Consdering a two – period problem,

$$U = \ln c_t + \beta \ln c_{t+1}, \qquad 0 < \beta < 1. \qquad (1)$$

$$c_{t+1} = R_t.\left(f\left(k_t\right) - c_t \right) \qquad (2)$$

Dividing both sides by $R_t = f'\left(k_{t+1}\right)$

$$c_t + R_t^{-1} c_{t+1} = f\left(k_t\right) = k_t^{\alpha}, \quad 0 < \alpha < 1 \qquad (3)$$

$$L = \ln c_t + \beta \ln c_{t+1} + \lambda\left[f\left(k_t\right) - \left(c_t + R_t^{-1} c_{t+1}\right)\right] \qquad (4)$$

The first-order conditions are

$$\frac{1}{c_t} = \lambda \qquad (5)$$

$$c_{t+1} = \frac{\beta R_t}{\lambda} \qquad (6)$$

Substituting the equation (5) into (6) *yields*

$$\frac{c_{t+1}}{c_t} = \beta R_t \qquad (7)$$

This is analogous to the Euler equation.

$Eq(7)$ *and* (3) *gives*

$$c_t + \beta c_t = f(k_t) = k_t^{\alpha} \qquad (8)$$

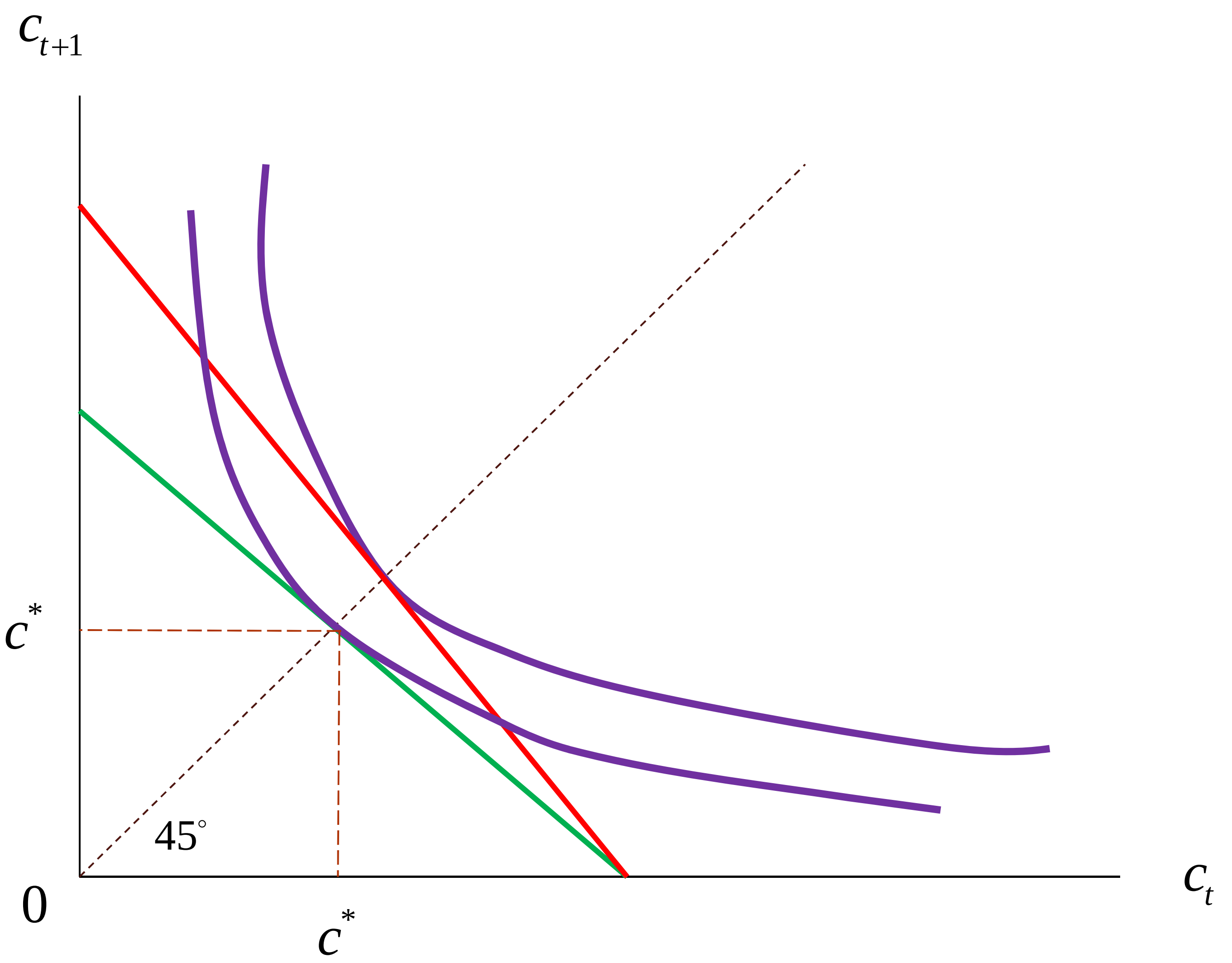
Or $$c_t = (1+\beta)^{-1} k_t^{\alpha} \qquad (9)$$

