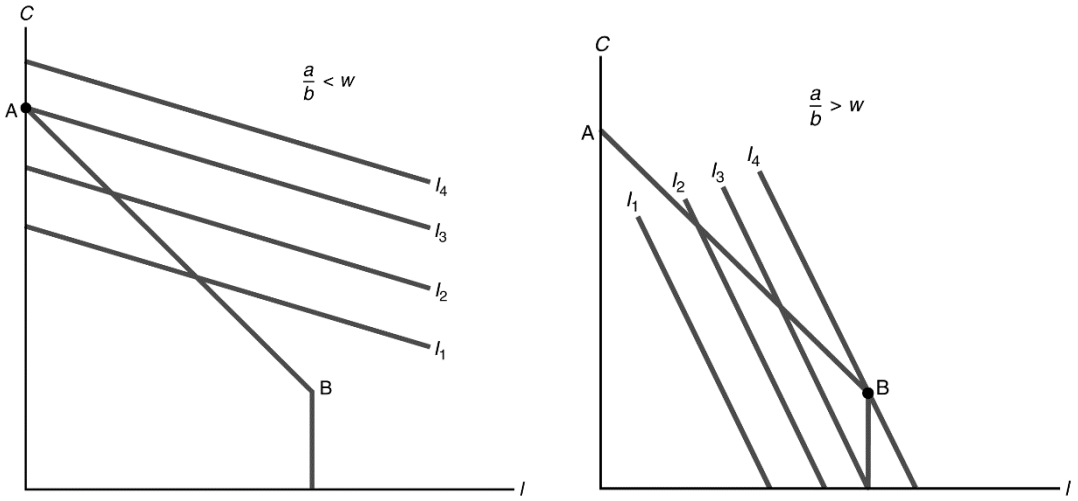
Problem set 1_solution

2. $u = al + bC$

(a) To specify an indifference curve, we hold utility constant at $\bar{u}$. Next rearrange in the form:

$$C = \frac{\bar{u}}{b} - \frac{a}{b} l$$

Indifference curves are therefore linear with slope, $-a/b$, which represents the marginal rate of substitution. There are two main cases, according to whether $a/b > w$ or $a/b < w$. The top panel of the left figure below shows the case of $a/b < w$. In this case the indifference curves are flatter than the budget line and the consumer picks point A, at which $l = 0$ and $C = wh + \pi - T$. The right figure shows the case of $a/b > w$. In this case the indifference curves are steeper than the budget line, and the consumer picks point B, at which $l = h$ and $C = \pi - T$. In the coincidental case in which $a/b = w$, the highest attainable indifference curve coincides with the indifference curve, and the consumer is indifferent among all possible amounts of leisure and hours worked.



(b) The utility function in this problem does not obey the property that the consumer prefers diversity, and is therefore not a likely possibility.

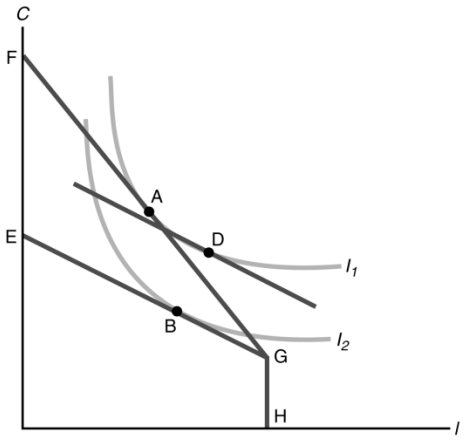
(c) This utility function does have the property that more is preferred to less. However, the marginal rate of substitution is constant, and therefore this utility function does not satisfy the property of diminishing marginal rate of substitution.

