Final Exam Study Sheet

Chapter Fifteen: The Term Structure of Interest Rates

I. In chapter 14 we assumed that the same constant interest rate is used to discount cash flows of any maturity. In reality, rates with different maturities are usually different. The relationship between time to maturity and YTM can vary dramatically from one period to another, though, usually, the longer-term securities have higher yields. In this chapter, we study the **term structure of interest rates,** the structure of interest rates for discounting cash flows of different maturities.

II. Term Structure Under Certainty

1. **Short interest rate**: the interest rate for a given time interval (usually one year). For a period of time, say, 3 years, the short interest rates for 1st year, 2nd year and 3rd year are usually different. On the contrary, the yield for a 3-year bond is the single interest rate that equates the present value of the bond's payments to the bond's price.

Year	1-year Short Interest rate	Time to maturity	Price of zeros	YTM of Zeros
0 (today)	8%	1	925.93	8%
1	10%	2	841.75	8.995%
2	11%	3	758.33	9.66%
3	11%	4	683.18	9.993%

2. **Yield curve:** a graph of YTM as a function of time to maturity.
   a. Yield curve can be upward-sloping, flat or downward sloping.
   b. The YTM on zeros is sometimes called the spot rate that prevails today for a period corresponding to the maturity of the zero. The yield curve (last column of the above table) presents the spot rates for four maturities. The spot rates, or YTM, do not equal the 1-year interest rates (2nd column of the above table) for each year.
   c. By definition, the price of a two-year zero is:

$$P_2 = \frac{\$1000}{(1.08)(1.10)} = \$841.75 = \frac{\$1000}{(1+y_2)^2}$$

- Doing the math: $(1+y_2)^2 = (1.08)(1.10)$ or $(1+y_2) = [(1.08)(1.10)]^{1/2}$
- More generally: $1+y_n = [(1+r_1)(1+r_2)...(1+r_n)]^{1/n}$
- YTM for long bond comes from **geometric mean** of gross returns to short bonds.

3. Pricing of coupon bonds:
   a. For coupon bonds, we simply discount each payment by the spot rate (YTM) corresponding to the time until that payment. To illustrate, consider a 3-year bond with par value $1000, paying an annual coupon rate of 8%. Using the YTM from the above table, the price of this bond is therefore:

$$P = \frac{80}{1.08} + \frac{80}{(1.08995)^2} + \frac{1080}{(1.09660)^3} = \$960.41$$

- In other words, we can treat each of the bond's payments as in effect a stand-alone zero-coupon security that can be valued independently. The total value of the bond is just the sum of the values of each of its cash flows.

- Notice that the YTM of this 3-year coupon bond is 9.58% (get it by yourself), is a bit less than that of a 3-year zero-coupon bond. This makes sense: if we think of **the coupon bond as a "portfolio" of three zeros** (corresponding to each of its three coupon payments), then its yield should be a weighted average of the three spot rates for years 1-3. Of course, the yield on the last payment will dominate, since it accounts for the overwhelming proportion of the value of the bond. But as a general rule, **yields to maturity can differ for bonds of the same maturity if their coupon rates differ**.

b. Bond traders often distinguish between the yield curve for zero-coupon bonds and that for coupon-paying bonds. The **pure yield curve** refers to the relationship between yield to maturity and time to maturity for zeros. The **coupon-paying curve** portrays this relationship for coupon bonds.

c. The most recently issued Treasuries are said to be **on the run**. On-the-run Treasuries have the greatest liquidity, so traders have keen interest in the on-the-run curve.

d. **Holding period returns**: In **a world with no uncertainty** all bonds must offer identical rates of return over any holding period. Otherwise, at least one bond would be dominated by the others in the sense that it would offer a lower rate of return than would combinations of other bonds; no one would be willing to hold the bond, and its price would fall. In fact, despite their different yields to maturity, each bond will provide a return over the coming year equal to this year's short interest rate.

