Exercise 1

Part I: Compute the following limits.

1. $\lim\limits_{x\to 25} \frac{5-\sqrt{x}}{25-x}$.
2. $\lim\limits_{x\to 0} \frac{x^2}{\sqrt{x^2+12}-\sqrt{12}}$.
3. $\lim\limits_{x\to 0} \frac{x\sin(x/2)}{|x|}$.
4. $\lim\limits_{x\to -2} \frac{|x|-2}{\sqrt{x^2-2x}-\sqrt{8}}$.
5. $\lim\limits_{x\to 2} \frac{|x-2|}{x-2}$.

Part II:

Let $f(x) = \begin{cases} x, & \text{for} \quad x < 0 \\ x^2, & \text{for} \quad 0 \le x < 2 \\ x, & \text{for} \quad x > 2 \end{cases}$.

1. Find each of the following limits (if exists).
   (a) $\lim\limits_{x\to 0^-} f(x)$ and $\lim\limits_{x\to 0^+} f(x)$.
   (b) $\lim\limits_{x\to 0} f(x)$.
   (c) $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$.
   (d) $\lim\limits_{x\to 2} f(x)$.
2. Find all the values of x at which the function $f(x)$ is discontinuous.

Limit and Continuity

1. Find each of the following limits (if it exists).
   (a) $\lim\limits_{x\to 8} \frac{x^2-5x-24}{x-8}$
   (b) $\lim\limits_{x\to 0} x^3(x^4+2x^3)^{-1}$
   (c) $\lim\limits_{x\to 0} \left(\frac{x^3+3x-1}{x} + \frac{1}{x}\right)$
   (d) $\lim\limits_{x\to 2} \left(\frac{1}{x-2} - \frac{6}{x^2+2x-8}\right)$
   (e) $\lim\limits_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}$
   (f) $\lim\limits_{x\to 5} \frac{|x-5|}{x-5}$

(g) $\lim\limits_{x\to 1} \frac{x-1}{\sqrt{5x^2-10x+5}}$

2. Determine the number (if any) at which each given function is discontinuous.

(a) $f(x) = x^3 + 2x - 1$

(b) $f(x) = \frac{x^2-1}{x^4-1}$

(c) $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{for} \quad x < 1 \\ 3, & \text{for} \quad 1 \le x < 2 \\ x^2 - 1, & \text{for} \quad x \ge 2 \end{cases}$

3. Find the values of a, b and c so that the given functions f and g are continuous.

(a) $f(x) = \begin{cases} ax, & \text{for} \quad x < 4 \\ x^2, & \text{for} \quad x \ge 4 \end{cases}$

(b) $g(x) = \begin{cases} bx, & \text{for} \quad x < 3 \\ c, & \text{for} \quad x = 3 \\ -2x + 7, & \text{for} \quad x > 3 \end{cases}$

4. **Trigonometric functions:** Find the the following limits.

(a) $\lim\limits_{x\to 0} \frac{x}{\sin(\pi x)}$

(b) $\lim\limits_{x\to 0} \tan(x)$

(c) $\lim\limits_{x\to 0} \frac{\tan(x/2)}{x}$

(d) $\lim\limits_{x\to 2} \frac{\sin(x-2)}{x^2+2x-8}$

(e) $\lim\limits_{x\to 0} \frac{3x+\tan(x)}{x\cos(5x)+\sin(7x)}$

(f) $\lim\limits_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$

(g) $\lim\limits_{x\to \pi/4} \frac{\tan(x)-1}{\sin(x)-\cos(x)}$

5. **Limit at infinity:** Find the the following limits.

(a) $\lim\limits_{x\to -\infty} \left(\frac{1}{5}x^7 - 5x^5 - x^2\right)$.

(b) $\lim\limits_{x\to -\infty} \frac{\sqrt{x^2-8}}{1+2x}$ (Note: $\sqrt{x^2} = |x|$)

(c) $\lim\limits_{x\to -\infty} \frac{2x-1}{\sqrt{3x^2-1}+2x}$

(d) $\lim\limits_{x\to \infty} \frac{2x^4-x^2-8x}{1-3x^4}$

(e) $\lim_{x\to-\infty} \frac{x^2-8}{1-2x}$

(f) $\lim_{x\to\infty} e^{1/x}$

(g) $\lim_{x\to\infty} \frac{3e^{5x}+2e^{-2x}}{e^{-2x}-e^{5x}}$ and $\lim_{x\to-\infty} \frac{3e^{5x}+2e^{-2x}}{e^{-2x}-e^{5x}}$

(h) $\lim_{x\to\infty} (\sqrt{x^2+5x}-x)$

Answers to Exercise 1

Part I: 1. 1/10; 2. $4\sqrt{3}$; 3. 0; 4. $2\sqrt{2}/3$; 5. DNE:
Part II:
1. (a) 0; (b)0;(c)4,2; (d) DNE;
2. $x = 2$;
Limit and Continuity
1. (a) 11; (b) 1/2; (c) 3; (d) 1/6; (e) 1/6; (f) DNE; (g) DNE;
2. (a) None; (b) $-1, 1$; (c) 1;
3. (a) $a = 4$; (b) $b = 1/3, c = 1$
4. (a) $1/\pi$; (b) 0; (c) 1/2; (d) 1/6; (e) 1/2; (f) 0; (g) $\sqrt{2}$
5. (a) $-\infty$; (b) $-1/2$; (c) $-2/(\sqrt{3}-2)$; (d) $-2/3$; (e) ∞; (f) 1; (g) $-3, 2$; (h) 5/2