4. When the government imposes a proportional tax on wage income, the consumer's budget constraint is now given by:

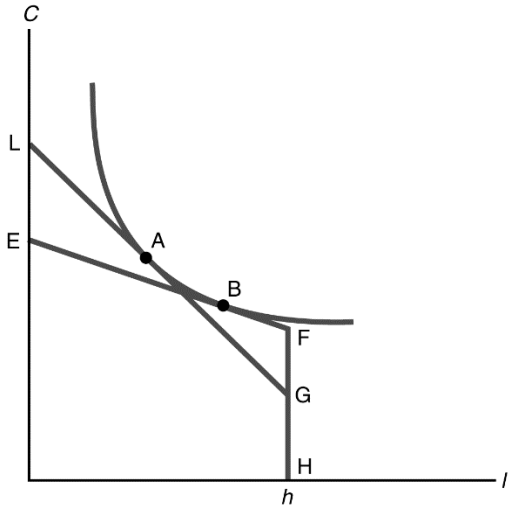
$$C = w(1-t)(h-l) + \pi - T,$$

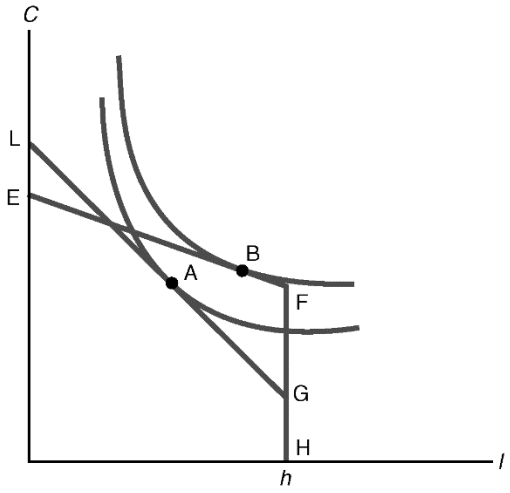
where t is the tax rate on wage income. In the figure below, the budget constraint for $t = 0$, is FGH. When $t > 0$, the budget constraint is EGH. The slope of the original budget line is $-w$, while the slope of the new budget line is $-(1-t)w$. Initially the consumer picks the point A on the original budget line. After the tax has been imposed, the consumer picks point B. The substitution effect of the imposition of the tax is to move the consumer from point A to point D on the original indifference curve. The point D is at the tangent point of indifference curve, I_1, with a line segment that is parallel to EG. The pure substitution effect induces the consumer to reduce consumption and increase leisure (work less).

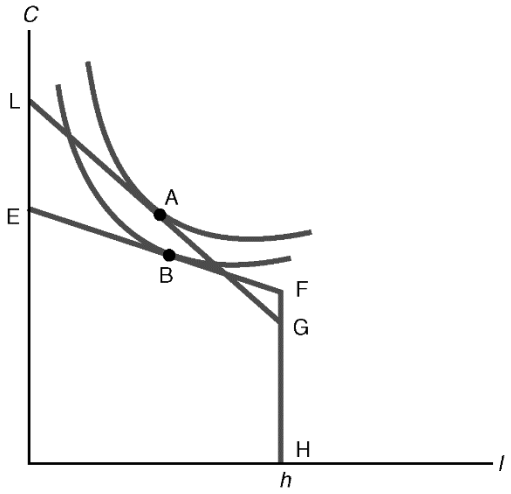
The tax also makes the consumer worse off, in that he or she can no longer be on indifference curve, I_1, but must move to the less preferred indifference curve, I_2. This pure income effect moves the consumer to point B, which has less consumption and less leisure than point D, because both consumption and leisure are normal goods. The net effect of the tax is to reduce consumption, but the direction of the net effect on leisure is ambiguous. The figure shows the case in which the substitution effect on leisure dominates the income effect. In this case, leisure increases and hours worked fall. *Although consumption must fall, hours worked may rise, fall, or remain the same.*



6. The increase in dividend income shifts the budget line upward. The reduction in the wage rate flattens the budget line. One possibility is depicted in the figures below. The original budget constraint HGL shifts to HFE. There are two income effects in this case. The increase in dividend income is a positive income effect. The reduction in the wage rate is a negative income effect. The drawing in the top figure shows the case where these two income effects exactly cancel out. In this case we are left with a pure substitution effect that moves the consumer from point A to point B. Therefore, consumption falls and leisure increases. As leisure increases, hours of work must fall. The middle figure shows a case in which the increase in dividend income, the distance GF, is larger and so the income effect is positive. The consumer winds up on a higher indifference curve, leisure unambiguously increases, and consumption may either increase or decrease. The bottom figure shows a case in which the increase in dividend income, the distance GF, is smaller and so the income effect is negative. The consumer winds up on a lower indifference curve, consumption unambiguously decreases, and leisure may either increase or decrease.







