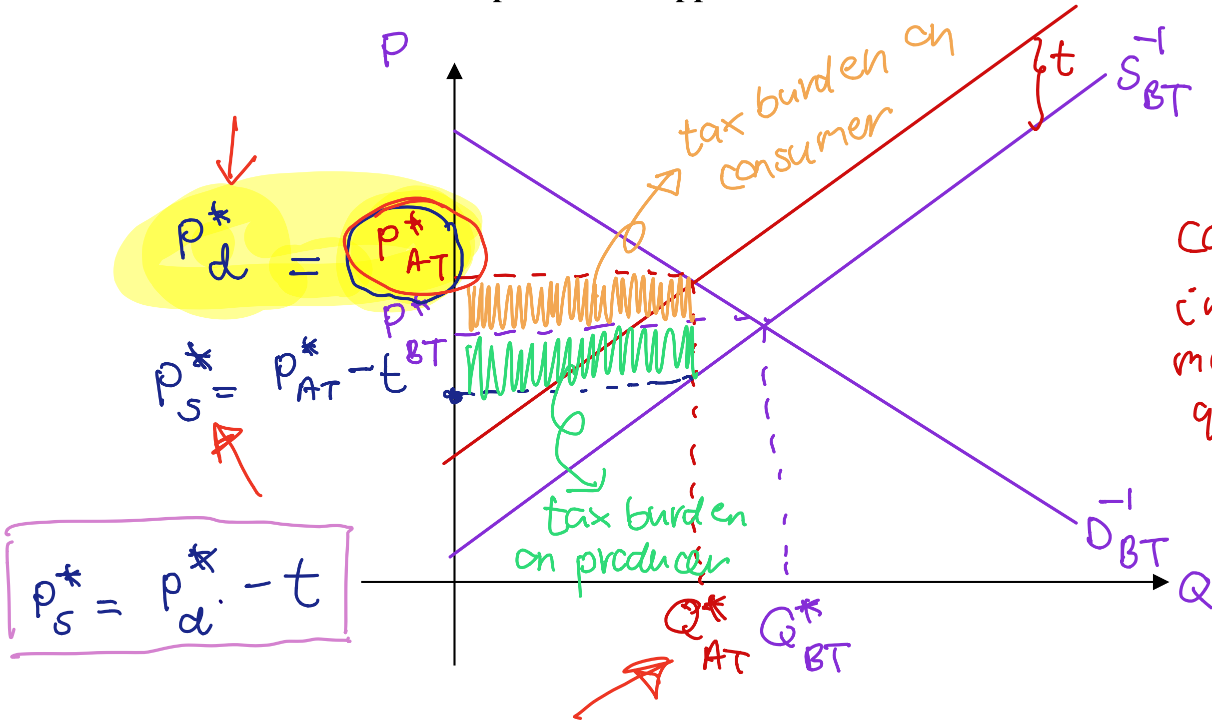
Comparative statics, as the name suggested, is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogeneous variables.

CASE 1: Specific tax

Collect tax t baht per unit of energy drink, e.g. t baht per bottle

The supply curve is firm's marginal cost and shifts in the Supply curve is caused by any changes in the the market that cause an increase in mc at every quantity level.

CASE 1.1: Collect from producers/suppliers



$$Q^D = Q^S$$
$$Q^D = a - bP_d$$
$$Q^S = -c + dP_s$$

Market equilibrium before tax

$$Q^D = Q^S$$
$$a - bp = -c + dp$$
$$p^* = \frac{a+c}{b+d}$$
$$Q^* = \frac{ad - bc}{b+d}$$

P_d is the price that consumers paid

P_s is the price that producers received

$P_d = P_s$, $t = 0$

$P_d = P_s = p$

Market equilibrium after tax

$$\boxed{\begin{aligned} Q^D &= Q^S \\ Q^D &= a - bP_d \\ Q^S &= -c + dP_s \\ P_s &= P_d - t = p - t \end{aligned}}$$

the price received by producers is the price paid by consumer minus tax

At market equilibrium $Q^D = Q^S$

$$a - bP_d = -c + d(P_d - t)$$

or

$$a - bp = -c + d(p - t)$$
$$a + c = (b+d)p - dt$$
$$p^*_{AT} = \frac{a + c + dt}{(b+d)} = p^*_d$$

