Probability weighting

Risk Preferences, part 6

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Recall: Prospect theory

- A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.
- Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

- A person evaluates a prospect $(x, p; y, q)$ according to the functional

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

Probability p vs. The weighting of probability $\pi(p)$

- The decision weights that people assign to outcomes might not be identical to the probabilities of these outcomes.

- The weighting is an operation of system 1.

Probability vs. The weighting of probability

- We have different sensitivity to change in probability depending on the level of probability being considered.

- Psychological effect:
  - possibility effect
  - certainty effect

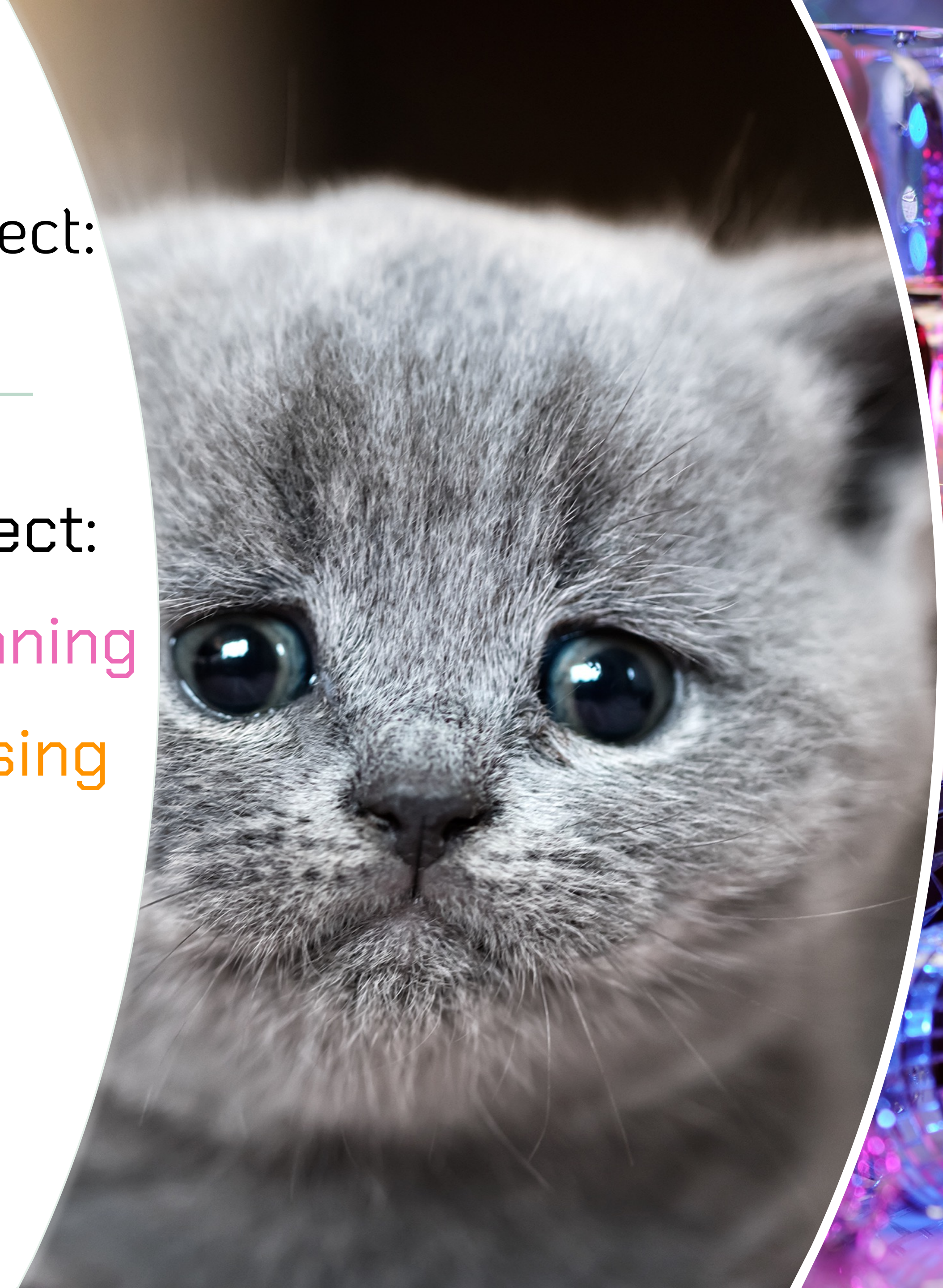
The possibility effect: 0% → 1%

- The move from a 0% chance to a 1% possibility of winning a prize or losing something transform the situation.
- It creates a possibility that did not exist earlier.
- The influence we give to the move from 0% probability to 1% illustrates the possibility effect.