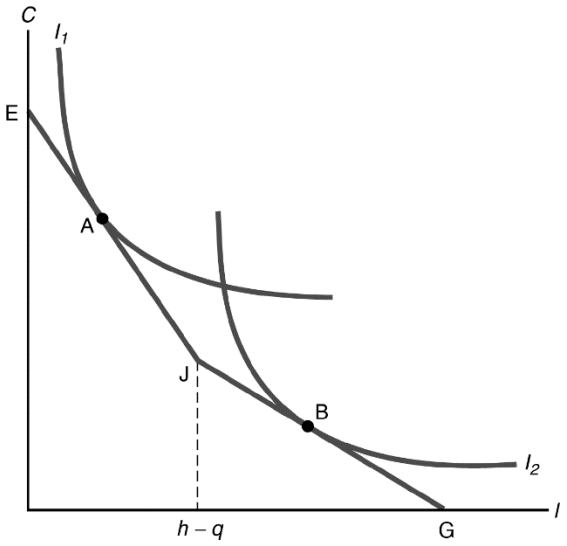
7. This problem introduces a higher, overtime wage for hours worked above a threshold, q. This problem also abstracts from any dividend income and taxes.

(a) The budget constraint is now EJG in the figure below. The budget constraint is steeper for levels of leisure less than h - q, because of the higher overtime wage. The figure depicts possible choices for two different consumers.

Consumer #1 picks point A on her indifference curve, I_1.

Consumer #2 picks point B on his indifference curve, I_2.

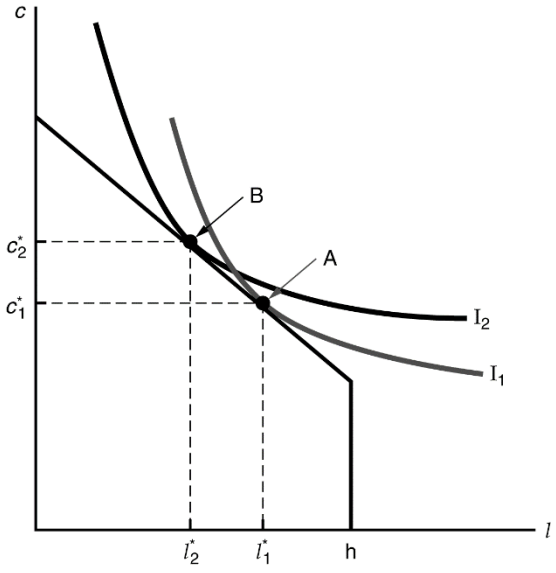
Consumer #1 chooses to work overtime; consumer #2 does not.



(b) The geometry of the figure above makes it clear that it would be very difficult to have an indifference curve tangent to EJG close to point J. In order for this to happen, an indifference curve would need to be close to right angled as in the case of pure complement. It is unlikely that consumers wish to consume goods and leisure in fixed proportions, and so points like A and B are more typical. For any other allowable shape for the indifference curve, it is impossible for point J to be chosen.

(c) An increase in the overtime wage steepens segment EJ of the budget constraint, but has no effect on the segment JG. For an individual like consumer #2, the increase in the overtime wage has no effect up until the point at which the increase is large enough to shift the individual to a point like point A. Consumer #2 receives no income effect because the income effect arises out of a higher wage rate on inframarginal units of work. An individual like consumer #1 has the traditional income and substitution effects of a wage increase. Consumer #1 increases her consumption, but may either increase or

reduce hours of work according to whether the income effect outweighs the substitution effect.



10. Supposing that the only options open to the consumer are working q hours and paying a tax T, or working zero hours and receiving an unemployment insurance benefit b, consumption will be $w(h-q)-T$ if the consumer works, and b if the consumer decides not to work. Then, either the consumer prefers not to work, as in the Figure 10.1, where the highest indifference curve is achieved at point A rather than at point B, or the consumer prefers to work, as in Figure 10.2. There is also another case where the consumer is just indifferent between working and not working, but that case is not important.

a) Think of the economy as consisting of many consumers, some of whom are in a situation as in the Figure 10.1 and some as in Figure 10.2. Some consumers do not work, and some choose to work. If the wage goes up, then that will make working preferable for some consumers who formerly did not choose to work. An increase in the wage will not discourage anyone from working, but those who were working already will not choose to vary hours of work (they cannot). But total employment in the economy will increase, as now more people are working. With the constraint on hours of work, there are no issues related to income and substitution effects. A higher wage always increases the total quantity of labor supplied.