in the infinite horizon model, *eq* (9) *becomes*

$$c_t = (1-\beta) k_t^{\alpha} \quad = h(k_t) \qquad (10)$$

In steady state equilibrium,

$$c^* = (1-\beta)(k^{*\alpha})$$


c_{t+1}
c^*
45°
0
c^*
c_t

$$U = \frac{c_t^{1-\theta}}{1-\theta} + \beta \frac{c_{t+1}^{1-\theta}}{1-\theta}, \qquad \theta > 0, \;\; 0 < \beta < 1. \qquad (11)$$

$$c_{t+1} = R_t.\left(f\left(k_t\right) - c_t \right) \qquad (12)$$

Dividing both sides by R_t

$$c_t + R_t^{-1} c_{t+1} = f\left(k_t\right) \; = k_t^{\alpha} \qquad (13)$$

$$L = \frac{c_t^{1-\theta}}{1-\theta} + \beta \frac{c_{t+1}^{1-\theta}}{1-\theta} + \lambda \left[k_t^{\alpha} - \left(c_t + R_t^{-1} c_{t+1} \right) \right] \qquad (14)$$

The first-order conditions are

$$c_t^{-\theta} = \lambda \qquad (15)$$

$$\beta c_{t+1}^{-\theta} = R_t^{-1}\lambda \qquad (16)$$

Substituting the first equation into the second yields

$$\beta c_{t+1}^{-\theta} = R_t^{-1} c_t^{-\theta} \qquad (17)$$

Or $$\frac{c_{t+1}}{c_t} = [\beta R_t]^{\frac{1}{\theta}} \qquad (18)$$

This is analogous to the Euler equation.

$$Eq(18) \text{ and } (13) \text{ give} \quad c_t + \beta^{\frac{1}{\theta}} R_t^{\frac{(1-\theta)}{\theta}} c_t = k_t^{\alpha} \qquad (19)$$

$$\text{Or} \quad c_t = \frac{1}{\left[1+\beta^{\frac{1}{\theta}} R_t^{\frac{(1-\theta)}{\theta}}\right]} k_t^{\alpha} \qquad (20)$$

Let $s(R_t)$ *denote the fraction of income saved. Then*

$$\Rightarrow \quad s(R_t) = \frac{\beta^{\frac{1}{\theta}} R_t^{\frac{(1-\theta)}{\theta}}}{\left[1+\beta^{\frac{1}{\theta}} R_t^{\frac{(1-\theta)}{\theta}}\right]} \qquad (21)$$

We can rewrite (20) *into* $\quad c_t = \left[1 - s(R_t)\right] k_t^{\alpha} \qquad (22)$

The saving is increasing in R_t *iff* $R_t^{\frac{(1-\theta)}{\theta}}$ *is increasing in* R_t.

$s(R_t)$ *is increasing in* R_t *if* $\theta < 1$.

The rise in R_t *has both the income and a substituting effects.*

For $\theta = 1, \quad \Rightarrow \quad R_t^{\frac{(1-\theta)}{\theta}} = \left[f'(k_{t+1})\right]^{\frac{(1-\theta)}{\theta}} = 1$