$d > 0 \rightarrow p^*_d > p^*_{BT}$

$= p^*_{AT}$

$$Q^*_{AT} = a - bp^*_d$$
$$= a - b\left(\frac{a+c+dt}{(b+d)}\right)$$

$ab + ad - ab - bc - bdt$

$$Q^*_{AT} = \frac{ad - bc - bdt}{b+d} < Q^*_{BT}$$

$$p^*_s = p^*_{AT} - t$$
$$= \frac{a+c+dt}{b+d} - t$$
$$= \frac{a+c-bt}{b+d} < p^*_{BT}$$

Tax revenue is equal to:

$$\text{Tax Revenue} = t\left[\frac{ad - bc - bdt}{b+d}\right]$$

consumer pay $\frac{dt}{b+d}$ more = tax burden per unit on consumer

Tax burden on consumers/buyers

producer receive $\frac{bt}{b+d}$ less = tax burden per unit on producer

$$CB = (P_d^* - P_{BT}^*)\, Q_{AT}^*$$

$$= \frac{dt}{b+d}\left[\frac{ad - bc - bdt}{b+d}\right]$$

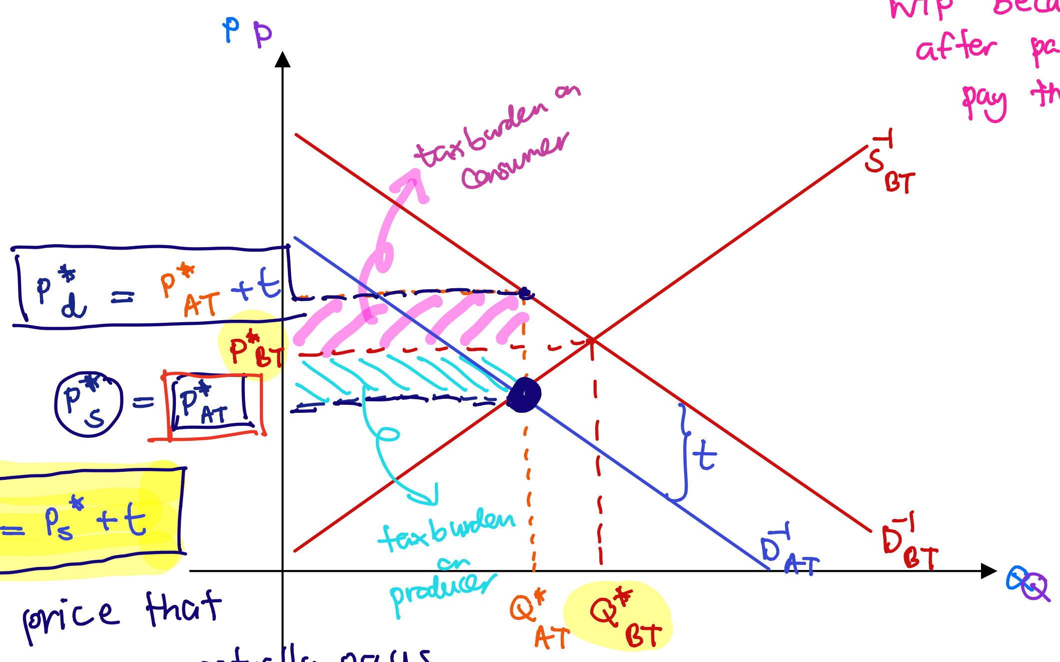
Tax burden on producers/sellers

$$PB = (P_{BT}^* - P_S^*)\, Q_{AT}^*$$

$$= \frac{bt}{b+d}\left[\frac{ad - bc - bdt}{b+d}\right]$$

CASE 1.2: Collect from consumers/buyers

At each quantity, consumer will have lower WTP because if after paying tax, they pay the same amount.



$$\boxed{P_d^* = P_s^* + t}$$

The price that the consumer actually pays is the price that the producer gets plus tax