4. Forward rates:

a. Compare two investment choices:

- buy-and-hold: invest \$100 in 3-year zero

Period 1	Period 2	Period 3

\$100 → $\$100(1+y_3)^3$

- roll-over: invest \$100 in 2-year zero. After 2 years, reinvest proceeds in 1-year zero

Period 1	Period 2	Period 3

\$100 → $\$100(1+y_2)^2$ → $\$100(1+y_2)^2(1+f_3)$

For some break-even value of f_3, roll-over strategy has same payoff as buy-and-hold.

b. Forward rate "f_3": break-even rate such that roll-over and buy-and-hold strategies have same payoff:

- f_3 solves: payoff to buy-and-hold = payoff to roll-over

- $\$100(1+y_3)^3 = \$100(1+y_2)^2(1+f_3)$

$$\rightarrow 1 + f_3 = \frac{(1+y_3)^3}{(1+y_2)^2}$$

Generally, $1+f_n = \dfrac{(1+y_n)^n}{(1+y_{n-1})^{n-1}}$

$$(1+y_n)^n = (1+f_n)(1+y_{n-1})^{n-1}$$

III. <u>Interest rate uncertainty and forward rates</u>

1. Different rates:

a. The ***n-period*** **<u>spot rate</u>** is the YTM on a zero-coupon bond with a maturity of n periods.

b. The **<u>short rate</u>** of period n is the ***one-period*** interest rate that will prevail in period n.

c. The **<u>forward rat</u>**e for period n is the short rate that would satisfy a "break-even condition" equating the total return on two n-period investment strategies.

d. Spot rates and forward rates are ***observabl***e today, but because interest rates evolve with uncertainty, ***future short rates are not***. In the special case in which there is no uncertainty in future interest rates, the forward rate calculated from the yield curve would equal the short rate that will prevail in that period.

2. short-term investors vs. long-term investors

Under uncertainty, we don't know short rates for sure. However, we can expect what short rates will be – expected short rate ($E(r_t)$) of period t.

Let's compare two different types of investors:

a. Short-term investors: want to make one-year investment. He can either buy one-year zero or buy two-year zero and sell at the end of the first year. If he buys one year zero, he can lock in a riskless return for one year. If he buys two-year zero, since he doesn't know the real interest rate at the end of the first year, therefore, the return of buying and holding two-year zero for one year is uncertain. If the real yield rises (drops) at the end of first year, the bond price (the price he can sell at the end of the first year) will decrease (increase). Thus his real return is less (more) than that of one-year zero. Since investors usually are risk-averse, they would not hold the 2-year bond unless it offered an expected rate of return greater than the riskless return available on the competing 1-year bond.

For short-term investors: $(1+r_1)(1+E(r_2)) < (1+E(r_{1B}))(1+E(r_2)) =$ → $(1+y_2)^2 = (1+r_1)(1+f_2) \rightarrow E(r_2) < f_2$

$E(r_2) < f_2$

b. Long-term investors: want to make two-year investment. He can either buy one 2-year zero or buy 1-year zero first and then reinvest the money in 1-year zero available at the end of the first year (roll-over). If he buys a 2-year zero, he can lock in a riskless return for two year. If he rolls over, since he doesn't know the real interest rate at the end of the first year, therefore, the return of buying and holding 1-year zero at the end of the first is uncertain. If the real yield rises (drops) at the end of first year, the payoff from reinvestment (the price he can sell at the end of the second year) will decrease (increase). Thus his real return is less (more) than that of 2-year zero. Since long-term investors usually are risk-averse, they would not roll over two 1-year bonds unless they offered an expected rate of total return greater than the riskless return available on the competing 2-year bond.

For long-term investors: $(1+r_1)(1+E(r_2)) > (1+y_2)^2 = (1+r_1)(1+f_2)$ →

$E(r_2) > f_2$

3. an example (where r1 is the real short rate for period 1, r2 is the real short rate for period 2):

Case 1: r1=8%, r2=10%					
	price at year 1	price at year 2	year 1 return	year 2 return	2-year YTM
2-year zero	108	118.8	8.00%	10%	**9.00%**
1-year zero	108	118.8	**8%**	10%	9.00%
Case 2: r1=8%, r2=11%					
	price at year 1	price at year 2	year 1 return	year 2 return	
2-year zero	107.027	118.8	7.03%	11%	**9.00%**
1-year zero	108	119.88	**8%**	11%	9.49%
Case 3: r1=8%, r2=9%					
	price at year 1	price at year 2	year 1 return	year 2 return	
2-year zero	108.9908	118.8	8.99%	9%	**9.00%**
1-year zero	108	117.72	**8%**	9%	8.50%