$s(R_t) = \frac{\beta}{1+\beta}, \qquad eq(22)$ *becomes*

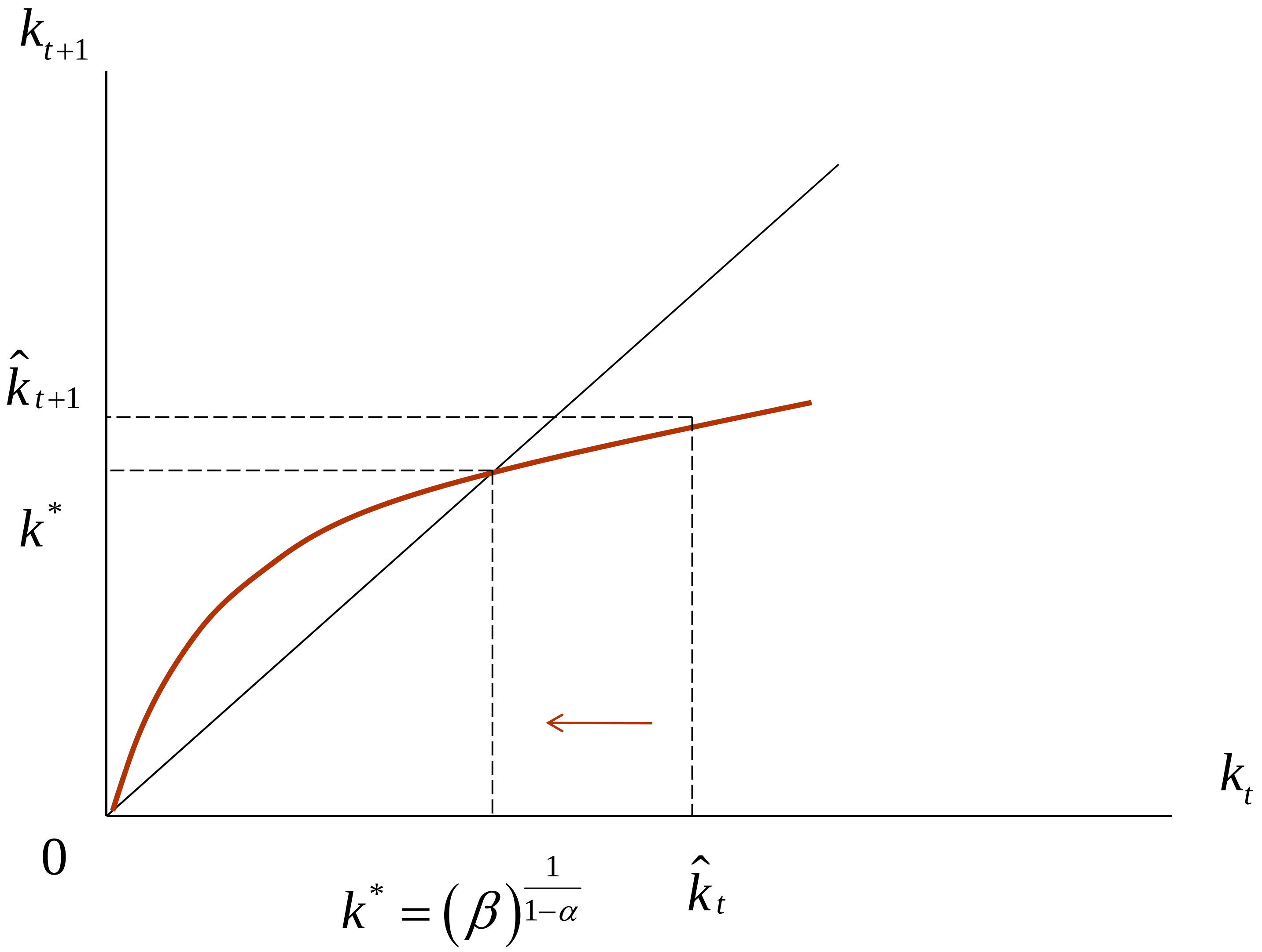
$$c_t = \left(\frac{1}{1+\beta}\right) k_t^{\alpha} \quad \Rightarrow \quad c_t = (1-\beta) k_t^{\alpha}, \quad \forall \ t \in [0, \infty)$$

$$k_{t+1} = k_t^{\alpha} - \left(\frac{1}{1+\beta}\right) k_t^{\alpha}$$

$$k_{t+1} = \left(\frac{\beta}{1+\beta}\right) k_t^{\alpha},$$

$$For \quad \inf inite \quad time\ horizon\ \mathrm{mod}\, el, \quad \left(\frac{\beta}{1+\beta}\right) \Rightarrow \beta,$$

$$k_{t+1} = \beta k_t^{\alpha}, \qquad \Rightarrow \quad k^* = \beta k^{*\,\alpha}, \qquad \Rightarrow \qquad k^* = (\beta)^{\frac{1}{1-\alpha}} > 0$$



k_{t+1}
$\hat{k}_{t+1}$
k^*
k_t
0
$k^* = (\beta)^{\frac{1}{1-\alpha}}$
$\hat{k}_t$