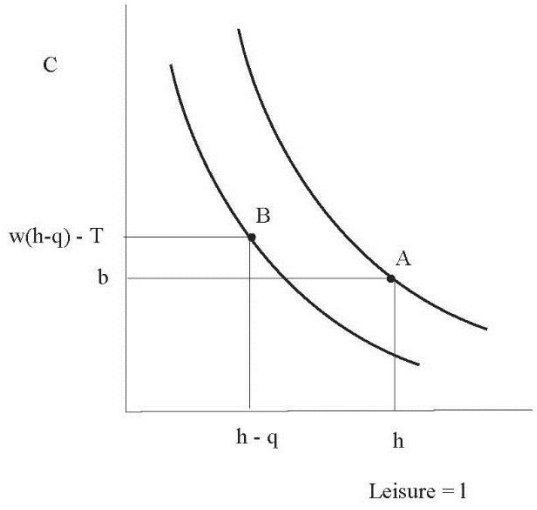


Figure 10.1

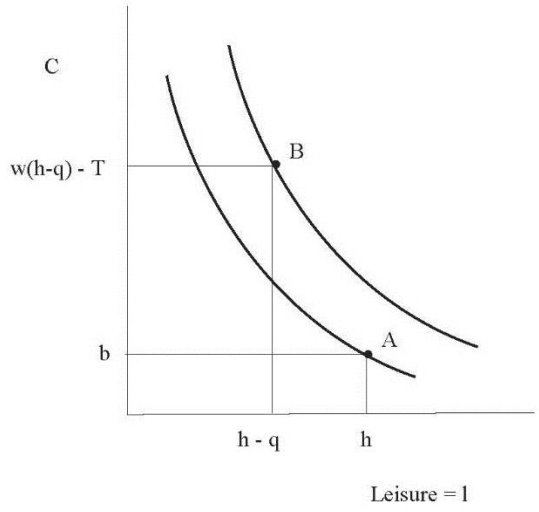


Figure 10.2

b) Similar to part (a), if the unemployment insurance benefit increases, this will make not working preferable to some consumers who were formerly working, and employment will fall. An increase in the unemployment insurance benefit unequivocally reduces the quantity of labor supplied.

12. The firm chooses its labor input N^d so as to maximize profits. When there is no subsidy, profits for the firm are given by

$$\pi = zF(K, N^d) - wN^d.$$

That is, profits are the difference between revenue and costs. In the top figure on the following page the revenue function is $zF(K, N^d)$ and the cost function is the straight line, wN^d. The firm maximizes profits by choosing the quantity of labor where the slope of the revenue function equals the slope of the cost function:

$$MP_N = w.$$

The firm's demand for labor curve is the marginal product of labor schedule in the bottom figure below.

With an employment subsidy, the firm's profits are given by:

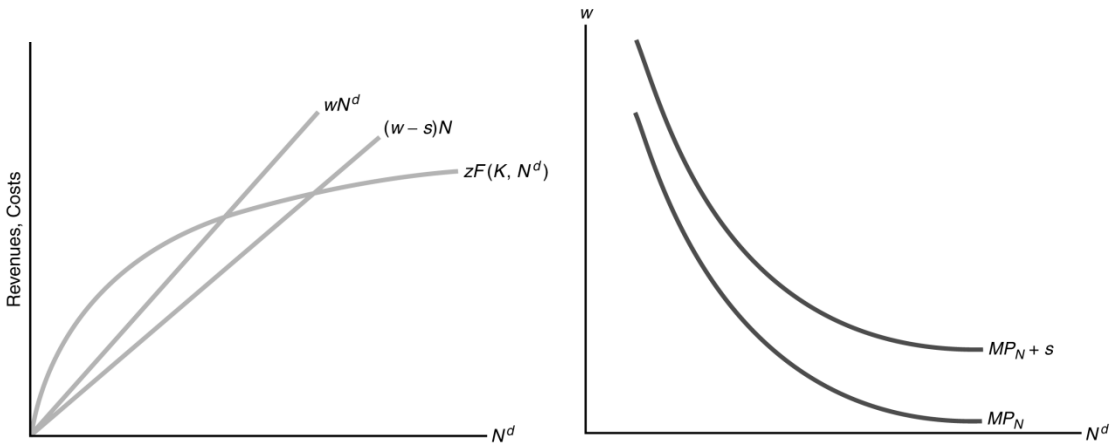
$$\pi = zF(K, N^d) - (w - s)N^d$$

where the term $zF(K, N^d)$ is the unchanged revenue function, and $(w - s)N^d$ is the cost function.