IV. <u>Theories of the term structure</u>

1. **<u>Liquidity preference:</u>** from the above discussion, short-term investors will be unwilling to hold long-term bonds unless the forward rate exceeds the expected short interest rate, $E(r_2) < f_2$, whereas long-term investors will be unwilling to hold short bonds unless $E(r_2) > f_2$. In other words, both groups of investors require a premium to induce them to hold bonds with maturities different from their investment horizons. Advocates of the **<u>liquidity preference theory</u>** of the term structure believe that short-term investors dominate the market so that, generally speaking, the forward rate exceeds the expected short rate. The f_2 over $E(r_2)$, the **<u>liquidity premium</u>**, is predicted to be positive. A liquidity premium can cause the yield curve to slope upward even if no increase in short rates is anticipated.
2. Another story – **<u>the Expectations Hypothesis:</u>** a common version of this hypothesis states that the forward rate equals the market consensus expectation of the future short interest rate; in other words $f_2 = E(r_2)$ and liquidity premiums are zero. Because $f_2 = E(r_2)$ we can use the forward rates derived from the yield curve to infer market expectations of future short rates. The YTM would thus be determined solely by current and expected future one-period interest rates. An upward-sloping yield curve would be clear evidence that investors anticipate increases in interest rates.

V. <u>Interpreting the term structure</u>

$$(1+y_n)^n = (1+f_n)(1+y_{n-1})^{n-1}$$

1. Mathematically, if the yield curve is rising, f_n must exceed y_{n-1}. In words, the yield curve is upward sloping at any maturity date, n-1, for which the forward rate for the coming period is greater than the yield at that maturity. But **what can account for that higher forward rate?**

2. there are two possible answers: since $f_n = E(r_n)$ + liquidity premium, either investors **expect rising interest rates**, meaning that $E(r_n)$ is high, **or** they **require a large premium** for holding longer-term bonds. Although it is tempting to infer from a rising yield curve that investors believe that interest rates will eventually increase, this is not a valid reasoning. (Please refer to figure 15.5 on page 500)
3. Although it is true that expectations of increases in future interest rates can result in a rising yield curve, the converse is not true: **A rising yield curve does not in and of itself imply expectations of higher future interest rates. This is the heart of difficulty in drawing conclusions from the yield curve**. The effects of possible liquidity premiums confound any simple attempt to extract expectations from the term structure. But estimating the market's expectations is a crucial task, because only by comparing your own expectations to those reflected in market prices can you determine whether you are relatively bullish or bearish on interest rates.
4. One rough approach to deriving expected future spot rates is to assume that liquidity premiums are constant. However, this approach has little to recommend it for two reasons. First, it is almost impossible to obtain precise estimates of a liquidity premium. Second, there is no reason to believe that the liquidity premium should be constant.
5. Still, **very steep yield curves** are interpreted by many market professionals as warning signs of impending rate increase. In fact, **the yield curve is a good predictor of the business cycle** as a whole, since long-term rates tend to rise in anticipation of an expansion in the economy. When the curve is steep, there is a far lower probability of a recession in the next year than when it is inverted or falling. For this reason, the yield curve is a component of the index of leading economic indicators.
6. Irregularity – why might interest rates fall? There are two factors to consider: the real rate and the inflation premium. Recall the Fisher equation:

1 + Nominal rate = (1 + Real rate)(1 + Inflation rate),

Or approximately, Nominal rate ≈ Real rate + inflation rate.

Therefore, an expected change in interest rates can be due to changes in either expected real rates or expected inflation rates.

7. **Term premium:** the spread between yields on long- and short-term bonds. It is generally are positive.

Chapter Twenty: Option Markets: Introduction

I. Derivatives are securities, whose prices are determined by, or "derive from", the prices of other securities. These assets are also called ***contingent claims*** because their payoffs are contingent on the prices of other securities. .

The asset can be a financial asset or a commodity. It may even be an index representing a broad asset class.

- **Users of derivative securities:**

o *Hedgers:* Market participants who want to eliminate the risk of holding a position in a security.

o *Speculators:* Market participants who want to speculate on market / asset price movements.

o *Arbitragers:* Market participants who want to take advantage of any mispricings in the markets.

