Group # 3 members: 6304640342
6304641001
6304641100
6304641167

Group Homework 1

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 3rd 2022 (before midnight /B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

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1. Consider the market for good x. Suppose that the market demand equation is given by

$$P_x = 8 - bQ_x^d + cP_y; \quad b > 0 \text{ and } c > 0$$

and the equation for market supply is given by

$$P_x = 20 + dQ_x^S; \quad d > 0$$

where P_x is price of good X, P_y is price of good Y, Q_x^d is quantity demanded for good X and Q_x^S is quantity supplied for good X. Answer the following questions.

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

2. (an old midterm exam question) Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 10 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \qquad \text{and}$$

$$P = 20 + k_4 T + 2Q_2^s,$$

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are STRICTLY positive.

Use the information given to answer the following questions:

a. Is "product x" considered as a *substitute* product or a *complementary* product?

b. Derive the market demand equation.

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	2	2	3	1

Variables	Y	P_x	w	T
Value	5	10	10	5

e. Solve for the equilibrium price *and* quantity.

f. Suppose that the government provides a subsidy of \$5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

3. Consider a simple macroeconomics model given below

$C = 0.3Y_d - k_1 r;\qquad k_1 > 0.$

$I = 0.5Y - k_2 r;\qquad k_2 > 0.$

$Y_d = Y - T$

$G = G_0$

$T = T_0$

$M^d = k_3 Y - k_4 r;\qquad k_3 > 0\ \ and\ \ k_4 > 0.$

$M^s = M_0$

where $Y = GDP,\ C = consumption,\ \ I = investment,\ G = government\ purchase,\ T = tax,\ r = interest\ rate,\ M^d = money\ demand,\ and\ M^s = money\ supply$

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.
b. Solve for the equilibrium solution (Y^*, r^*)
c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

4. Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by \$t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

   a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.
   b. State the condition that links between consumer's and producer's price
   c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)
   d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?
   e. Find the expression of the revenue that the government can collect from the market under the equilibrium.
   f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

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Question 1:

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

Ans: Endogenous variables include Q^d, Q^s, and P_x

Exogenous variable includes P_y

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y

Ans: $P_x = 8 - Q^d_x + cP_y$; $b > 0$ / $c > 0 \Rightarrow M^d$ $\boxed{Q^d_x = \frac{8 + cP_y - P_x}{b}}$

$P_x = 20 + dQ^s_x$; $d > 0 \Rightarrow M^s$ $\boxed{Q^s_x = \frac{P_x - 20}{d}}$

Equilibrium ; $Q^d = Q^s$

$$\frac{P_x - 20}{d} = \frac{8 + cP_y - P_x}{b}$$

$$b(P_x - 20) = (8 + cP_y - P_x)d$$

$$bP_x - 20b = 8d + dcP_y - dP_x$$

$$bP_x + dP_x - 20b - 8d = dcP_y$$

$$\frac{(dP_x - 8d) + (bP_x - 20b)}{dc} = P_y$$

$$\frac{\cancel{d}(P_x - 8) + b(P_x - 20)}{\cancel{d}c} = P_y$$

$$\frac{(P_x - 8) + b(P_x - 20)}{c} = P_y \rightarrow c > 0 \ \#$$

$\therefore$ For good x to exist in the market, c must not be equal or lower than zero #

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

Ans: Equilibrium ; $Q^d = Q^s$

$$\frac{P_x - 20}{d} = \frac{8 + cP_y - P_x}{b}$$

$$bP_x - 20b = 8d + dcP_y - dP_x$$

$$bP_x + dP_x = 8d + dcP_y + 20b$$

$$P_x(b + d) = d(8 + cP_y) + 20b$$

$$P^*_x = \frac{d(8+cP_Y)+20b}{b+d}$$ #

$P^*_x = \frac{d(8+cP_Y)+20b}{b+d}$ sub in demand equation :

$$Q^d_x = \frac{\left(\frac{d(8+cP_Y)+20b}{b+d}\right)-20}{d}$$

$$= \frac{8d+dcP_Y+20b-20(b+d)}{b+d} \cdot \frac{1}{d}$$

$$= \frac{\cancel{8d}+dcP_Y\cancel{+20b}-\cancel{20b}\overset{-12d}{-\cancel{20d}}}{b+d} \cdot \frac{1}{d}$$

$$= \frac{\cancel{d}(cP_Y-12)}{b+d} \cdot \frac{1}{\cancel{d}}$$

$$Q^*_x = \frac{cP_Y-12}{b+d}$$ #

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

Ans : $\frac{\Delta Q^*}{\Delta P_Y} = \frac{c}{b+d}$

If Price of good increases by \$1 , equilibrium quantity increases by $\frac{c}{b+d}$ unit(s). #

Question 2: a. Is "product x" considered as a *substitute* product or a *complementary* product?

Ans: $P = 10 + k_1 P_x + k_2 Y - Q^d_j$

$Q^d_j = 10 + k_1 P_x + k_2 Y - P$

Market demand $= 10\,(Q_i)$; $\sum_{j=1}^{10} Q^d_j = 10\,(10 + k_1 P_x + k_2 Y - P)$

$= 100 + 10 k_1 P_x + 10 k_2 Y - 10P$

producer 1: $P = 5 + k_3 W + Q^s_1$

$Q^s_1 = -5 - k_3 W + P$

$Q^d_j = 10 + k_1 P_x + k_2 Y - P$

producer 2: $P = 20 + k_4 T + 2Q^s_2$

$Q^s_2 = \frac{-20 - k_4 T + P}{2}$

$Q^d_j = 10 + k_1 P_x + k_2 Y - P$

$\frac{\Delta Q^d}{\Delta P_x} = k_1$ When price of good x rises by \$1, quantity demand of good x rises by k_1 units.