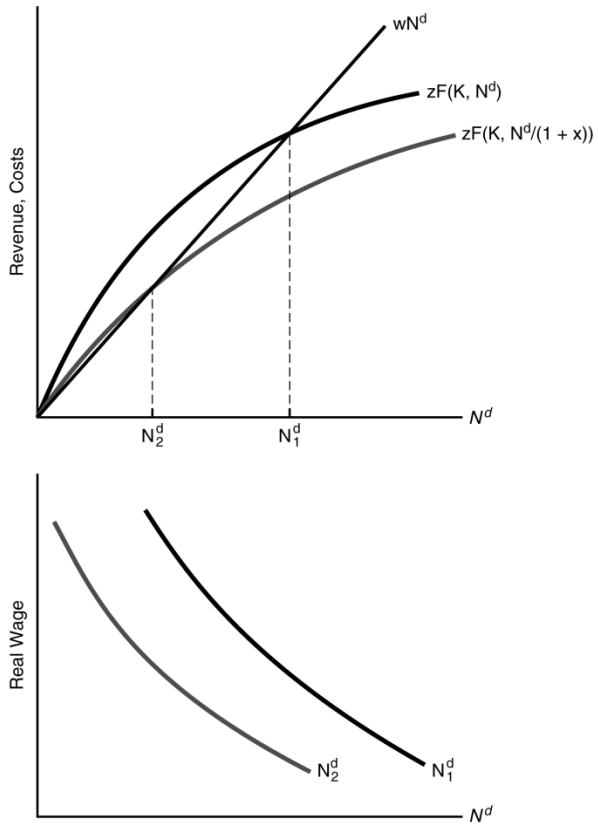
The subsidy acts to reduce the cost of each unit of labor by the amount of the subsidy, s. In the top figure below, the subsidy acts to shift down the cost function for the firm by reducing its slope. As before, the firm will maximize profits by choosing the quantity of labor input where the slope of the revenue function is equal to the slope of the cost function, $(t - s)$, so the firm chooses the quantity of labor where

$$MP_N = w - s.$$

In the bottom figure below, the labor demand curve is now $MP_N + s,$ and the labor demand curve has shifted up. The subsidy acts to reduce the marginal cost of labor, and the firm will hire more labor at any given real wage.


Revenues, Costs
wN^d
$(w - s)N$
$zF(K, N^d)$
N^d
w
$MP_N + s$
MP_N
N^d

15. As the firm has to internalize the pollution, it realizes that labor is less effective than it previously thought. It now needs to hire $N(1 + x)$ workers where N were previously sufficient. This is qualitatively equivalent to a reduction of z, total factor productivity. The figure below highlights the resulting outcome: the firm now hires fewer people for a given wage and thus its labor demand is reduced.



16. $Y = zK^{0.3}n^{0.7}$

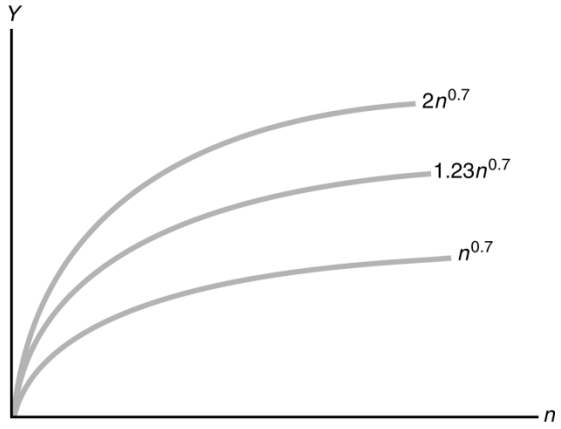
(a) $Y = n^{0.7}$. See the top figure below. The marginal product of labor is positive and diminishing.

