Quiz 4

1. Find
$$\int \left[\frac{\sqrt{\ln(x)}}{x} + \frac{\cos(x)}{\sqrt{2-\sin(x)}} \right] dx.$$

<u>Solution</u>

Consider
$$\int \left[\frac{\sqrt{\ln(x)}}{x} + \frac{\cos(x)}{\sqrt{2-\sin(x)}} \right] dx. = \int \frac{\sqrt{\ln(x)}}{x}\, dx + \int \frac{\cos(x)}{\sqrt{2-\sin(x)}}\, dx.$$

We can apply substitution technique for each of the terms $\int \frac{\sqrt{\ln(x)}}{x}\, dx$ and $\int \frac{\cos(x)}{\sqrt{2-\sin(x)}}\, dx$ separately.

(I) Let $u = \ln(x)$. Then $du = \frac{1}{x}dx$ or $dx = xdu$.

$$\begin{aligned} \int \frac{\sqrt{\ln(x)}}{x}\, dx &= \int \frac{\sqrt{u}}{x}\, xdu \\ &= \int u^{1/2}\, du \\ &= \frac{u^{3/2}}{3/2} + C \\ &= \frac{2}{3}[\ln(x)]^{3/2} + C \end{aligned}$$

(II) Let $u = 2 - \sin(x)$. Then $du = -\cos(x)dx$ or $dx = \frac{1}{-\cos(x)}du$.

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{2-\sin(x)}}\, dx &= \int \frac{\cos(x)}{u^{1/2}} \frac{1}{-\cos(x)} du \\ &= -\int u^{-1/2} du \\ &= -\frac{u^{1/2}}{1/2} + C \\ &= -2[2-\sin(x)]^{1/2} + C \end{aligned}$$

From(I) and (II),

$$\int \left[\frac{\sqrt{\ln(x)}}{x} + \frac{\cos(x)}{\sqrt{2-\sin(x)}} \right] dx = \frac{2}{3}[\ln(x)]^{3/2} - 2[2-\sin(x)]^{1/2} + C.$$

■