Market equilibrium before tax

the same

Market equilibrium after tax

$$Q^D = Q^S$$

$$Q^D = a - bP_d$$

$$Q^S = -c + dP_S$$

$$P_d = P_S + t = P + t$$

At mkt eq. $Q^D = Q^S$

$$a - b(p+t) = -c + dp$$

$$P^*_{AT} = \frac{a+c-bt}{b+d} = P^*_S < P^*_{BT}$$

$$Q^*_{AT} = -c + dP^*_S$$

$$= -c + d\left[\frac{a+c-bt}{b+d}\right]$$

$$Q^*_{AT} = \frac{ad - bc - bdt}{b+d} < Q^*_{BT}$$

$$P^*_d = P^*_{AT} + t$$

$$= P^*_S + t$$

$$= \frac{a+c-bt}{b+d} + t$$

$$= \frac{a+c+dt}{b+d} > P^*_{BT}$$

Tax revenue is equal to:

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Tax burden on consumers/buyers

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Tax burden on producers/sellers

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OBSERVATION:

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everything is the same, except p^*_{AT}.
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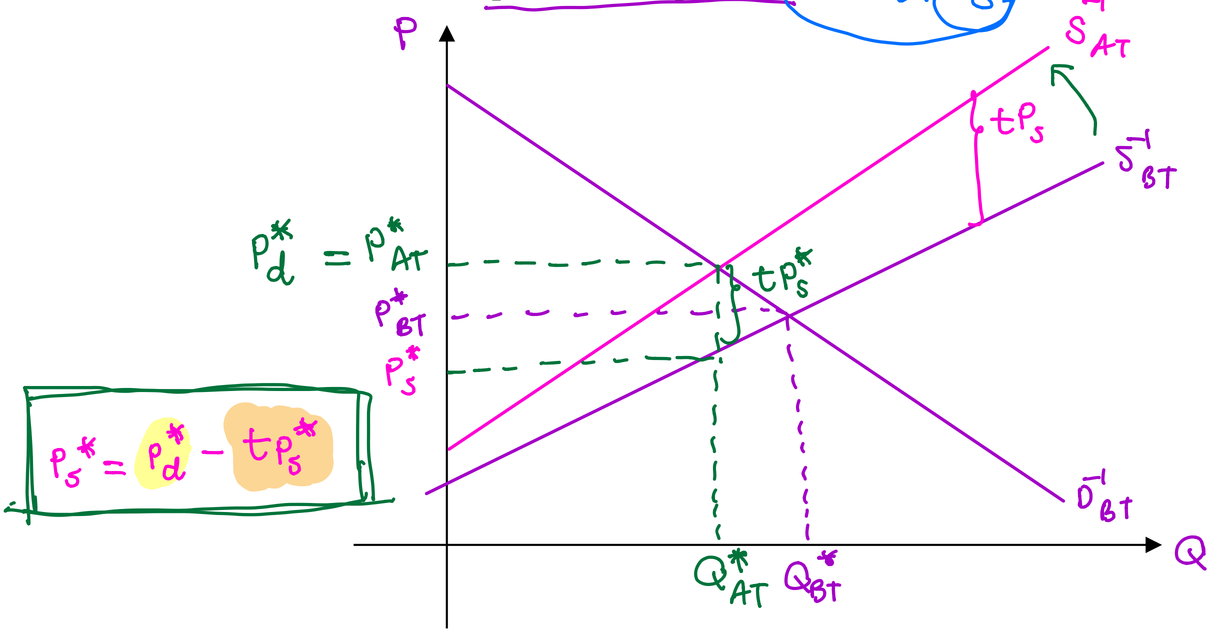
CASE 2: Ad Valorem tax

Collect tax t% of price, e.g. 7% of price per bottle

t% of price

CASE 2.1: Collect from producers/suppliers t% of P_s



Market equilibrium before tax

$t = -s$
$s > 0$

Market equilibrium after tax

$$P_s = P_d - tP_s \qquad P_{AT} = P = P_d$$

$$P_s = \frac{1}{(1+t)} P_d = \frac{1}{1+t} P$$

At market eq. : $Q^D = Q^S$

market price after tax to solve for

$$a - bP = -c + d\left(\frac{P}{1+t}\right)$$

$$\vdots$$

$$P^*_{AT} = \frac{(1+t)(a+c)}{(1+t)b+d} = P^*_d > P^*_{BT}$$

$$Q^*_{AT} = a - bP^*_d < Q^*_{BT}$$

$$= \frac{ad - bc(1+t)}{(1+t)b+d}$$

$b(1+t)+d$

$$p_s^* = \frac{p_d^*}{1+t} = \frac{p_{AT}^*}{1+t} = \frac{a+c}{(1+t)b+d} < p_{BT}^*$$

