4 Testing Hypotheses about a Single Linear Combination of the Parameter

Consider

$$\log(wage) = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 \exp er + u$$

where jc = number of years attending a two-year college
$univ$ = number of years at a four-year college
$\exp er$ = months in the workforce.
We want to test whether $\beta_1 = \beta_2$.

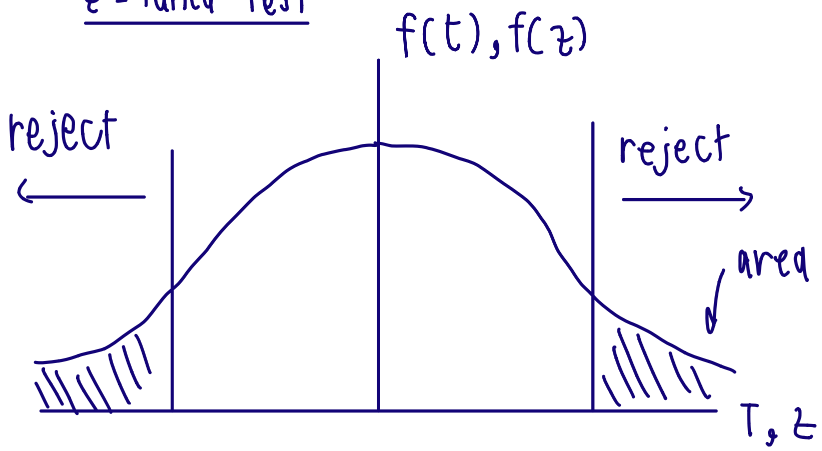
if the returns from 1 more year of education at a junior college is the same as that of the university.

$H_0: \beta_1 = \beta_2 \rightarrow \beta_1 - \beta_2 = 0$

against

$H_a: \beta_1 \neq \beta_2 \rightarrow \beta_1 - \beta_2 \neq 0$

<u>2-tailed test</u>



$$t = \frac{(\hat{\beta}_1 - \hat{\beta}_2) - 0}{s.e.(\hat{\beta}_1 - \hat{\beta}_2)}$$

we compute this t-statistic and compare with the critical value

calculate using STATA

where $s.e.(\hat{\beta}_1 - \hat{\beta}_2) = \sqrt{var(\hat{\beta}_1 - \hat{\beta}_2)}$

$$= \sqrt{\widehat{var}(\hat{\beta}_1) + \widehat{var}(\hat{\beta}_2) - 2\widehat{cov}(\hat{\beta}_1, \hat{\beta}_2)}$$

not very straight forward to calculate.

~ we use a variable transformation trick ~ see notes.

another possible hypothesis test (one-tailed alternative)

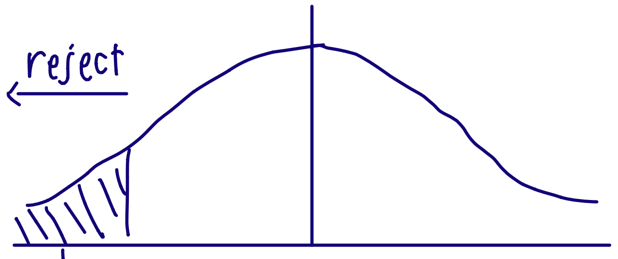
$H_0 : \beta_1 = \beta_2 \rightsquigarrow H_0 : \beta_1 - \beta_2 = 0$

$H_a : \beta_1 < \beta_2 \rightsquigarrow H_a : \beta_1 - \beta_2 < 0$

- It is assumed that β_1 would not be more than β_2 (returns to a 2-years college would never be more than returns to university education)

$$t = \frac{(\hat{\beta}_1 - \hat{\beta}_2) - 0}{s.e.(\hat{\beta}_1 - \hat{\beta}_2)}$$

