2.2.3b) Exponential and Logarithm

Another two functions commonly used in economics analysis are exponential and logarithm. The two functions are inverse function of each other.

Exponential : $y = a^x$

Logarithm : $y = \log_a x$

Example 2.J: Bond pricing formula

A Two-year bond offers a stream of payment for \$10 in every single year until it is matured. At the maturity date, this bond also delivers the principal amount at \$100. If market interest rate is r% per annual, what would be the fair price of the two-year bond?

Principle of Valuation.

Intrinsic Value = Value of the remaing Cash-flow that the asset will be delivered

$V_0 = \frac{10}{1+r} + \frac{110}{(1+r)^2}$

↑ time zero. $P_0 = \frac{10}{1+r} + \frac{110}{(1+r)^2}$

Present Value. "t" ; F

$\Rightarrow \frac{F}{(1+r)^t} = PV(F)$

(r discount rate

$\frac{1}{1+r}$ => discount factor

Chapter 3: Equilibrium *and* Comparative static analysis: Linear model

3.1 Linear economics model

What we have been discussing so far centers on

- How to characterize relationship of economic variables using mathematical function/equation.
- Properly define the function/equation in economics analysis.

What we are going next is to

- Write down some basic linear economic models (in mathematical form)
  - Set of system of equations.
  - Interlink among variables in the model.
- Solve for the solution of the model => *Static analysis*
  - (Simultaneous) Solution to the system of equations
- Analyze behavior of the endogenous equilibrium solutions => *Comparative static analysis*

List of the economic models that we will be taking about include

- Micro-market equilibrium model
  - Single market equilibrium (Partial analysis)
  - Multi-market equilibrium (General analysis)
- Macroeconomic model
  - Keynesian cross
  - IS-LM model

3.2 System of equations

- Comprises of *N-unknown* variables **and** *M-equations.*
  - Equations could be linear, nonlinear, or mixed.
  - But we now focus on linear system.

- Set of N-unknown variables that simultaneously satisfies all the M equations, at the same time, is called solution of the system.

- For now, we will only focus on
  - 2 x 2 case
  - 3 x 3 case

- These two cases are solvable by *pencil and paper.*

- For any larger scale of the system of equations, we have to apply matrix algebra to solve for the solution. (In the later chapter)

Example 3.A: Solving for the solution of the system of equations

$X + Y = 7$

$3X - 6y = -15;$

$x + y = 7 \quad -(1)$

$3x - 6y = -15 \quad -(2)$

$(1) \times 6 \rightarrow 6x + 6y = 42 \quad -(3)$

$(2) + (3) \quad 3x - 6y + 6x - 6y = -15 + 42$

$9x = 27 \rightarrow x = 3$

$y = 4$

Example 3.B: Distinguishing the type of solution

- System 1: $2x + 3y = 7$ and $x - y = 5$. Unique
- System 2: $y = -2x + 6$ and $4x + 2y = 12$. Infinite Sol's
- System 3: $x + 2y = 14$ and $3x + 6y = 8$

Even $n = m$ $\not\Rightarrow$ Sol's exists

↳ no Sol's

① Exist (Sol's)

. Unique one point two lines crossed

. Multiple (Infinite Sol's)

↳ two equations represent the same line

② No-exist problem.

Parallel "//"

3.3 Market Equilibrium model

- What is the market equilibrium model?
  - The model is used to predict equilibrium price and output.
    - At what price, people trade?
    - How much do they trade?
  - Two concepts: ***partial equilibrium (PE) vs general equilibrium (GE)***
    - PE: One particular market, ignore the rest.
    - GE: All the interlinked markets are taken into consideration at the same time.

Example 3.C Compare and contrast between PE and GE model

Partial	General
$Q_d^x = a - bP_x + cP_y$	$Q_d^x = a - bP_x + cP_y$
$Q_s^x = d + eP_x$	$Q_s^x = d + eP_x$
P_y are given exogenously	$Q_d^y = f - gP_y + hP_x$
Treat as given	$Q_s^y = j + kP_y$
	(P_x, P_y) are solved for together.