Tax revenue is equal to:

$$\text{Tax Revenue} = t\, p_s^* Q_{AT}^*$$

Tax burden on consumers/buyers

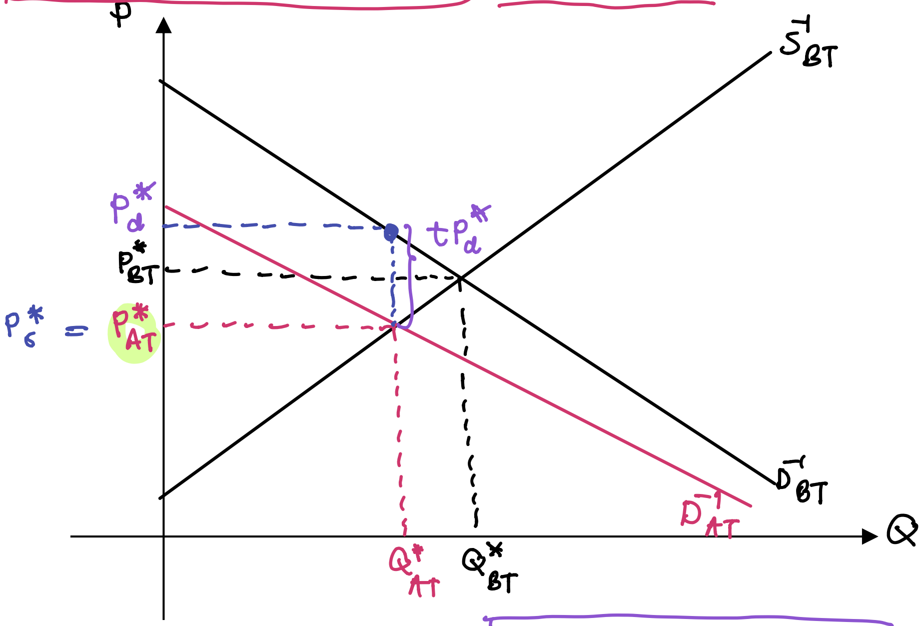
$$CB = \underbrace{(p_d^* - p_{BT}^*)}_{\text{CB per unit}} Q_{AT}^*$$

Tax burden on producers/sellers

$$PB = \underbrace{(p_{BT}^* - p_s^*)}_{\text{PB per unit}} Q_{AT}^*$$

CASE 2.2: Collect from consumers/buyers

$\boxed{t\% \text{ of } P_d}$



$\boxed{P_d = P_s + tP_d}$

Market equilibrium before tax

Market equilibrium after tax

$P_d = P_s + tP_d$, $P_s = P_{AT} = P$

$P_d = P + tP_d$

$(1-t)P_d = P$

$P_d = \frac{1}{1-t} P$ ($P = P_s$)

previous case, note $P_s = \frac{P}{1+t}$ ($P = P_d$)

Mkt Clearing condition : $Q_d = Q_s$

$a - b\left(\frac{P}{1-t}\right) = -c + dP$

$\vdots$

$P_{AT}^* = P_s^* = \frac{(a+c)(1-t)}{b+(1-t)d} < P_{BT}^*$

$Q_{AT}^* = -c + dP_s^*$

$= \frac{ad(1-t) - bc}{b + d(1-t)} < Q_{BT}^*$

$P_d^* = \frac{1}{1-t} P_s^* = \frac{a+c}{b+(1-t)d} > P_{BT}^*$

The relationship after tax & before tax actually depends on elasticity of supply & demand

Tax revenue is equal to:

Tax burden on consumers/buyers

Tax burden on producers/sellers

OBSERVATION:

Collecting Ad Valorem tax from producer vs. consumer might not yield the same results.

Tax incidence (or incidence of tax) is an economic term for understanding the division of a tax burden between buyers and sellers or producers and consumers.

Tax incidence is related to the price elasticity of supply and demand. When supply is more elastic than demand, the tax burden falls on the buyers. If demand is more elastic than supply, producers will bear the cost of the tax.

Digression

What is elasticity ? How can we derive elasticity?