The possibility effect:
0% → 1%

The weights reflect:

- the hope of winning
- the worry of losing



The possibility effect: 0% → 1%

- The possibility effect causes highly unlikely outcomes to be weighted disproportionately more than they “deserve” if we evaluate the change in probability objectively.

- Overweighting of small probabilities increases the attractiveness of both lottery ticket and insurance policies.

The certainty effect: 99% → 100%

- Outcomes that are almost certain are given less weight than their probabilities justifies.

Example

Prob(%) of winning a gamble	0	1	2	5	10	20	50	80	90	95	98	99	100
Decision weight	0	5.5	8.1	13.2	18.6	26.1	42.1	60.1	71.2	79.3	87.1	91.2	100

- The decision weights are identical to the corresponding probabilities at the extremes: the impossible $\pi(0) = 0$ and the sure thing $\pi(1) = 1$.
- The decision weights *depart sharply* from probabilities near these points.
- Inadequate sensitivity to intermediate probabilities.

Example from the work of Kahneman and Tversky

Prob(%) of winning a gamble	0	1	2	5	10	20	50	80	90	95	98	99	100
Decision weight	0	5.5	8.1	13.2	18.6	26.1	42.1	60.1	71.2	79.3	87.1	91.2	100

- At 2%, the rare event is overweighted by a factor of 4.
- At 98%, a 2% risk of not winning the prize reduces the weight by 13%, from 100 to 87.

Features of Probability Weighting $\pi(p)$

- $\pi(p)$ is an increasing function of p, $\pi(0) = 0$, $\pi(1) = 1$
- Overweighting of small probabilities $\pi(p) > p$
- Underweighting of large probabilities $\pi(p) < p$
- Subcertainty $\pi(p) + \pi(1-p) < 1$

