HW

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1. Find cournot equilibrium when there are 3 firms in the market

$P = a - bQ$; $Q = q_1 + q_2 + q_3$

$C_1 = C_2 = C_3 = C$

What is equilibrium price? : P^*

What are firms' profit? $\pi_1 = \pi_2 = \pi_3 = ?$

Firm 1 : $\pi = P \cdot Q - C_1$

$= (a - b(q_1+q_2+q_3))q_1 - C_1$

$= aq_1 - q_1^2 b - q_1 q_2 b - q_1 q_3 b - C_1$

$\frac{d\pi}{dq_1} = a - 2q_1 b - q_2 b - q_3 b$

$2q_1 b = a - q_2 b - q_3 b$

$q_1 = \frac{a - q_2 b - q_3 b}{2b}$

$q_1 = \frac{a - \left(\frac{a - q_3 b}{3\cancel{b}}\right)\cancel{b} - q_3 b}{2b}$

$q_1 = \frac{3a - a + q_3 b - 3q_3 b}{6b}$

$= \frac{2a - 2q_3 b}{6b} = \frac{a - q_3 b}{3b} = \frac{a - \left(\frac{a}{4\cancel{b}}\right)\cancel{b}}{3b}$

$= \frac{4a - a}{12b} = \frac{3a}{12b} = \frac{a}{4b}$ ※

Firm 2 : $\pi = P \cdot Q_2 - C_2$

$= (a - b(q_1+q_2+q_3))q_2 - C_2$

$= aq_2 - q_1 q_2 b - q_2^2 b - q_2 q_3 b - C_2$

$\frac{d\pi}{dq_2} = a - q_1 b - 2q_2 b - q_3 b$

$0 = a - q_1 b - 2q_2 b - q_3 b$

$2q_2 b = a - q_1 b - q_3 b$

$2q_2 b = a - \left(\frac{a - q_2 b - q_3 b}{2\cancel{b}}\right)\cancel{b} - q_3 b$

$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$

$4q_2 b = a + q_2 b - q_3 b$

$3q_2 b = a - q_3 b$

$q_2 = \frac{a - q_3 b}{3b}$

$= \frac{a - \left(\frac{a}{4\cancel{b}}\right)\cancel{b}}{3b}$

$= \frac{4a - a}{12b} = \frac{\cancel{3}^{1} a}{\cancel{12}_{4} b} = \frac{a}{4b}$ ※

Firm 3 : $\pi = P \cdot Q_3 - C_3$

$= (a - b(q_1+q_2+q_3))q_3 - C_3$

$aq_3 - bq_1 q_3 - bq_2 q_3 - bq_3^2 - C_3$

$\frac{d\pi}{dq_3} = a - bq_1 - bq_2 - 2bq_3$

$2bq_3 = a - bq_1 - bq_2$

$q_3 = \frac{a - bq_1 - bq_2}{2b}$

$= \frac{a - \cancel{b}\left(\frac{a - q_3 b}{3\cancel{b}}\right) - \cancel{b}\left(\frac{a - q_3 b}{3\cancel{b}}\right)}{2b}$

$= \frac{3a - a + q_3 b - a + q_3 b}{6b}$

$q_3 = \frac{a + 2q_3 b}{6b}$

$6bq_3 = a + 2q_3 b$

$4bq_3 = a$

$q_3 = \frac{a}{4b}$ ※

Equilibrium price: $P = a - bQ$

$= a - b(q_1 + q_2 + q_3)$

$= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$

$= a - \cancel{b}\left(\frac{3a}{4\cancel{b}}\right) = \frac{4a - 3a}{4} = \frac{a}{4}$

$\pi_1 = P \cdot Q - C_1$

$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_1 = \frac{a^2}{16b} - C_1$

$\pi_2 = P \cdot Q - C_1$

$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_2 = \frac{a^2}{16b} - C_2$

$\pi_3 = P \cdot Q - C_1$

$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_3 = \frac{a^2}{16b} - C_3$

2. If there are N firm

$q_i^* = f(N)$, $P = (N)$, $\pi_i = f(N)$

$P = a - bQ$; $Q = q_N$

$C_N = C_\mu = C_\mu = C$

Assume that : $q_1 + q_2 + \ldots + q_n = A$

$P = a - b(q_1 + q_2 + \ldots + q_n)$

$P = a - bq_1 - bq_2 - \ldots - bq_n$

$\pi_i = (a - bq_1 - bq_2 - \ldots - bq_n) q_i - C_i$

$\vdots$

$\pi_n = (a - bq_1 - bq_2 - \ldots - bq_n) q_n - C_n$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - \ldots - bq_n = 0$

$\frac{a}{2b} - 0.5(q_2 + q_3 + \ldots + q_n) = q_1$

$\frac{a}{2b} - 0.5(q_1 + q_2 + \ldots + q_n) = q_n$

to include $0.5q_1$ in

$q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \ldots q_n)$

$0.5q_1 = \frac{a}{2b} - 0.5A$

$q_1 = \frac{a}{b} - A$

$q_2 = \frac{a}{b} - A$

$\vdots$

$q_n = \frac{a}{b} - A$

from $q_1 + q_2 + \ldots + q_n \rightarrow A = n\left(\frac{a}{b} - A\right) = nq$

$= n\frac{a}{b} - nA$

$(n+1)A = n\frac{a}{b}$

$* A = nq \longrightarrow A = \frac{na}{(n+1)b}$

$q_i = \frac{a}{(n+1)b} *$

$P = a - b(A) = a - \cancel{b}\left(\frac{na}{(n+1)\cancel{b}}\right)$

$P = a - \frac{n}{n+1}(a) = \frac{a(n+1) - na}{n+1} = \frac{na + a - na}{n+1}$

$P = \frac{a}{n+1}$

$\pi_i = P \cdot q_i - C_i = \frac{a}{n+1} * \frac{a}{(n+1)b} - C_i$

$\pi_i = \frac{a^2}{(n+1)^2 b} - C_i$

3. From Q_2, What happen if $N \rightarrow \infty$

N = 1

if $n \rightarrow \infty$: $q_i = \frac{a}{(n+1)b} \longrightarrow 0$ The quantity that each firm will sell approach to zero

$A = nq_i \longrightarrow \infty$ When sum the quantity that every firm will sell ∞ unit The quanity approach to ∞ unit

$P = \frac{a}{n+1} \longrightarrow 0$: Supply increase cause demand decrease and approach to zero

$\pi = \frac{a^2}{(n+1)^2 b} - C_i \longrightarrow -C_i \longrightarrow$ Profit in each firm will negative because it less than fixed cost

if $n \rightarrow 1$: $q_i = \frac{a}{2b}$ ※ Monopoly will decrease the quantity that they sell $q = \frac{a}{2b} < q = \frac{na}{(n+1)b}$

$A = nq_i = Q$ * $n = 1$ ∴ firm is monopoly

$P = \frac{a}{n+1} = \frac{a}{2}$ * $P_m = \frac{a}{2} > P = \frac{a}{n+1}$: Profit of monopoly will more than $n = \infty$

$\pi = \frac{a^2}{(n+1)^2 b} - C_i = \frac{a^2}{4b} - C_i$; $\pi_M > \pi_i = \frac{a^2}{(n+1)^2 b} - C_i$. Profit of monopoly more than $n = \infty$