∗ then go to the extra note

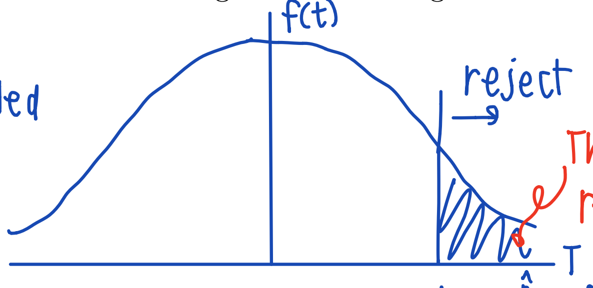


area = sig. level

5 Computing p-Values for t-Tests

- What is the significance level given the computed t-statistics?

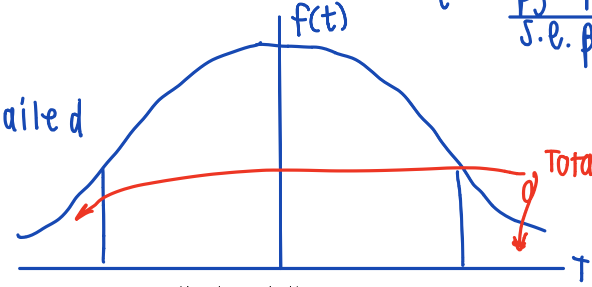
1-tailed



This shaded area in the rejection region is the p-value

$t = \frac{\hat{\beta}_j - \beta_j}{s.e.\ \beta_j}$

2-tailed



Total area from 2 sides.

- p-value : $P(|T| > |t|)$

t = computed $t = \frac{\hat{\beta}_j - \beta_j}{s.e.\ \beta_j}$

T = t-distributed random variable with d.f. $= n-k-1$

t = computed t-statistic

⤳ p-value = probability that a random T value will be greater (in the | | term) than our t in the H_0 test.

In-class exercise

consider the multiple regression model, assume MLR 1-6 are satisfied.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

You would like to test the $H_0 : \beta_1 - 3\beta_2 = 1$

1st) write the t-statistic for testing H_0

$$t = \frac{(\hat{\beta}_1 - 3\hat{\beta}_2) - 1}{s.e.(\hat{\beta}_1 - \hat{\beta}_2)}$$

2nd) Define $\theta_1 = \hat{\beta}_1 - 3\hat{\beta}_2$ → $H_0 : \theta_1 = 1$, $H_a\ \theta_1 \neq 1$

$$t = \frac{\hat{\theta}_1 - 1}{s.e.(\hat{\theta}_1)}$$

→ we need our regression to have θ_1 in it. So, STATA or OLS estimation will automatically give θ_1 & s.e. θ_1

Now, $\hat{\beta}_1 = \hat{\theta}_1 + 3\hat{\beta}_2$

$\beta_1 = \theta_1 + 3\beta_2$

Substitute in the main regression and get

$$y = \beta_0 + (\theta_1 + 3\beta_2) x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

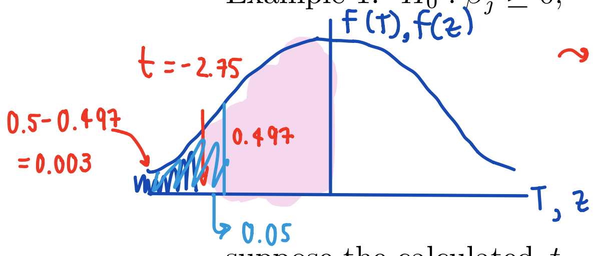
$$= \beta_0 + \theta_1 x_1 + 3\beta_2 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

$$= \beta_0 + \theta_1 x_1 + \beta_2 (3x_1 + x_2) + \beta_3 x_3 + u$$

* now, the explanatory variables are going to be x_1, $x_2 + 3x_1$, and x_3

- we can calculate $t = \dfrac{\hat{\theta}_1 - 1}{s.e.\,\hat{\theta}_1}$

Example 1: $H_0 : \beta_j \geq 0$, $H_a : \beta_j < 0$, d.f.= 140. → z table



→ p-value = what should be the significant level, given the critical value of -2.75??