From linear demand function in (2):

$Q^d = a - bP$ $\quad\quad \frac{\Delta Q^d}{\Delta P} = -b$

With P on Y-axis and Q on x-axis, the inverse demand function is:

$P^d = \frac{a}{b} - \frac{1}{b}Q$ $\quad\quad \frac{\Delta P}{\Delta Q^d} = -\frac{1}{b}$

From linear supply function in (3):

$Q^S = -c + dP$ $\quad\quad \frac{\Delta Q^S}{\Delta P} = d$

The inverse supply function is:

$P^S = \frac{c}{d} + \frac{1}{d}Q$ $\quad\quad \frac{\Delta P}{\Delta Q^S} = \frac{1}{d}$

$-b$ is

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$-d$ is

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To be able to compare how responsive of change in quantity to change in price across different commodities, the concept of "elasticity" comes in handy.

Elasticity is the measurement of the percentage change of one economic variable in response to one percentage change in another.

$$\boxed{E_{y,x} = \frac{\%\Delta y}{\%\Delta x} = \frac{\frac{\Delta y}{y} \times 100}{\frac{\Delta x}{x} \times 100}} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y} \quad (4)$$

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The price elasticity of demand is:

$$E_p^d = \frac{\%\Delta Q^d}{\%\Delta p} = \frac{\frac{\Delta Q^d}{Q^d} \times 100}{\frac{\Delta p}{p} \times 100}$$

$$E_p^d = \frac{\Delta Q^d}{\Delta p} \times \frac{p}{Q^d}$$

$$= -b \times \frac{p}{Q^d}$$

$$= -\frac{1}{1/b} \times \frac{p}{Q^d}$$

The price elasticity of supply is:

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Inelastic vs. Elastic vs. Unit elastic

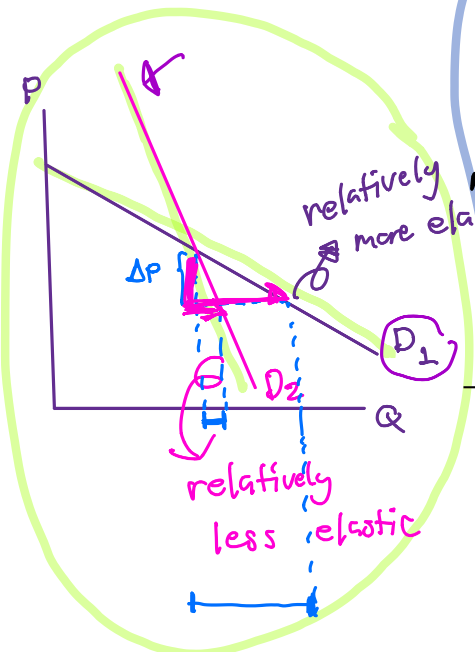
In absolute term (not thinking about sign)

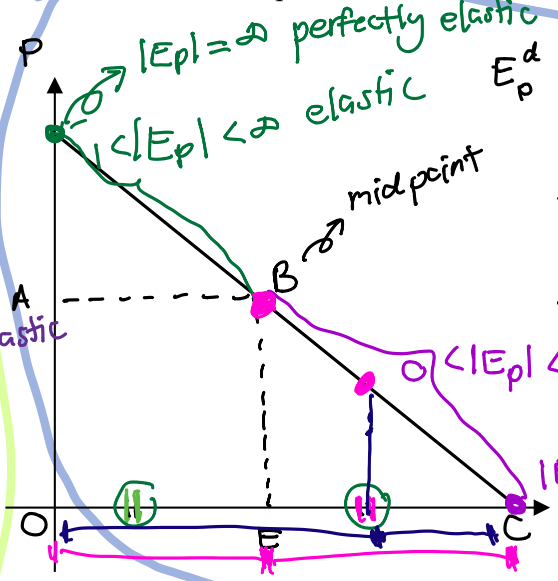
$|E_p| < 1$ inelastic $\quad \%\Delta P > \%\Delta Q$

$|E_p| > 1$ elastic $\quad \%\Delta P < \%\Delta Q$

$|E_p| = 1$ unit elastic $\quad \%\Delta P = \%\Delta Q$

Price Elasticity of Demand at different point on inverse demand function:

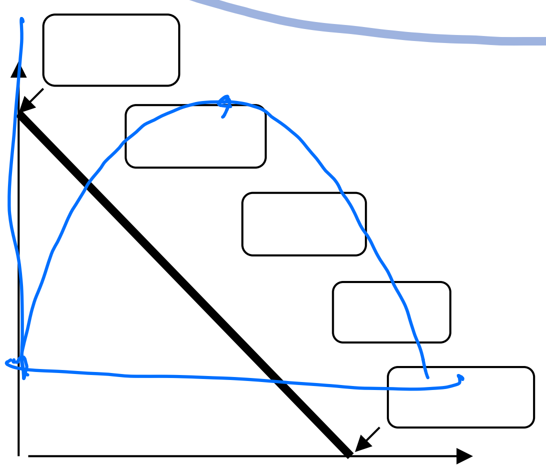




$$E_p^d = \frac{\Delta Q^d}{\Delta P} \cdot \frac{P}{Q^d}$$

$$= -\frac{EC}{OA} \cdot \frac{OA}{OE}$$

$$= -\frac{EC}{OE}$$

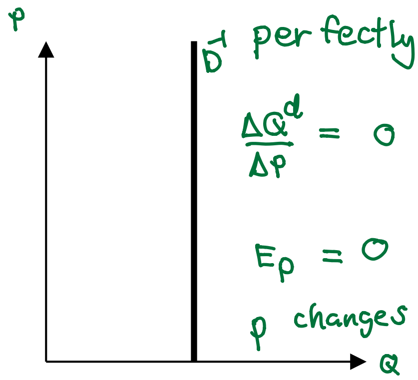


The price elasticity of demand at each point of linear demand function, with negative slope, is not equal to other point. This is because:

$$\frac{P}{Q}$$

$$\frac{\Delta Q}{\Delta P} = -b$$

In which case are the price elasticities of demand for different points equal?



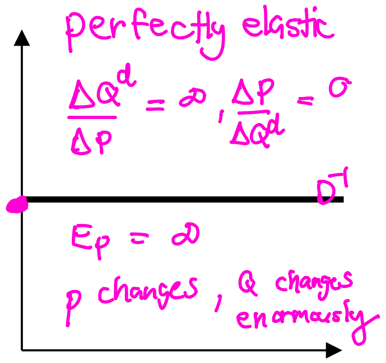
perfectly inelastic

$\frac{\Delta Q^d}{\Delta P} = 0$, $\frac{\Delta P}{\Delta Q^d} = \infty$

slope of inverse demand curve

$E_P = 0$

P changes , Q doesn't change

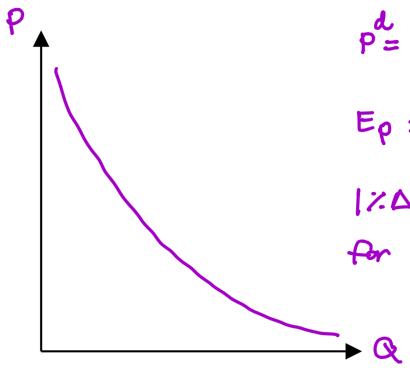


perfectly elastic

$\frac{\Delta Q^d}{\Delta P} = \infty$, $\frac{\Delta P}{\Delta Q^d} = 0$

$E_P = \infty$

P changes , Q changes enormously



$P^d = \frac{a}{Q}$, i.e. $Q^D = \frac{a}{P}$

$E_P = -\frac{a}{P^2} \cdot \frac{P}{a/P} = -1$

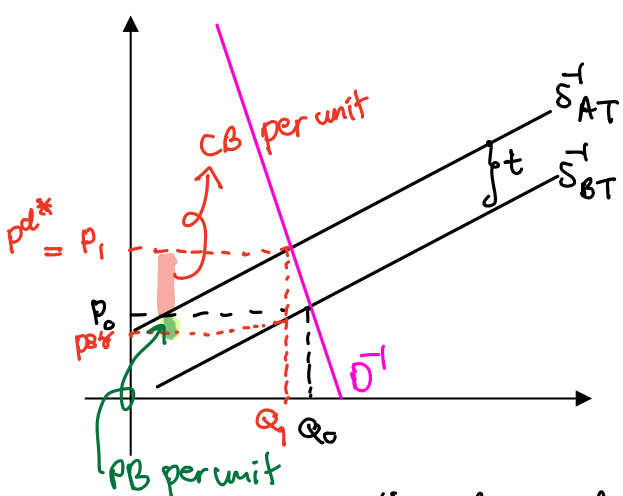
$|\% \Delta P| = |\% \Delta Q^d|$ for each point for (Q, P) , TR will be constant

