1.a) $y_i = \beta_1 + \beta_2 X_i + U_i,\ U_i \sim NIID\ (0, 6^2)$

find the estimators of β_1 & β_2

$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X}$ $\qquad \hat{\beta}_2 = \dfrac{\Sigma (X_i - \bar{X})(y_i - \bar{y})}{\Sigma (X_i - \bar{X})^2}$

$\hat{\beta}_2 = \dfrac{-174.20}{1,098.8} = \dfrac{-13}{82} = -0.158537$ #

$\tilde{y}_i = 22.9641 - 0.159\ X_i$

$\hat{\beta}_1 = 21.03 - \left(\dfrac{-13}{82}\right)(12.20) = 22.964$ #

$\hat{\beta}_1$ & $\hat{\beta}_2$ are the point estimators which $\hat{\beta}_1$ is the estimator of β_1 and $\hat{\beta}_2$ is the estimator of β_2

when X_i is equal to 0, y_i

1.b) $r^2 = \dfrac{ESS}{TSS} = \dfrac{\Sigma (\hat{y}_i - \bar{y})^2}{\Sigma (y_i - \bar{y})^2}$ or $1 - \dfrac{RSS}{TSS} = 1 - \dfrac{\Sigma \hat{U}_i^2}{\Sigma (y_i - \bar{y})^2}$

$= 1 - \dfrac{873.14}{882.97}$

$= 0.0111$ # the measurement of goodness of fit of the fitted regression line comparing to an estimator.

1.c if $X_i = 5$, $y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{U}_i \implies y_i?$

w/ normally distributed $\qquad \hat{y}_i = 22.964 - 0.1585\ (5) = 22.172$ #

1.d) $Var(U_i) = \hat{6}^2 \qquad \hat{6}^2 = \dfrac{\Sigma \hat{U}_i^2}{n-k} = \dfrac{\Sigma (y_i - \hat{y})^2}{n-k} = \dfrac{873.14}{30-2} = 31.184$ #

$Var(\hat{\beta}_1) = 6^2_{\hat{\beta}_1} = \dfrac{\Sigma X_i^2}{n \Sigma x_i^2} 6^2 = \dfrac{5564}{30 \cdot (1098.8)} \times 31.184 = 5.264$ #

$= \Sigma (x_i - \bar{x})^2$

$Var(\hat{\beta}_2) = \dfrac{6^2}{\Sigma x_i^2}$

$\Sigma (X_i - \bar{X})^2$

$= \dfrac{31.184}{1098.8} = 0.028$ #

1.e) the hypothesis coefficients are different from 0

β_2 at 0.05 level of significant

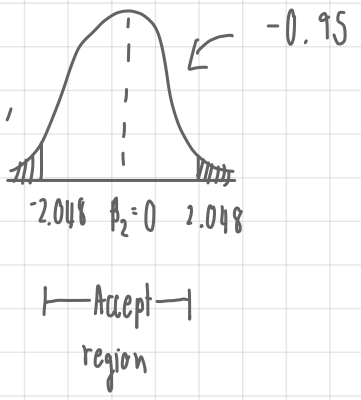
$H_0 : \beta_2 = 0$ – null hypothesis

$H_a : \beta_2 \neq 0$ – Alternative hypothesis

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma \hat{\beta}_2} = \frac{-0.159 - 0}{\sqrt{0.028}} = -0.95$$

$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}}$ lower bond $= -2.048$

$n-k = 28$ Upper bond $= 2.048$



we cannot reject the null hypothesis, at the significance level of 95%.

β_1 at 0.05 level of significant

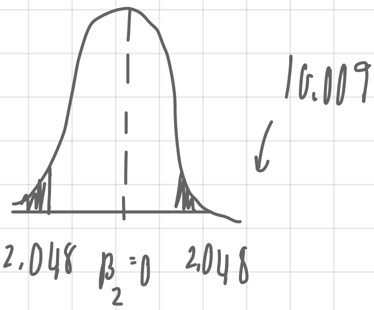
$H_0 : \beta_1 = 0$ – null hypothesis

$H_a : \beta_1 \neq 0$ – Alternative hypothesis

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma} \beta_1} = \frac{22.964 - 0}{\sqrt{5.264}} = 10.009 \#$$

$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}}$ lower bond $= -2.048$

Upper bond $= 2.048$



We can reject the null hypothesis, at the significant level of 95%. or

We are sure that β_1 is not 0 95 out 100

1.f less than zero

$\downarrow$

one tail test

$n = 30$ $k = 2$

$n - k = 28$

β_2

$H_0 : \beta_2 \leq 0$

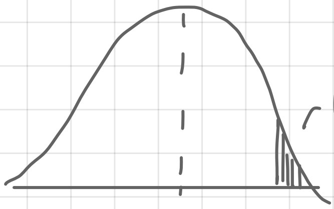
$H_1 : \beta_2 > 0$

$$t_{cal} = -0.950 = \frac{-0.159 - 0}{\sqrt{0.028}}$$

Upper band $\beta_2 + t_{\alpha} \cdot \hat{\sigma}_{\hat{\beta}_2} = 0 + (2.763)\sqrt{(0.028)}$

0.01

$= 0.462$



0.462

$\beta_2 = 0$

-0.95

Accept

we cannot reject the null hypothesis, at the significance level of 99% #

β_1

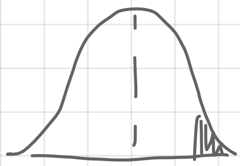
$H_0 : \beta_1 \leq 0$

H_1 $\beta_2 > 0$

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\beta_1}} = \frac{22.964 - 0}{\sqrt{5.264}} = 10.009 \#$$

Upper band $\beta_1 + t_{\alpha} \cdot \hat{\sigma}_{\hat{\beta}_1} = 0 + (2.763)(\sqrt{5.264})$

$= 6.339 \#$



10.009

6.339

$\beta_2 = 0$

Accept

we can reject the null hypothes at the significant level of 99% #

2.

2.a) Yes, because it shows that the more of x, the less of y of q

2.b) averagly price ? when car is = 5

(1) $Var(\hat{y}_i) = \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma (x_i - \bar{x})^2} \right]$

$n = 11$

$n - k = 9$

$= 212{,}877 \left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73} \right]$

$\hat{y}_0 = 7836 - 502.4 (5)$

$= 5324$ #

$= 35{,}582.534$ #

(2) find $\sigma_{\hat{y}_0} = 188.633$ #

(3) find the 95 % CI for $E(y | X_0 = 5)$

Upper $= \hat{y}_0 + (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{y}_0}) = 5324 + (2.262 \cdot 188.633) = 5750.688$ #

lower $= \hat{y}_0 - (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{y}_0}) = 5324 - (2.262 \cdot 188.633) = 4{,}897.312$ #

2.c) multiply all x w/ 10

$\hat{y}_i = 78{,}360 - 5024 X_i$

se = (520) (4118)

x rises by 10 years, $\hat{y}$ also increases by 5024 USD

2.d) $\frac{dy}{dx} \cdot \frac{X}{y}$

$-502.4 \cdot \frac{(10)}{2812} = -1.787$ #