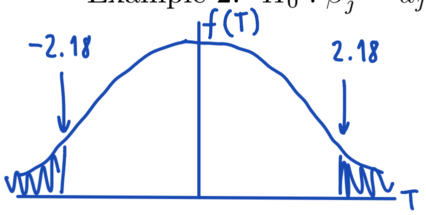
→ find the shaded area!

suppose the calculated $t_{\hat{\beta}_j} = -2.75$ → $t_{\hat{\beta}_j} = \frac{(\hat{\beta}_j - \beta_j)}{s.e.\ \hat{\beta}_j}$

- From the z-table, the value -2.75 corresponds to area = .003.
- Thus, p-value = 0.003.
- Would we reject H_0 if we use the significance level = 5%? Yes

  * rule! we reject H_0 if p-value < sig. level

Example 2: $H_0 : \beta_j = a_j$, $H_a : \beta_j \neq a_j$, d.f.= 18. ← use t-table



suppose the calculated $t_{\hat{\beta}_j} = -2.18$

- From the t-table, the value -2.18 corresponds to area = 0.02 to 0.05.
- Thus, p-value = is between 0.02 - 0.05.
- Would we reject H_0 if we use the significance level = 5%?

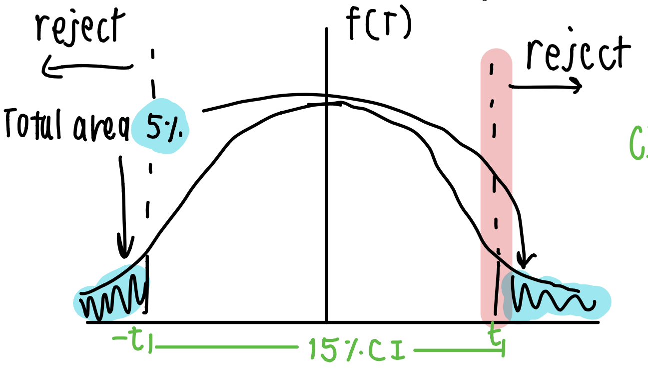
  Yes, reject H_0 because the area is less than 0.05 or p-value < 0.05

6 Confidence Intervals (CI)

- Confidence Intervales for the POPULATION PARAMETER (β_j)

  → The range of values that would capture the true β_j at a 95% chance

- A 95% CI of β_j is given by



CI → $\hat{\beta}_j \pm c \times s.e.(\hat{\beta}_j)$

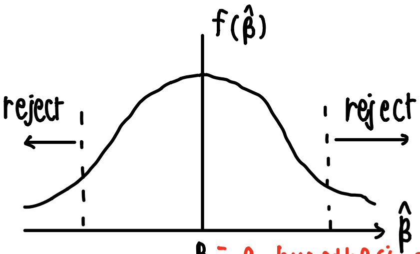
c is the $\boxed{97.5}$ percentile in the t-distribution with n-k-1 d.f.

Inference ⤳ hypothesis testing about "β" the true parameter

wage $= \beta_0 + \beta_1$ educ $+ \beta_2$ experience $+ \dots + u$

we want to test hypotheses about the true impact (β) of each X variables (educ, experience) on the dependent variable (Y)

BUT. we don't know what the true β are. so, we use $\hat{\beta}$ (estimator) and s.e.($\hat{\beta}$) to test the hypotheses.



β = a hypothesized value

e.x. $\beta = 0$, $\beta = 1$, etc.