Tax incidence and Elasticity

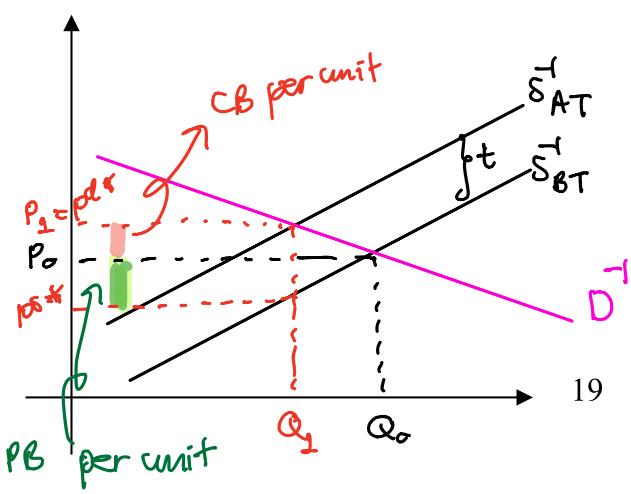
Suppose Gov. impose unit tax t on supplier

Whoever cannot adjust themselves instantaneously to change in price will have to bear a larger portion of the tax burden.

Inelastic demand

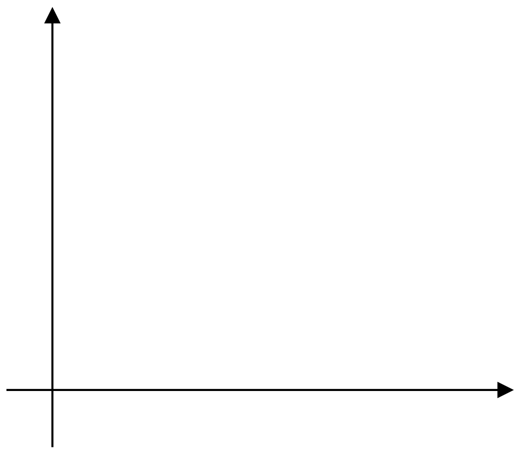


Elastic demand

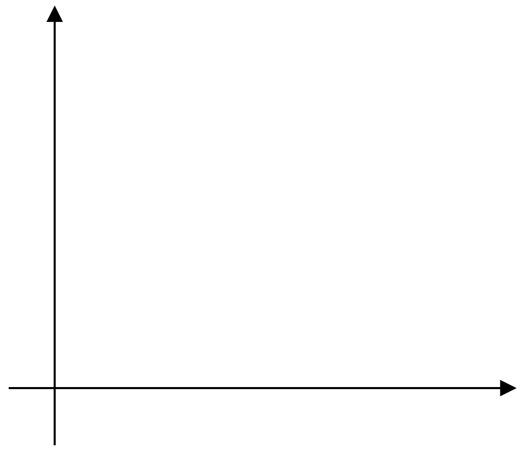


more inelastic demand, more consumer burden per unit

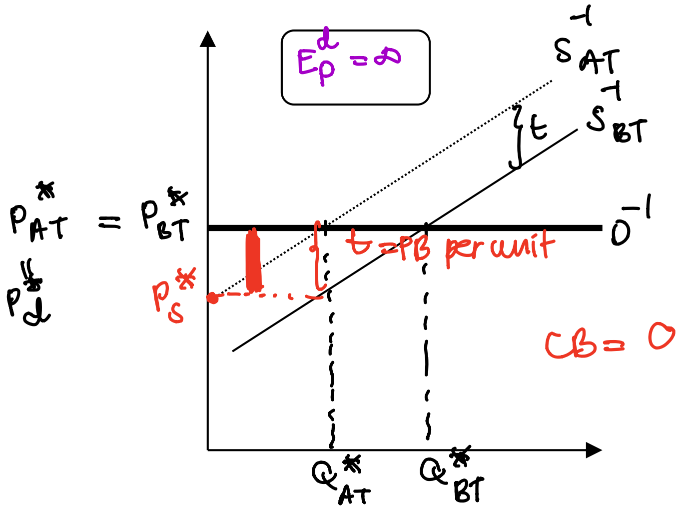
Inelastic supply



Elastic supply



perfectly elastic demand



perfectly inelastic demand