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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\bar{x} = \left[\dfrac{63+72+78+81+87+75+75+90}{8}\right]$

$\bar{x} = 77.625$

$\bar{y} = \dfrac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8}$

$\bar{y} = 3.2125$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_1 = \sum x_i y_i - n\bar{x}\bar{y}$

$\hat{\beta}_1 = \dfrac{2012.4 - 8(77.625)(3.2125)}{48717 - 8(77.625)^2} = \dfrac{2012.4 - 1994.9625}{48717 - 48205.125} = \dfrac{31}{910}$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_0 = 3.2125 - 0.034066(77.625)$

$= 0.5681$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{u}_i = y_i - \hat{y}_i$

$i=1,\ \hat{u}_1 = 2.8 - 2.714 = 0.086$

$2,\ \hat{u}_2 = 3.4 - 3.021 = 0.379$

$3,\ \hat{u}_3 = 3 - 3.225 = -0.225$

$4,\ \hat{u}_4 = 3.5 - 3.327 = 0.173$

$5,\ \hat{u}_5 = 3.6 - 3.532 = 0.068$

$6,\ \hat{u}_6 = 3 - 3.123 = -0.123$

$7,\ \hat{u}_7 = 2.7 - 3.123 = -0.423$

$8,\ \hat{u}_8 = 3.7 - 3.634 = 0.066$

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$i=1,\ \hat{y}_1 = 0.5681 + 0.034(63) = 2.714$

$=2,\ \hat{y}_2 =$ ——— $(72) = 3.021$

$=3,\ \hat{y}_3 =$ ——— $(78) = 3.225$

$=4,\ \hat{y}_4 =$ ——— $(81) = 3.327$

$=5,\ \hat{y}_5 =$ ——— $(87) = 3.532$

$=6,\ \hat{y}_6 =$ ——— $(75) = 3.123$

$=7,\ \hat{y}_7 =$ ——— $(75) = 3.123$

$=8,\ \hat{y}_8 =$ ——— $(90) = 3.634$

$\sum_{i=0}^{N} \hat{u}_i = 0.086 + 0.379 + (-0.225) + 0.173 + 0.068 + (-0.123) + (-0.423) + 0.066 = 0$

1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.435}{8-2} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sum x_i^2}{n\sum(x_i-\bar{x})^2}(\sigma^2) = \frac{48717(0.07245)}{8(511.875)} = 0.863$$

$$var(\hat{\beta}_0) = \frac{\sigma^2}{\sum(x_i-\bar{x})^2} = \frac{0.0725}{511.875} = 0.000141$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{} = \frac{2214 - 10(20)(9.1)}{4440 - 10(20)^2} = \frac{2214 - 1820}{4440 - 4000} = 0.895$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 9.1 - 0.895(20) = -8.81$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$i=1, \hat{y}_1 = 0.1453, \hat{u}_1 = -0.145$

$=2, \hat{y}_2 = 1.9364, \hat{u}_2 = 0.064$

$=3, \hat{y}_3 = 3.727, \hat{u}_3 = 1.273$

$=4, \hat{y}_4 = 5.518, \hat{u}_4 = 0.482$

$=5, \hat{y}_5 = 7.309, \hat{u}_5 = -0.309$

$=6, \hat{y}_6 = 10.891, \hat{u}_6 = -0.891$

$=7, \hat{y}_7 = 12.682, \hat{u}_7 = -2.682$

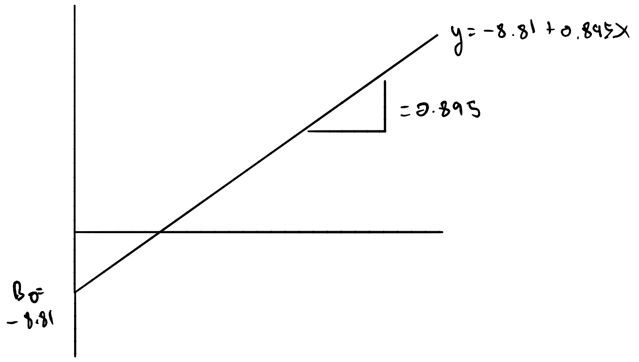
$=8, \hat{y}_8 = 14.4727, \hat{u}_8 = 0.527$

$=9, \hat{y}_9 = 16.2636, \hat{u}_9 = -0.264$

$=10, \hat{y}_{10} = 18.0549, \hat{u}_{10} = 1.945$

$\sum_{i=0}^{n} \hat{u}_i = (-0.145) + 0.064 + 1.273 + 0.482 + (-0.309) + (-0.891) + (-2.682) + 0.527 + (-0.264) + 1.945 = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



sub $\bar{x}$ to regression fn,

$y = -8.81 + 0.895(20)$

$= 9.1 = \bar{y}$

$\therefore$ line pass $(\bar{x}, \bar{y})$

2.4 If X_i =16, what is the predicted Y?

$\hat{y}_i = \beta_0 + \beta_1 x_1 = -8.81 + 0.895(16)$

$= 5.51$

2.5 Find $var(\widehat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{10-2} = 1.7614$

$var(\hat{\beta}_0) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2} = \frac{1.7614}{440} = 0.004$

$var(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2}(\sigma^2) = \frac{4440(1.7614)}{10(440)} = 1.777$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

SLA 1-4, $\hat{\beta}_1$ = unbiased

$$\hat{\beta}_1 = \frac{\sum (y_i - y)(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = \sum_{i=1}^{n} (y_i - \bar{y}) k_i$$

$$= \sum_{i=1}^{n} (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i$$

$$= \sum_i \beta_1 (x_i - \bar{x}) k_i + \sum_i u_i k_i$$

$$= \beta_1 \sum_i x_i k_i + \sum_i u_i a_i$$

$$= \beta_1 \sum_i x_i \left(\frac{x_i}{\sum_i x_i^2} \right) + \sum_i u_i a_i$$

$$= \beta_1 \frac{\sum_i x_i^2}{\sum_i x_i^2} + \sum_i u_i k_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_i u_i k_i] = \beta_1 + E\left[\sum_i u_i k_i\right]$$

$$E(\hat{\beta}_1) = \beta_1 + \sum_i k_i E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$

$\therefore$ unbiased.