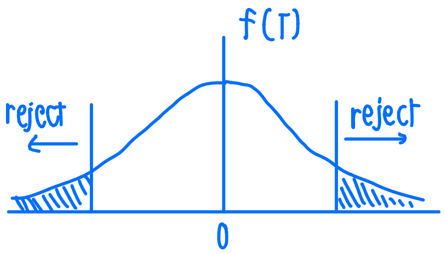
1.) test if β = some number

e.g. $\beta_j = 0$ ⤳ x_j has no impact on y

$\beta_j = 1$ ⤳ 1 unit ↑ in x_j correspond to 1 unit ↑ in y

→ t-test ✩ → how?

$$\frac{\hat{\beta}_j - \beta_j}{s.e.(\hat{\beta}_j)} \sim t_{d.f.}$$



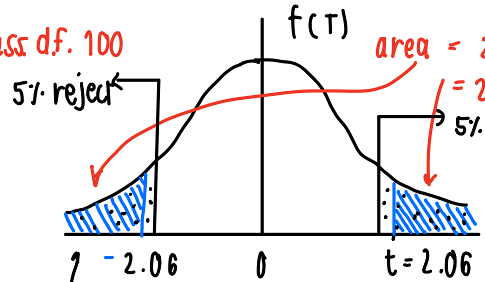
$$T = \frac{\hat{\beta}_j - \beta_j}{s.e.(\hat{\beta}_j)}$$

$\boxed{\text{significant level = area in the rejection region}}$

ass d.f. 100

area $= 2 \times (0.5 - 0.4803)$

$= 2 \times 0.0197 = 0.0394$ = p-value!



$$T = \frac{(\hat{\beta}_j - \beta)}{s.e.\ \hat{\beta}_j}$$

significant level which we reject H_0

• suppose, we calculate a t-statistic $= \dfrac{\hat{\beta}_j - \beta_j}{s.e.(\hat{\beta}_j)} = 2.06$

• suppose, we are testing $H_0: \beta_j = 0$, $H_a: \beta_j \neq 0$

↳ 2 tailed test

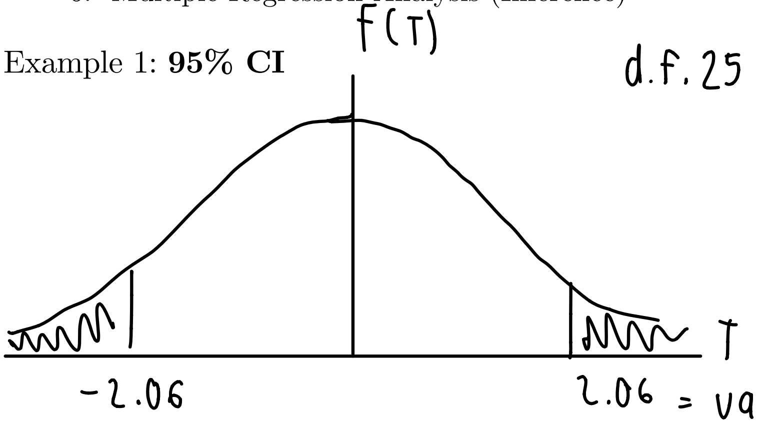
• p-value = total shaded area

p-value = significant level which we will reject the H_0 or prob that we will reject H_0

if p-value < significant level ⟶ reject H_0

Example 1: **95% CI**

d.f. 25

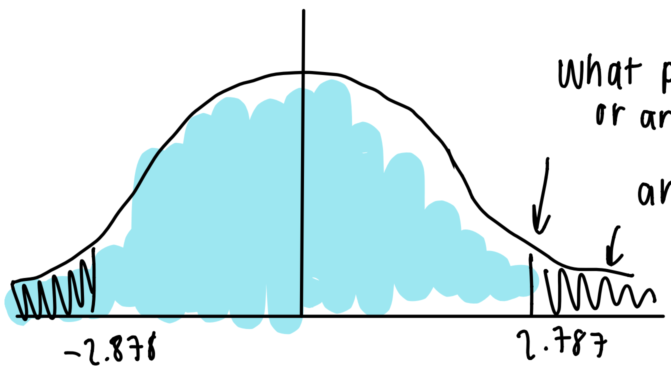


= value of 97.5th percentile in the t_{25} distribution

The 95% CI for $\hat{\beta}_j = [\hat{\beta}_j - 2.06 \cdot s.e.(\hat{\beta}_j), \hat{\beta}_j + 2.0 \cdot s.e.(\hat{\beta}_j)]$