In addition, investors may use derivatives to increase or decrease risk exposure of their portfolios. The liquidity of some derivative contracts offer investors an attractive alternative for quickly gaining or reducing exposure to the market as a whole even if they are not modifying their overall risk.

II. Derivative markets:

1. Over the Counter

Most forward contracts and customized option contracts are traded "over the counter".

2. Exchanges

Futures and standardized options contracts are traded on organized exchanges. Trading can be done in different formats:

III. The Option contract:

1. A **call option** on a security gives its holder the right, but **not** the obligation, to purchase the security for a stated amount, K, the **exercise price** or **strike price** on or before some specified expiration date.

2. Since the call option gives the holder the right to buy the security for K, if the price of the security at the time of exercise, S, is greater than the strike price, K, the holder will exercise the option, i.e., buy the security for K and sell it for S. The difference, then, S-K, is the value of the option. Since it is a right and not an obligation, if $S < K$, holders will not exercise their right of purchase and the option will expire without being exercised. Thus, the formula for the payoff, or value, on a call option at maturity is:

$C(S; K) = Max[S-K, 0]$

3. The purchase price of the option is called the **premium**.
4. The **net profit** on the call is the value of the option minus the price originally paid to purchase it.
5. A **put option** on a security gives its holder the right, but **not** the obligation, to sell the security for a stated amount, K, the **exercise price** or **strike price** on or before some specified expiration date.
6. If the value of the security at maturity, S, falls short of the exercise prices, then the holder will exercise the put, purchasing the security for its market price of S and selling it to the writer, i.e., the seller of the put for the exercise price of K. Sometimes,

this is called putting it to the writer. This results in a payoff of K – S. If on the other hand, K is below the security price, S, the holder has no incentive to exercise his/her right to sell it for K and the option will expire. Thus, the payoff of a put option at maturity is:

$P(S; K) = Max[K - S, 0]$

7. While the holder of an option has the right to exercise it, the seller or writer of the option has **an obligation.** Thus, if the holder of a call (put) wishes to exercise, the writer is obligated to deliver (buy) the security for the exercise price. In practice, most options are **settled in cash,** which means that rather than buying or selling the security, the cash value of the option at maturity, say absolute value of (S – K), is paid by the writer to the holder.
8. An option is said to be **in the money** if it is in the interest of the holder to exercise it. Thus, a call (put) is in the money if S> (<) K. Similarly, options are **out of the money** if the holder does not have a payoff gain, i.e., if a call (put) has S< (>) K. When the price is at or near the exercise price, K, the option is said to be trading **at the money.**
9. Option trading:

a. Some options **trade on over-the-counter markets**. The OTC market offers the advantage that the terms of the option contract – the exercise price, maturity date, and number of shares committed – can be tailored to the needs of the traders. The costs of establishing an OTC option contract, however, are higher than for exchange-traded options.

b. Options contracts **traded on exchanges** are standardized by allowable maturity dates and exercise prices for each listed option. Standardization of the terms of listed option contracts means all market participants trade in a limited and uniform set of securities. This increases the depth of trading in an option, which lowers trading costs and results in a more competitive markets.

10. **American options** allow exercise on or before the exercise date. **European options** allow exercise only on the expiration date. Most traded options in the US are American in nature.

IV. Values of options at expiration

1. Payoff to call holder (value at expiration) = Max $[S_T – K, 0]$. The profit to the option holder is the value of the option at expiration minus the original purchase price.
2. Payoff to call writer = $Min[K-S_T, 0]$. The profit to the writer is the payoff plus the original sell price.
3. Payoff to put holder (value at expiration) = Max $[K - S_T, 0]$. The profit to the option holder is the value of the option at expiration minus the original purchase price.
4. Payoff to put writer = Min $[S_T - K, 0]$. The profit to the writer is the payoff plus the original sell price.
5. Options can be used either to lever up an investor's exposure to an asset price or to provide insurance against volatility of asset prices.

V. Option strategies

1. Protective put strategy: buy a stock and a put on the stock.
   a. The total payoff to this portfolio = $max[K - S_T, 0] + S_T = max[K, S_T]$
   b. The total profit to this portfolio = payoff – $(S_0 + P)$
   c. Notice that protective put offers some insurance against stock price declines in that it limits losses. Therefore, protective put strategies provide a form of

portfolio insurance. The cost of the protection is that, in the case of stock price increases, the profit is reduced by the cost of the put, which turned out to be unneeded.