3.3.1) Partial equilibrium model

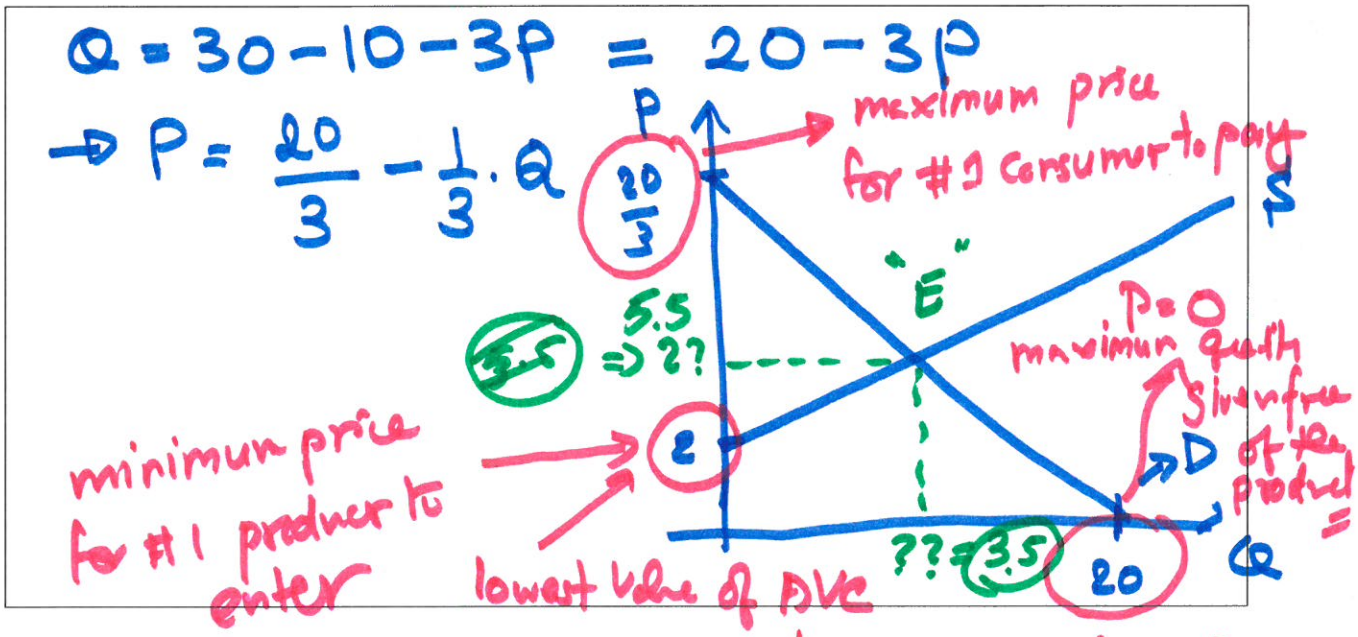
Example 3.D Suppose that *market* demand and supply equation can be given by the following two equations:

$$Q_d = 30 - 5Y - 3P$$

and $P = 2 + Q_s$

P : vertical
Q : horizontal

a. Graph the demand and supply curve for Y = 2



b. Is the product normal/inferior goods?

→ Inferior product.

→ maximum Consumer price of # 1 consumer

→ minimum produce price of #1 producer

→ maximum quantity purchased in the market by demander.

→ to have min AVC for the most efficient firm.

$$Q^d = 20 - 3P \quad \longrightarrow \quad P = 2 + Q^s \qquad \Big\} \Rightarrow Q^d = Q^s = Q^*$$

$$Q^* = 20 - 3(2 + Q^*)$$

$$Q^* = 20 - 6 - 3 \cdot Q^*$$

$$4Q^* = 14 \rightarrow Q^* = \frac{14}{4} = 3.5 \text{ units}$$

$$P^* = 2 + Q^* = 2 + 3.5 = 5.5 \text{ bitcoin.}$$

c. At what level of income, does the demand function coincide with the equation $Q_d = 10 - 3P$?

Original Equation $Q = 30 - 5y - 3P$

$Q_d = 10 - 3P$

$y = 4 \Rightarrow$ when

$Q = 10 - 3P$

$30 - 5y = 10$

$20 = 5y$

$y = 4$

↳ $P_{curve} \Rightarrow$ left P_{curve} ($y = 2$; b/c inferior goods)

d. Find the market equilibrium for quantity of output and price, using the demand equation given in "c".

$Q = 10 - 3P$

$P = 2 + Q$

$Q = 10 - 3(2 + Q)$

$Q = 10 - 6 - 3Q$

$4Q = 4 \rightarrow Q = 1$ (previously, 3.5)

$P = 3$ (previously, 5.5)

($Q \downarrow, P \downarrow$ ($y \downarrow$; b/c inferior goods))

e. **What happen to equilibrium output/price when income is $1 higher than the level associated to that in "c"? Interpret your result.

$y = 5 \rightarrow Q = 30 - 5(5) - 3P$

$= 5 - 3P \rightarrow P$

$P = 2 + Q \rightarrow S$

$Q = 5 - 3(2 + Q)$

$= 5 - 6 - 3Q$

$Q = -\frac{1}{4}$ → market doesn't exist

→ why?

→ Price $\frac{5}{3}$ (< 2)

Existence → Max Price (consumer) > min price (produ)