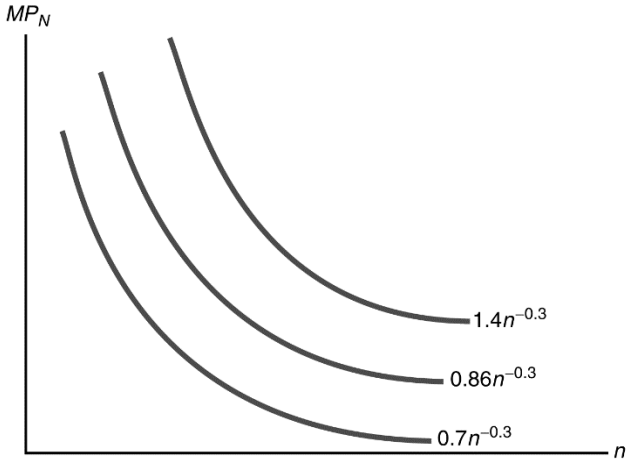
(b) $Y = 2n^{0.7}$. See the figures below.

(c) $Y = 2^{0.3}n^{0.7} \approx 1.23n^{0.7}$. See the figures below.

(d) See the bottom figure below.

$$z = 1, K = 1 \Rightarrow MP_N = 0.7n^{-0.3}$$
$$z = 2, K = 1 \Rightarrow MP_N = 1.4n^{-0.3}$$
$$z = 1, K = 2 \Rightarrow MP_N = 2^{0.3} \times 0.7n^{-0.3} \approx 0.86n^{-0.3}$$





2. Suppose that labor supply is strictly increasing; no part of the supply curve has backward bending property. An increase in the pay-roll tax would always cause an increase in the equilibrium real wage and a decrease in the total labor employment. Is the statement "TRUE", "FALSE" or "UNCERTAIN"?

An increase in pay-roll tax distorts household decision in that household tends to work less and choosing more of the leisure. This distortionary effect is strong only substitution effect dominates the income effect, which is the case for this problem. Thus, in terms of the graphical adjustment, individual labor supply curve should be shifting left, and hence the aggregate supply curve. To conclude about the equilibrium adjustment, it remains to be seen if the demand for labor is downward or fixed. Under both situations, wage would increase; however, equilibrium employment would only drop in the former.

Question 3

Suppose that there is a progressive tax on labor income. We model this by supposing that labor income wN below a threshold "M*" then the household faces a tax rate τ , but if $wN > M^*$ the tax rate increases to $\tau' > \tau$ for *that portion of income above the threshold*.

(Hint: For simplicity, we will assume that the price of consumption goods is normalized to one, i.e. $p = 1$. This assumption can be made without the loss of generality.)

a. Suppose a worker has unearned income π, has standard preferences over consumption and leisure, and faces this tax schedule.
    i. Characterize the worker's budget set. Properly define what it looks like for the budget set.
    ii. Characterize the worker's optimal choice. How many possibility could there be for the outcomes of optimal choices?

b. Suppose that preferences (MRS) differ across workers but that they all face the same taxes and wage rates, and characterize the optimal choices for different individuals. Is there likely to be a mass of workers with labor income equal to M*?

c. Now suppose that there is an increase in the top labor tax rate τ', but τ is unchanged. What happens to the labor supply choices of different workers?

d. Now suppose that there is an increase in the lower labor tax rate τ , but τ' is unchanged. What happens to the labor supply choices of different workers?

Question 4

A representative worker (household) has preferences:

$$u(C, L) = 2\sqrt{C} - a\frac{(1 - L)^{1.5}}{1.5}$$

Suppose that we normalized price of the consumption goods to 1. The budget constraint is: C = w(1 − L), where 1 is the hours in the day, so 1 − L is labor supply. Capital is fixed at 1, and the representative firm technology is: $Y = z\sqrt{N}$

a. Find the household labor supply function.

b. Find the labor demand function

c. Find expressions for the equilibrium values of the labor input and the wage.

d. How much does the representative worker (household) consume in the equilibrium?

e. Suppose that "a" increases but "z" is unchanged. What is the effect of this change on labor and the wage? Provide some economics intuitions.

   "a" can be interpreted as the coefficient of laziness.

f. Do the same exercise in "e", but now suppose instead that "z" decreases but "a" stays the same. Provide some economics intuitions.