

Quiz#3
EE320 Introductory Mathematical Economics
Section 046403

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Question 1 (100 points) Only two firms are competing in the market with inverse demand function $P = 50 - 2Q$. Firm 1 and firm 2 cost functions are $C_1 = 10 + 2q_1$ and $C_2 = 12 + 8q_2$, respectively.

1) (50 points) Derive profit functions, reaction functions and equilibrium quantities of firm 1 and 2.

$$\begin{aligned} \checkmark \quad \pi_1 &= P \cdot q_1 - C_1 = [50 - 2(q_1 + q_2)]q_1 - 10 - 2q_1 \\ &= 50q_1 - 2q_1^2 - 2q_1q_2 - 2q_1 - 10 \end{aligned}$$

$$\pi_1 = -2q_1^2 + 48q_1 - 2q_1q_2 - 10$$

$$\begin{aligned} \pi_2 &= P \cdot q_2 - C_2 = [50 - 2(q_1 + q_2)]q_2 - 12 - 8q_2 \\ &= 50q_2 - 2q_1q_2 - 2q_2^2 - 12 - 8q_2 \end{aligned}$$

$$= -2q_2^2 + 42q_2 - 2q_1q_2 - 12$$

$$\checkmark \quad \text{Reaction functions: } \frac{\partial \pi_1}{\partial q_1} = -4q_1 + 48 - 2q_2 = 0$$

$$-4q_1 = 2q_2 - 48$$

$$\boxed{q_1^* = 12 - \frac{q_2}{2}}$$

$$\frac{\partial \pi_2}{\partial q_2} = -4q_2 + 42 - 2q_1 = 0$$

$$-4q_2 = 2q_1 - 42$$

$$\boxed{q_2^* = 10.5 - \frac{q_1}{2}}$$

$$\checkmark \quad \text{Find } q_1^* \text{ and } q_2^* : \quad q_1^* = 12 - \frac{10.5}{2} + \frac{q_1}{4}$$

$$\frac{3}{4}q_1 = \frac{24 - 10.5}{2}$$

$$\frac{3}{4}q_1 = \frac{13.5}{2}$$

$$\begin{array}{c} 1 \quad 54.0 \\ \Rightarrow \quad \boxed{q_1^* = \frac{13.5 \cdot 4}{2 \cdot 3} = 9} \Rightarrow q_2^* = 6 \end{array}$$

2) (5 points) Which firm makes more profit and by how much?

$$\pi_1 = -2(q_1^2) + 48(q_1) - 2(q_1)(6) - 10 = -162 + 482 - 108 - 10 = 152$$

$$\pi_2 = -2(6^2) + 42(6) - 2(9)(6) - 12 = -72 + 252 - 108 - 12 = 60$$

4) (25 points) Show that both firms maximize profit in the equilibrium.

Check SOC

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= -4q_1 + 48 - 2q_2 \\ \frac{\partial \pi_1}{\partial q_2} &= -2q_1 \Rightarrow \frac{\partial^2 \pi_1}{\partial q_2^2} = 0 \end{aligned} \quad \begin{aligned} \frac{\partial^2 \pi_1}{\partial q_1^2} &= -4 \\ \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} &= -2 \\ \frac{\partial^2 \pi_1}{\partial q_2 \partial q_1} &= 0 \end{aligned} \quad \left. \begin{aligned} \pi_{1,11} &= -4 \\ \pi_{1,12} &= -2 \\ \pi_{1,22} &= 0 \end{aligned} \right\} \begin{aligned} \pi_{1,11} &= -4 < 0 \\ \pi_{1,11} \pi_{1,22} - \pi_{1,12}^2 &= 0 - (-2)^2 = -4 < 0. \end{aligned}$$

read to be $\oplus$

$\Rightarrow \therefore$ not satisfy max condition.

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_2} &= -4q_2 + 42 - 2q_1 \\ \frac{\partial \pi_2}{\partial q_1} &= -2q_2 \Rightarrow \frac{\partial^2 \pi_2}{\partial q_1^2} = 0 \end{aligned} \quad \begin{aligned} \frac{\partial^2 \pi_2}{\partial q_2^2} &= -4 \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} &= -2 \\ \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} &= 0 \end{aligned} \quad \left. \begin{aligned} \pi_{2,22} &= -4 \\ \pi_{2,21} &= -2 \\ \pi_{2,11} &= 0 \end{aligned} \right\} \begin{aligned} \pi_{2,22} &= -4 < 0 \\ \pi_{2,22} \pi_{2,11} - \pi_{2,12}^2 &= (-4)(0) - (-2)^2 = -4 < 0. \end{aligned}$$

$\therefore$ not satisfy max condition

5) (20 points) If firm 1 can set his quantity first, how much more profit he can make?

"Firm 1 moves first" means firm 1 takes what firm 2 will do into account and choose its own quantity produced accordingly:

$$\begin{aligned} \pi_1 &= P \cdot q_1 - C_1 = [50 - 2(q_1 + q_2)]q_1 - 10 - 2q_1 \quad \text{where } q_2^* = 10.5 - \frac{q_1}{2} \\ &= -2q_1^2 + 48q_1 - 2q_1q_2 - 10 \\ &= -2q_1^2 + 48q_1 - 2q_1(10.5 - \frac{q_1}{2}) - 10 \\ &= -2q_1^2 + 48q_1 - 21q_1 + q_1^2 - 10 \end{aligned}$$

$$\pi_1 = -q_1^2 + 27q_1 - 10$$

$$\frac{\partial \pi_1}{\partial q_1} = 2q_1 + 27 = 0 \Rightarrow q_1^* = 13.5 \Rightarrow q_2^* = 10.5 - \frac{13.5}{2} = 3.75$$

$$\pi_1 = -q_1^2 + 27q_1 - 10 = -(13.5)^2 + 27(13.5) - 10 = -182.25 + 364.5 - 10 = 172.25$$

$$\Rightarrow \boxed{\Delta \pi = 172.25 - 152 = 20.25}$$