Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_iY_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5,564$	$\sum_{i=1}^{n} X_i Y_i = 7,524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\beta_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{-174.20}{1098.8} \approx -0.1585 \ \#$$

$$\beta_1 = \bar{Y} - \beta_2 \bar{X}$$

$$= 21.03 - (-0.1585(12.20))$$

$$= 22.9637 \ \#$$

$\therefore$ $\beta_1 = 22.9637$: It mean that when X_i equal to 0, y_i will be 22.9637.

$\beta_2 = -0.1585$: when X_i increase 1 unit, Y_i will increase -0.1585.

b) Find r^2 and explain its meaning.

$$r^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma (Y_i - \bar{Y})^2}$$

$$= 1 - \frac{873.14}{882.97}$$

$$= 0.0111 \#$$

$\therefore$ 0.0111 of the variation in y is explained by the variation in x, the remaining 0.9889 is unexplained (residual).

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = 22.9637 + (-0.1585 (5))$$

$$= 22.1712$$

$\therefore$ when $X_i = 5$, $\hat{Y}_i$ will be 22.1712.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$$\text{var}(u_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-K}$$

$$= \frac{873.14}{30-2}$$

$$= 31.1835 \#$$

$$\Sigma x_i^2 = \Sigma (X_i - \bar{X})^2 = 1098.8$$

$$\text{var}(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2 = \frac{5564 \cdot 31.1835}{30 \cdot 1098.8} = 5.2634 \#$$

$$\text{var}(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2} = \frac{\sigma^2}{\Sigma x_i^2} = \frac{31.1835}{1098.8} = 0.02840 \#$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

$H_0 : \beta_2 = 0$: null hypothesis

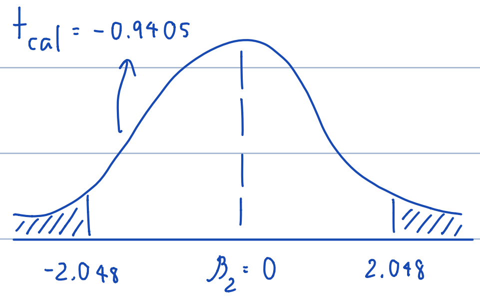
$H_1 : \beta_2 \neq 0$: Alternative hypothesis

$\alpha = 0.05$, $d.f = 28$

$$t = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$$

$$= \frac{-0.1585 - 0}{\sqrt{0.02840}}$$

$$= -0.9405$$



the lower bound : $t_{\frac{\alpha}{2}} = -2.048$

the upper bound : $t_{\frac{\alpha}{2}} = 2.048$

$\therefore$ t_{cal} lies within acceptance region, we cannot reject H_0,
we cannot say for sure that β_2 is not 0 95 out of 100 times.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

$H_0 : \beta_2 \leq 0$ – null hypothesis

$H_1 : \beta_2 > 0$ – Alternative hypothesis

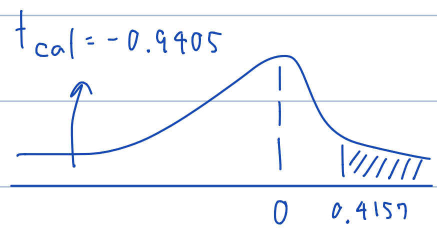
$\alpha = 0.01$, $d.f = 28$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{-0.1585 - 0}{\sqrt{0.02840}}$$

$$= -0.9405$$

the upper bound : $\beta_2 + t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2} = 0 + (2.467 \cdot \sqrt{0.02840})$

$$= 0.4157$$



$\therefore t_{cal}$ lies within acceptance region,

we cannot reject H_0,

we cannot say for sure that

β_2 is more than 0 99 out of 100 times.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$N = 11$

$\bar{X} = 7.45$,

d.f $= 11 - 2$
$= 9$

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

Yes, the regression function is negative slope when X (car aged) is increased, Y (market price of car) will be decreased.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

when his car is 5 years old, $\hat{Y}_0 = 7{,}836 - 502.4(5) = 5324.$ #

market price range : $\sqrt{var(\hat{Y}_0)} = \sqrt{\sigma^2\left[\frac{1}{n} + \frac{(X_0-\bar{X})^2}{\sum(x-\bar{X})^2}\right]} = \sqrt{212877\cdot\left[\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right]}$

$\hat{\sigma}_{\hat{Y}_0} = 188.6333$

95% of CI : $P(\hat{Y}_0 - t_{\frac{\alpha}{2}}\cdot\sigma_{\hat{Y}_0} \leq Y_0 \leq \hat{Y}_0 + t_{\frac{\alpha}{2}}\cdot\sigma_{\hat{Y}_0}) = 0.95$

$t_{\frac{\alpha}{2}} = 2.262$; $P(4897.3114 \leq Y_0 \leq 5750.6885) = 0.95$

$\therefore$ Market price at $\hat{Y}_0 = 5324$ will be within the specific range.

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

SRF when x(10) : $\hat{y} = 7836 - 5024(X)$

$(52) \quad (4118)$

d) Calculate the elasticity of market price when a car is 10 years old.

$X_i = 10$

$y_i = 2812$

Slope $= -502.4$; elasticity $= \frac{dy}{dx} \times \frac{X}{Y} = -502.4 \cdot \frac{10}{2812} = -1.7866$ #