2. Covered all: long a share of stock with a simultaneous short a call on that stock. The call is “covered” because the potential obligation to deliver the stock is covered by the stock held in the portfolio.
   a. The total payoff to this portfolio = S_T - max[S_T - K , 0] = min[K, S_T]
   b. The total profit to this portfolio = payoff – S_0 + C, where C is the premium of the call.

3. Straddle: A long straddle is established by buying both a call and a put on a stock, each with the same exercise price, K, and the same expiration date, T. Straddles are useful strategies for investors who believe a stock will move a lot in price but are uncertain about the direction of the move.
   a. The total payoff to this portfolio = max[S_T - K , 0] + max[K - S_T, 0] = absolute value of (S_T – K).
   b. The total profit to this portfolio = payoff – C - P
4. Spreads: A spread is a combination of two or more call options (or two or more puts) on the same stock with differing exercise prices or times to maturity. A money spread involves the purchase of one option and the simultaneous sale of another with a different exercise price. A time spread refers to the sale and purchase of options with differing expiration dates. E.g. long a call at K1 and short a call at K2 (K2 > K1), this strategy called bullish spread.

VI. Put-call parity relationship

1. Put-call parity theorem for European options: A riskless bond that pays K for sure plus a call with an exercise price of K on an asset is equivalent to or replicated by a put with an exercise price of K and a holding of the underlying asset:

$$\frac{X}{(1+r_f)^T} \bullet (1+r_f)^T + C(S_0, X) = S_0 + P(S_0, X) \quad \frac{X}{(1+r_f)^T} + C(S_0, X) = S_0 + P(S_0, X)$$

Where S_0 is the current price of the asset and r_f is the riskless rate.

2. Proof:

At time T, payoffs the LHS and RHS:

LHS		RHS	
Riskless Bond	= $\frac{K}{(1+r_f)^T} \bullet (1+r_f)^T =$ K	Underlying security	= S_T
Call	= Max[S_T - K , 0]	Put	=Max[K - S_T, 0]
If S_T > K	Bond + call = S_T	If S_T > K	Security + put = S_T
If S_T < K	Bond + call = K	If S_T < K	Security + put = K

Since LHS and RHS always provide equal payoffs at time T, then they must cost the same amount to establish. Therefore, the call-plus-bond portfolio must cost the same as the stock-plus-put portfolio.
Proved

3. For **European** call options on dividend-paying stocks, the put-call parity is:

$C + PV(K) + PV(dividends) = P + S_0$.

VII. Optionlike securities

1. **Callable bonds**: many corporate bonds are issued with call provisions entitling the issuer to buy bonds back at some time in the future at a specified call price. A callable bond arrangement is essentially a sale of a **straight bond** to the investor and the concurrent issuance of a **call option** by the investor to the bond-issuing firm.
2. **Convertible bonds:** they give investors the right to exchange each bond or share of preferred stock for a fixed number of shares of common stock, regardless of the market prices of the securities at the time.
   a. A bond's **conversion value** equals the value it would have if you converted it into stock immediately. Clearly, a bond must sell for at least its conversion value.
   b. The **straight bond value**, or **"bond floor"**, is the value the bond would have if it were not convertible into stock. The bond must sell for more than its straight bond value because a convertible bond has more value; it is in fact a straight bond plus a valuable call option. Therefore, the convertible bond has two lower bounds on its market price: the conversion value and the straight bond value.
   c. When stock prices are low, the straight bond value is the effective lower bound, and the conversion option is nearly irrelevant. The convertible will trade like straight debt. When stock prices are high, the bond's price is determined by its conversion value. With conversion all but guaranteed, the bond is essentially equity in disguise.
3. Warrants: they are essentially call options issued by a firm.
   a. One difference between calls and warrants is that exercise of a warrant requires the firm to issue a new share of stock – the total number of shares outstanding increases. Also unlike options, warrants result in a cash flow to the firm when the warrant holder pays the exercise price.
   b. Like convertible debt, warrant terms may be tailored to meet the needs of the firm. Also like convertible debt, warrants generally are protected against stock splits and dividends.