Example 2: **99% CI**

d.f. = 25



what pctile ?? 99.5

or area = 1−0.0005 = 0.995

area = $\frac{1\%}{2} = \frac{0.01}{2}$ = 0.005

The 99% CI for $\hat{\beta}_j = [\hat{\beta}_j - 2.787 \cdot s.e.\hat{\beta}_j, \hat{\beta}_j + 2.787 \cdot s.e.\hat{\beta}_j]$

F-test motivation

→ we want to test the significance of a group of hypotheses (multiple hypotheses)

$$Grade_{325} = \beta_0 + \beta_1 \#times\_front + \beta_2 \#times\_back + \beta_3 hr\_study + \beta_4 past\_GPA + \beta_5 gender + u$$

H_0 : seat position doesn't have impact on GPA

$\beta_1 = 0$ and $\beta_2 = 0$ → $\beta_1 = \beta_2 = 0$

H_a : seat position matters

$$\left.\begin{array}{llll} & \beta_1 \neq 0 & \text{and} & \beta_2 \neq 0 \\ \text{or} & \beta_1 \neq 0 & \text{and} & \beta_2 = 0 \\ \text{or} & \beta_1 = 0 & \text{and} & \beta_2 \neq 0 \end{array}\right\} \text{at least one of the } \beta_1, \beta_2 \neq 0$$

7 Testing Multiple Linear Restrictions: The F-test

Suppose the model is specified by

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\ H_0 &: \beta_2 = 0 \text{ and } \beta_3 = 0 \\ H_a, H_1 &: H_0 \text{ is not true} \end{aligned}$$

→ we want to test if x_1 and x_2 BOTH have no impact on y

We can use the F-test to test this type of "multiple hypotheses".

1. Our full model is called the "unrestricted" model (ur). Suppose it can be expressed as:

   Big model

   $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ is true → reject H_0

   $$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_k x_k + u$$

2. The model which takes out x (which we think its associated $\beta = 0$) is called the restricted model (r). ← small model

   $y = \beta_0 + \beta_1 X_1 + u$ is true → do not reject H_0

Suppose there are "q" number of β that we would like to perform a joint-test of =0

→ e.g. in this model q=2

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_{k-q} X_{k-q} + u$$

$H_0 = \beta_{k-q+1} = \beta_{k-q+2} = ... = \beta_k = 0$

(the last q $\beta_s = 0$)

$H_a = H_0$ is not true.

(

$$y = \underbrace{\underbrace{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_{k-q} X_{k-q}}_{r} + \beta_{k-q-1} X_{k-q-1} + \beta_{k-q+2} X_{k-q+2} + ... + \beta_k X_k + u}_{ur}$$