Inverse S-Shaped Probability Weighting Functions

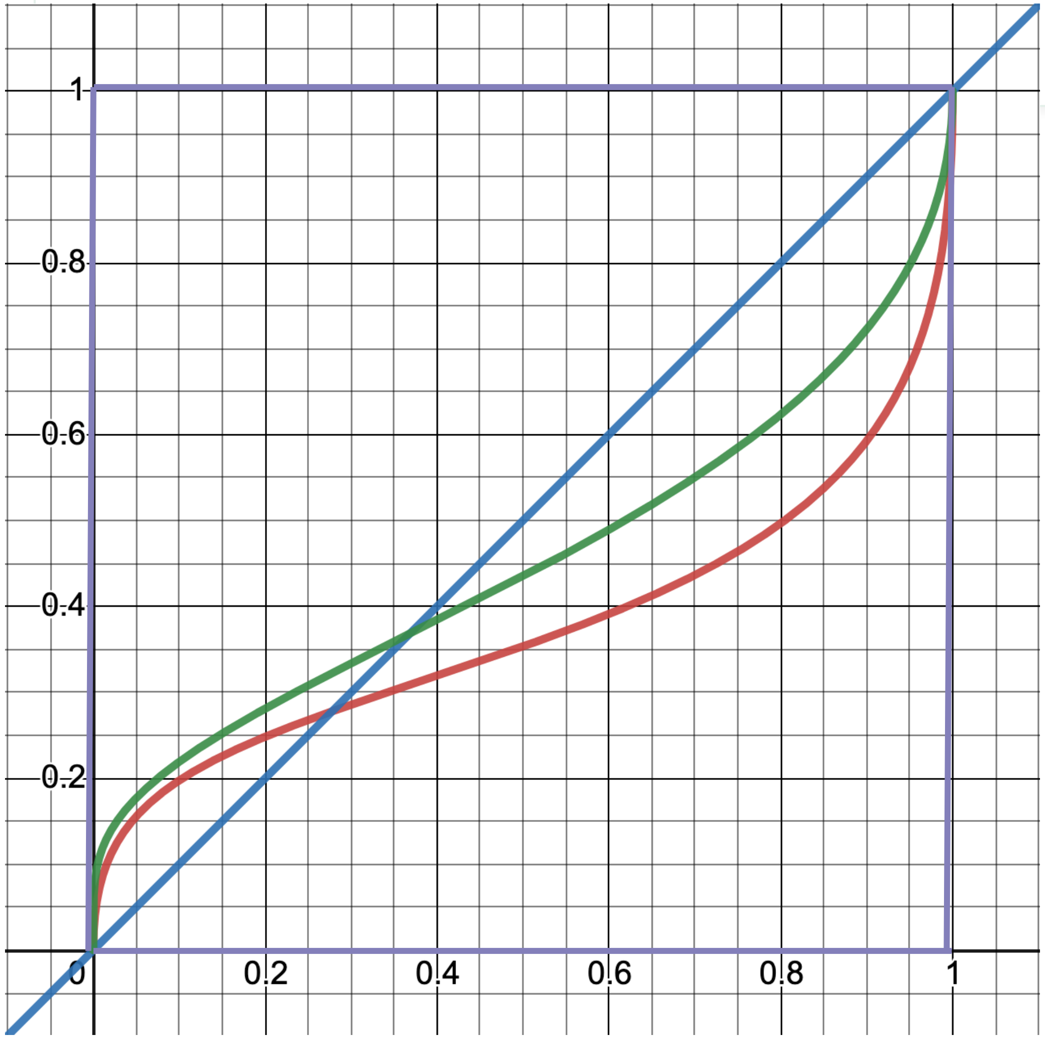
- Tversky & Kahneman (JRU 1992) suggest

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \qquad \textit{for some } \gamma \in (0.279,1)$$

- Prelec (ECTA 1998) suggests

$$\pi(p) = \exp(-(-lnp)^\alpha) \qquad \textit{for some } \alpha \in (0,1)$$

Inverse S-Shaped Probability Weighting Functions



$$\pi(p) = \exp(-(-ln p)^{0.5})$$

$$\pi(p) = \frac{p^{0.5}}{(p^{0.5} + (1-p)^{0.5})^{1/0.5}}$$

https://www.desmos.com/

A Simple Functional Form

- $\pi(p) = \begin{cases} 0 & if\ p = 0 \\ \alpha + \beta p & if\ p \in (0,1) \\ 1 & if\ p = 1 \end{cases}$
- If $0 < \alpha < 1$ and $\beta < 1 - 2\alpha$, then this functional form generates overweighting of small probabilities, underweighting of large probabilities, subcertainty, and a discontinuity at the endpoints.

Differential Weighting for Gains vs. Losses

- Tversky & Kahneman (JRU 1992) propose differential probability weighting for gains and losses.
- Weights on losses and weights on gains derived from two separate probability-weighting functions π^- and π^+ .

Example

Let

$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$ and $\gamma = 0.65$

$$v(x) = \begin{cases} x^{0.88} & if\ x > 0 \\ -2.25(-x)^{0.88} & if\ x < 0 \end{cases}$$

Example

Consider the lottery scenario choosing between prospects: (\$5,000, 0.001) and (\$5, 1.0)

$V(5000, 0.001) = \pi(0.001)v(5000)$	$V(5,1) = \pi(1)v(5)$
$V(5000, 0.001) = \frac{0.001^{0.65}}{(0.001^{0.65}+(1-0.001)^{0.65})^{1/0.65}} \times (5000)^{0.88}$	$V(5,1) = 1 \times 5^{0.88} = 4.12$
$V(5000, 0.001) = 0.011 \times 1799 = 19.8$	

Example

- The fact that a typical decision-maker likes to buy a lottery ticket is driven by the fact that the decision weight on $p = 0.001$ is about 10 times higher than the objective probability.

Economic Applications

Barberis (Management Science 2011): Rank-dependent probability weighting can generate a preference for casino gambling despite negative expected returns because certain gambling strategies can generate an asset with positively skewed returns.

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A Model of Casino Gambling

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We show that prospect theory offers a rich theory of casino gambling, one that captures several features of actual gambling behavior. First, we demonstrate that for a wide range of preference parameter values, a prospect theory agent would be willing to gamble in a casino even if the casino offers only bets with no skewness and with zero or negative expected value. Second, we show that the probability weighting embedded in prospect theory leads to a plausible time inconsistency: at the moment he enters a casino, the agent plans to follow one particular gambling strategy; but after he starts playing, he wants to switch to a different strategy. The model therefore predicts heterogeneity in gambling behavior: how a gambler behaves depends on whether he is aware of the time inconsistency; and, if he is aware of it, on whether he can commit in advance to his initial plan of action.

Key words: gambling; prospect theory; time inconsistency; probability weighting
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Probability weighting vs. Probability misperception

- We cannot distinguish probability weighting from probability misperception.