Chapter Twenty-one: Option Valuation

Determinants of option values

a. Stock price and strike price: A call option should increase in value with the stock price and decrease in value with the exercise price because the payoff to a call, if exercised, equals $S_T - K$.

b. Call option values also increase with the volatility of the underlying stock price. Because extremely poor outcomes cannot worsen the payoff below zero, while extremely good stock outcomes can improve the option payoff with limit. This asymmetry means that volatility in the underlying stock price increases the expected payoff to the option, thereby enhancing its value.

c. Longer time to expiration increases the value of a call option. For more distant expiration dates, there is more time for unpredictable future events to affect prices, and the range of likely stock prices increases. This has an effect similar to that of increased volatility. Moreover, as time to expiration lengthens, the PV of the exercise price falls, thereby benefiting the call option holder and increasing the option value. As a corollary to this issue:

d. Call option values are higher when interest rate rise (holding the stock price constant) because higher interest rates also reduce the PV of the exercise price.

e. Dividend payout will decrease call option's value.

Binomial option pricing

Binomial model is a numerical approach to valuation of options on stocks. In this approach the stock price evolution over time is modeled with a binomial tree or lattice.

1. One-step binomial model (two-stage)

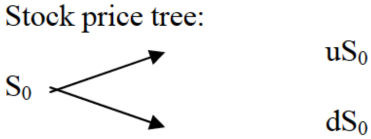
Let us consider the following scenario:

Stock price at $t = 0$: S_0

Stock price at $t = T$ in "up" state: uS_0 $(u>1)$

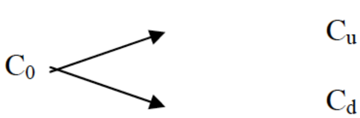
Stock price at $t = T$ in "down" state: dS_0 $(d<1)$

Stock price tree:



Now, let us consider a call option on the stock. The payoff at $t = T$ is $C_u = \max[uS_0 - K, 0]$ in the up state, or $C_d = \max[dS_0 - K, 0]$ in the down state.

Call price tree:



Now consider a portfolio of a long position in Δ shares and a short position in one call option such that it's value remains the same both in the up and down state.

Then the value of the portfolio at $t = 1$ is:

$\Delta * uS_0 - C_u = \Delta * dS_0 - C_d$

→ $\Delta = (C_u - C_d)/(uS_0 - dS_0)$ -------------------------------------- (1)

The value of portfolio in up and down states is same, which means that this is a risk free investment. The cost of setting up the portfolio at t = 0 is then the present value of the portfolio value at t =T.

$\Delta S_0 - C_0 = (\Delta * uS_0 - C_u)/(1+r_f)^T$

If using continuously compounding, $\Delta S_0 - C_0 = e^{-r_f T} (\Delta * uS_0 - C_u)$

Since $\Delta = (C_u - C_d)/(uS_0 - dS_0)$, we can get:

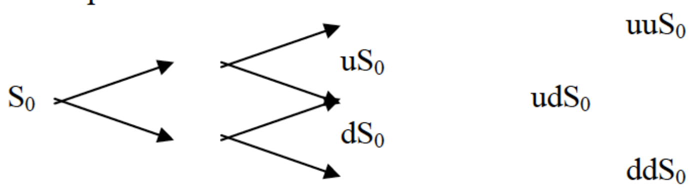
$$C_0 = \frac{1}{(1+r_f)^T}(pC_u + (1-p)C_d), \text{ ------------------------------------ (2)}$$

Where, $p = \dfrac{(1+r_f)^T - d}{u-d}$

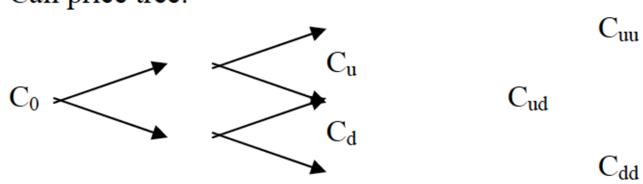
2. Hedge ratio: Δ, is the ratio of the swings in the possible end-of-period values of the option and the stock (or, the first derivative of option price relative to the stock price).
3. Two step model:

Assume the each time step is dT

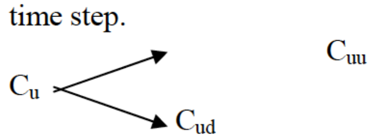
Stock price tree:



Call price tree:



Now consider the "up" and "down" states following the "up" state at the end of first time step.