$$F = \frac{\left(SSR_r - SSR_{ur} \right) / q}{SSR_{ur} / (n-k-1)}$$

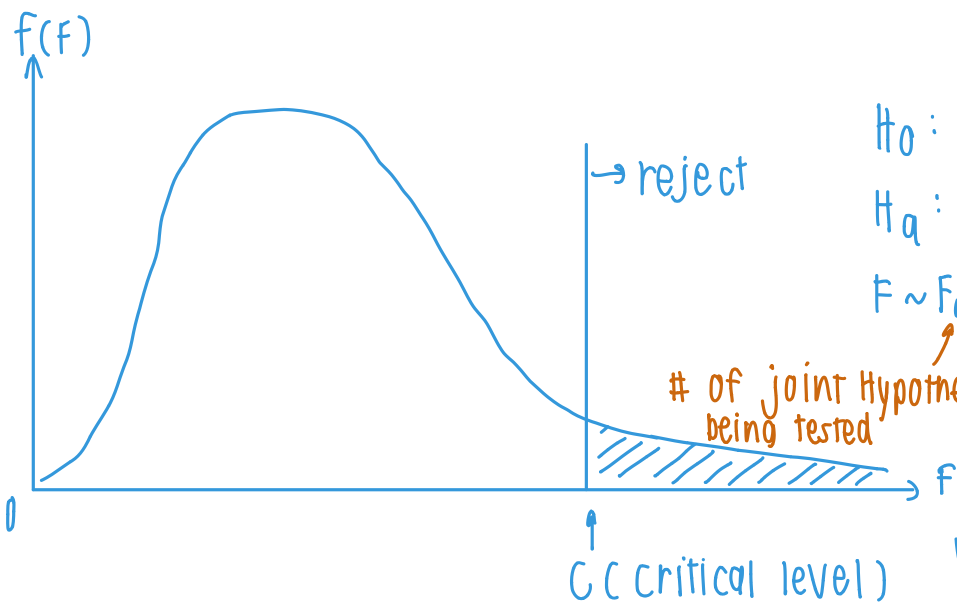
This is always (+) b/c $SSR_{ur} < SSR_r$

Every time you add 1 more x, the model will be better explained.

d.f. of the "ur" model

- So, if every time you add 1 more X variables, the SSR↓ and R^2↑, why don't we just keep the additional X in the model??

→ because every time we add 1 more X, $var(\hat{\beta}_s)$ will increase, making the prediction of β less precise. So, we only keep the addition X_s if it/they can improve the model enough can significantly ↓ SSR and ↑ R^2



$H_0: \beta_2 = \beta_3 = \ldots = 0$

H_a: H_0 not true

$F \sim F_{q, n-k-1}$

q: # of joint Hypotheses being tested

$n-k-1$: d.f. of the ur model

We reject H_0 of jointly no effect if $F > C$

3. Some useful facts

① $R^2_{ur} > R^2_r$ because any additional X would increase R^2 (improve fit)

$\rightarrow SSR_{ur} < SSR_r$

② By including more X, the model is certainly better explained. However, we would like to reject H_0 if the inclusion of extra variables does not improve the model enough.

4. Other ways to calculate the F-statistics:

$\rightarrow$ From $R^2 = 1 - \dfrac{SSR}{SST}$ RSS TSS

We have $$F = \frac{(R^2_{ur} - R^2_r)/q}{(1-R^2_{ur})/(n-k-1)}$$

# of β that are set to "0"

# of obs.

# of slope β

# of intercept

$\rightarrow$ If we want to test the overall significance of the model

$H_0: \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_k = 0$, H_a = otherwise

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

R^2 of the model w/ ur

the "r" model has no X at all.

Example: Suppose we are interested in understanding the determinant of a baseball player's salary.

y *salary* = season salary

r { ur { *years* = years in major leagues
gamesyr = games per year in the league
bavg = career batting average
hrunsyr = homeruns per year
rbisyr = runs batted in per year

If we want to test whether performance has any impact on salary.

$H_0: \beta_{bavg} = \beta_{hrunsyr} = \beta_{rbisyr} = 0$

H_a: otherwise

- the unrestricted model (ur) is defined by ur model

Y X

```
. regress log_salary years gamesyr bavg hrunsyr rbisyr
```

Source	SS	df	MS		
Model	308.989208	5	61.7978416	Number of obs =	353
Residual	183.186327	347	.527914487	F(5, 347) =	117.06
				Prob > F =	0.0000
				R-squared =	0.6278
				Adj R-squared =	0.6224
Total	492.175535	352	1.39822595	Root MSE =	.72658

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years 1	.0688626	.0121145	5.68	0.000	.0450355	.0926898
gamesyr 2	.0125521	.0026468	4.74	0.000	.0073464	.0177578
q { bavg 3	.0009786	.0011035	0.89	0.376	-.0011918	.003149
hrunsyr 4	.0144295	.016057	0.90	0.369	-.0171518	.0460107
rbisyr 5	.0107657	.007175	1.50	0.134	-.0033462	.0248776
_cons	11.19242	.2888229	38.75	0.000	10.62435	11.76048

When considering each of the performance X one-by-one, none of them has a significant impact at 5%.