Using the approach we described for one step binomial tree,

$$C_u = \frac{1}{(1+r_f)^{dT}}(pC_{uu} + (1-p)C_{ud})$$

$$C_d = \frac{1}{(1+r_f)^{dT}}(pC_{ud} + (1-p)C_{dd})$$

$$C_0 = \frac{1}{(1+r_f)^{dT}}(pC_u + (1-p)C_d), \text{ where } p = \frac{(1+r_f)^{dT} - d}{u-d}$$

Generally speaking, the value of call at node I is:

$$C_i = \frac{1}{(1+r_f)^{dT}}(pC(i+1)_u + (1-p)C(i+1)_d)$$

Black Scholes Option Evaluation

In the binomial model, if we let the period-length get smaller and smaller, we obtain the Black-Merton-Scholes option pricing formula:

$C_0 = S_0N(d_1) – K e^{-rT}N(d_2)$

$P_0 = K e^{-rT}N(-d_2) - S_0N(-d_1)$

Where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

And where

C0 = current cal option value

S0 = Current stock price.

N(.) is the normal cumulative density function.

K = exercise price.

R = risk-free interest rate (the annualized continuously compounded rate on a safe asset with the same maturity as the expiration of the option, which is to be distinguished from rf, the discrete period interest rate).

T = time to maturity of option, in years.

Ln = Natural logarithm function.

σ = Standard deviation of the annualized continuously compounded rate of return of the stock.

An interpretation of the Black-Scholes formula:

- The call is equivalent to a levered long position in the stock.
- The replicating strategy:
  - $S_0N(d_1)$ is the amount invested in the stock.
  - $K e^{-rT}N(d_2)$ is the dollar amount borrowed
  - The option delta is $N(d_1)$

The above Black-Scholes formula applies to stocks that do not pay dividends. When dividends are to be paid before the option expires, we need to adjust the formula.

1. Black calls for adjusting the stock price downward by the present value of any dividends that are to be paid before option expiration. Therefore, we would simply replace S_0 with S_0 – PV(dividends) in the formula (be careful! You should replace S_0 with S_0 – PV(dividends) in computing d_1 and d_2 too)

Using the Black Scholes Formula

Hedge ratio: is the change in the price of an option for a $1 increase in the stock price. The hedge ratio is commonly called the option's delta. Remember, if we form a portfolio by long delta shares of stock and short one call, the portfolio is risk-free (delta = 0, or **"delta neutral"**). This basic idea – delta hedging, can be used in risk management.

Chapter Twenty-two: Futures Markets

I. A **forward contract** is a commitment to purchase at a future date (delivery or maturity date) a given amount of a commodity or an asset at a price (futures price) agreed on today.

1. Terminology and Contract Specifications

- Buyer (Long) – the party agreeing to buy the underlying instrument or commodity.
- Seller (Short) – the party agreeing to sell the underlying instrument or commodity.
- Spot Price: The current price of the underlying asset.
- Delivery price: The price at which the underlying asset is exchanged between the buyer and the seller.
- Delivery date: The date at which the delivery takes place.
- Delivery location: Place where the delivery takes place.

In addition, the amount and quality of underlying asset to be delivered may also be specified.

2. Features of forward contracts:

- Custom tailored
- Traded over the counter (not on exchanges)
- No money changes hand until maturity
- Non-trivial counter-party risk.

II. Futures contract:

1. Forward contracts have two limitations:

a. illiquidity.
b. Counter-party risk.

Futures contracts are designed to address these two limitations.

2. **Definition:** A futures contract is an exchange-traded, standardized, forward-like contract that is marked to market daily.

3. Features of futures contracts:

a. Standardized contracts:
(1) underlying commodity or asset
(2) quantity
(3) maturity
b. Exchange traded
c. Guaranteed by the clearing house – no counter-party risk.
d. Gains/losses settled daily
e. Margin account required as collateral to cover losses.

4. Terminology and Contract Specifications

- Buyer (Long) – the party agreeing to buy the underlying instrument or commodity.
- Seller (Short) – the party agreeing to sell the underlying instrument or commodity.
- Open Interest: The total number of contracts outstanding at any time (long and short positions are not counted separately, meaning that open interest can be defined as the number of either long or short contracts outstanding).
- Volume: The number of contracts traded in a given trading session or a day.
- Spot price: The current price of the underlying asset.

- Delivery price: The price at which the underlying asset is exchanged between the buyer and the seller.
- Delivery date: The date at which the delivery takes place.
- Delivery location: Place where the delivery takes place.
- Expiry date: The date at which the contract expires. The delivery date could be different from expiry date because a contract can have a time period during which the delivery can take place.

In addition, the amount and quality of underlying asset to be delivered are also specified. The exchanges also specify minimum price change (tick size) and price limits.

5. Clearing house

- A clearinghouse facilitates trades in futures contracts by acting as counterparty to every futures contract.
- It acts as a seller to every buyer and buyer to every seller.
- It facilitates the trading of futures contracts by making it easy for buyers and sellers to open or close their positions.
- It eliminates counter party risk.

It requires all positions to recognize profits or losses as they accrue daily.

6. Initial margin

When a contract is established, the buyer/seller is required to deposit an amount in a **margin account**. The amount that must be deposited at the time the contract is entered into is known as **initial margin**. It could be in the form of cash or treasury bills. The initial margin is usually set between 5% and 15% of the total value of the contract. Contracts written on assets with more volatile prices require higher margins.

7. Maintenance and variation margin

- A futures contract is marked to market every day. If the margin falls below a level established by an exchange (maintenance margin), then the investor gets a margin call and is expected to top up the margin account to the initial margin (not maintenance margin) level the next day. This call is required to be fulfilled with cash.

Maintenance margin is somewhat lower than the initial margin.

- The variation margin is the additional money deposited by the investor.
- The purpose of various margins is to avoid default by the buyer/seller of a contract.

8. Settlement

- *Physical settlement*

Buyer takes delivery of underlying instrument or commodity from seller.

- *Cash settlement*

For contracts where physical delivery is not possible or not convenient, cash settlement is used. In this case the buyer (long) pays or receive cash from the seller (short) as the case may be.

9. **Futures Contracts vs. Forward Contracts**
   - Futures contracts traded on organized exchanges. Forward contracts are agreements between parties.
   - Futures contracts have standardized terms except for flex contracts. Forward contracts are customized contracts.
   - Futures contracts marked to market on a daily basis. Forward contracts are not marked to market.
   - Futures contracts can be closed at any time by offsetting transactions. This convenience does not exist for forward contracts.
   - Most futures contracts are closed (offset) before contract expiration date. Almost all forward contracts are closed after the delivery of the underlying at expiration date.
   - Unlike forward contracts futures contracts do not have counter party risk.

10. Futures price on the delivery date will equal the spot price of the commodity on that date. As a maturing contract calls for immediate delivery, the futures price on that day must equal the spot price – the cost of the commodity form the two competing sources is equalized in a competitive market (no arbitrage). Therefore, the futures price and the spot price must converge at maturity. This is called the **convergence property.**

Table: Operation of margins for a long position in two gold futures contracts. Each contract size is 100 ounces. The initial margin is $2000 per contract, or $4000 in total, and the maintenance margin is $1500 per contract, or $3000 in total. The contract is entered into on June 13, 2005 and closed out on June 17, 2005. the numbers in the second column except the first and the last, represent the futures prices at the close of trading.

Day	Futures price per ounce ($)	Daily gain (loss) ($)	Cumulative gain (loss) ($)	Margin account balance ($)	Margin call ($)
	400			4000	
June 13	397	(600)	(600)	3400	
June 14	393	(800)	(1400)	2600	1400
June 15	392	(200)	(1600)	3800	
June 16	393.5	300	(1300)	4100	
June 17	396	500	(800)	4600	

Forwards and Futures Prices

- Spot-Futures Parity Theorem, with and without dividends
- Parity for Spreads
- Arbitrage opportunities
- Hedging with futures
- Futures Prices vs Expected Spot Prices: Expectations Hypothesis, Normal Backwardation, Contango, Modern Portfolio Theory