- the restricted model (r) is defined by

Y X

```
. regress log_salary years gamesyr
```

Source	SS	df	MS		
SSE Model	293.864058	2	146.932029	Number of obs =	353
SSR Residual	198.311477	350	.566604221	F(2, 350) =	259.32
				Prob > F =	0.0000
				R-squared =	0.5971
				Adj R-squared =	0.5948
SST Total	492.175535	352	1.39822595	Root MSE =	.75273

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years	.071318	.012505	5.70	0.000	.0467236	.0959124
gamesyr	.0201745	.0013429	15.02	0.000	.0175334	.0228156
_cons	11.2238	.108312	103.62	0.000	11.01078	11.43683

Now, our H_0 and H_a becomes

$$F \equiv \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)}$$

$$\equiv \frac{198.311 - 183.186/3}{183.186/(353-5-1)} \approx 9.55$$

Let's use 5% level of sig.

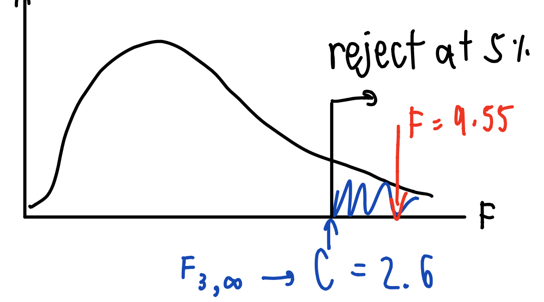
- But when performing an F-test, performances have joint impact.

HW

$$F \equiv \frac{(R^2/q)}{(1-R^2)/n-k-1}$$

$$\equiv \frac{(0.6278/3)}{(1-0.6278)/353-5-1} \approx 195.0981$$

f(F)



Since F = 9.55 > 2.6, we reject H_0 at 5% level and conclude that performances have joint effects on salary.

TABLE D.3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION

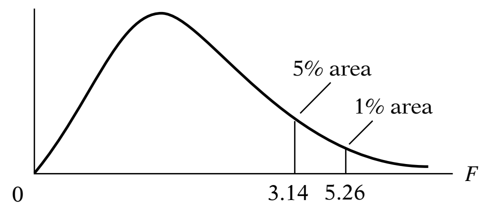
Example

$\Pr(F > 1.59) = 0.25$

$\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$

$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$

$\Pr(F > 5.26) = 0.01$



or the # of joint hypotheses tested (q)

(n-k-1)

df for denominator N_2	Pr	1	2	3	4	5	6	7	8	9	10	11	12
		df for numerator N_1											
25%	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
1 10%	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
5%	.05	161	200	216	225	230	234	237	239	241	242	243	244
	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
2	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
3	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
4	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
5	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
6	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
7	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
8	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
9	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians,* vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika.*

TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (*Continued*)

df for denominator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

8 How the Hypothesis Testing is done in Practice

1. Check the values of $t - statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t - statistics$ are to test $H_0 : \beta_i = 0$

   z-table

   $\Rightarrow$ If the d.f. > 30, then when $t \ > 1.96$, we can reject H_0 with 5% sig level

   $\Rightarrow$ **When** $t \ $ **> 1.96**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When** $t \ $ **< 1.96** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t \ < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $H_0 : \beta_i = 5$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

sales

other company performance

CEO characteristics

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	——	.112 (.050)	.100 (.049)
profmarg	——	−.0023 (.0022)	−.0022 (.0021)
ceoten	——	——	.0171 (.0055)
comten	——	——	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353

like a simple regression with 1 X