$\therefore$ since k_1 is positive, the product x is substitute. #

b. Derive the market demand equation.

Ans: $Q^d_j = 10 + k_1 P_x + k_2 Y - P$

Market Demand: $Q^d = 10\,(Q_i)$

$= 10\,(10 + k_1 P_x + k_2 Y - P)$

$= 100 + 10 k_1 P_x + 10 k_2 Y - 10P$ #

$$Q^d = \begin{cases} 10\,(10 + k_1 P_x + k_2 Y - P), & P < 10 + k_1 P_x + k_2 Y \\ 0, & P \geq 10 + k_1 P_x + k_2 Y \end{cases}$$ #

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

Ans: $Q^s_1 = P - (5 + k_3 W)$; $Q^s_1 > 0$; $P > (5 + k_3 W)$

$Q^s_1 = 0$; $P \leq (5 + k_3 W)$

$Q^s_2 = P - (20 + k_4 T)\frac{1}{2}$; $Q^s_2 > 0$; $P > (20 + k_4 T)$

$Q^s_2 = 0$; $P \leq (20 + k_4 T)$

$$Q_S = \begin{cases} 0+0 & , 0 \leq P \leq (20+K_4T) \\ P-(20+K_4T)\frac{1}{2} & , (20+K_4T) < P \leq (5+k_3W) \\ Q^S_1 + Q^S_2 & , P > (5+k_3W) \end{cases}$$

\#

Firstly, producer 2 has lower reservation cost than producer 1. Thus, producer 2 has higher cost advantages among two producers.

d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Ans:

$$Q^d = \begin{cases} 10(10+k_1P_x+k_2Y-P) & , P < 10+k_1P_x+k_2Y \\ 0 & , P \geq 10+k_1P_x+k_2Y \end{cases}$$

$$Q^S = \begin{cases} 0+0 & , 0 \leq P \leq (20+K_4T) \\ P-(20+K_4T)\times\frac{1}{2} & , (20+K_4T) < P \leq (5+k_3W) \\ Q^S_1 + Q^S_2 & , P > (5+k_3W) \end{cases}$$

∴ For the market to cease including only single firm active in the business, the price has to be more than $(20+K_4T)$ or less than and equal to $(5+k_3W)$ \#

e. Solve for the equilibrium price *and* quantity.

Ans:

$$Q^d = \begin{cases} 10(10+k_1P_x+k_2Y-P) & , P < 10+k_1P_x+k_2Y \\ 0 & , P \geq 10+k_1P_x+k_2Y \end{cases} \Rightarrow Q^d = \begin{cases} 400-10P & , P < 40 \\ 0 & , P \geq 40 \end{cases}$$

$$Q^S = \begin{cases} 0+0 & , 0 \leq P \leq (20+K_4T) \\ P-(20+K_4T)\times\frac{1}{2} & , (20+K_4T) < P \leq (5+k_3W) \\ Q^S_1 + Q^S_2 & , P > (5+k_3W) \end{cases} \Rightarrow Q^S = \begin{cases} 0+0 & , 0 \leq P \leq 25 \\ \frac{P-25}{2} & , 25 < P \leq 35 \\ \frac{3P-95}{2} & , P > 35 \end{cases}$$

Consumers would accept at most = \$40
(Reservation price)

Producers would accept at least = \$35
(Reservation price)

Therefore, there is market equilibrium,

#1 Case with only one producer: (producer 2)

$$400 - 10P = \frac{P - 25}{2}$$

$$800 - 20P = P - 25$$

$$825 = 21P$$

$$P^* = 39.29 \ \#$$

Case 1 has no equilibrium since price (P^*) is 39.29 which is more than 35. #

#2 Case with two producers: (market equilibrium)

$$400 - 10P = \frac{3P - 95}{2}$$

$$800 - 20P = 3P - 95$$

$$895 = 23P$$

$$P^* = \frac{895}{23}$$

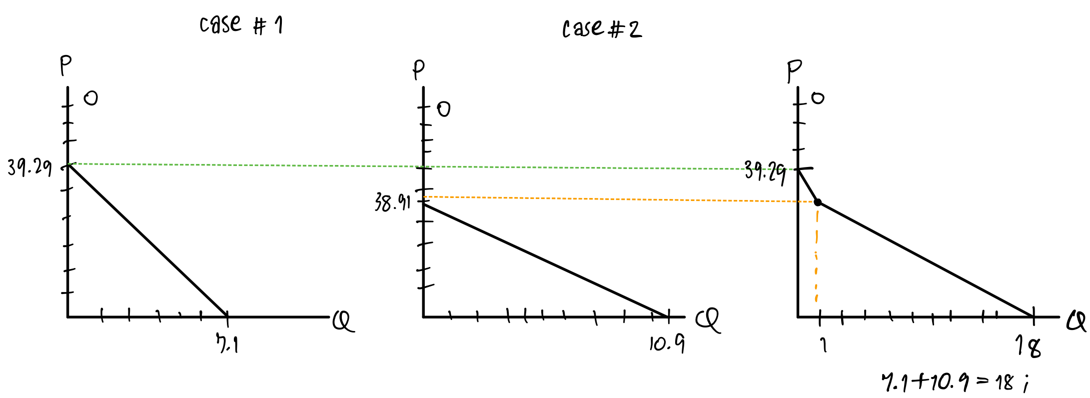
$$P^* = 38.91 \ \#$$

Then, sub in Q^d equation,

$$Q^* = 400 - 10P$$

$$Q^* = 400 - 10(38.91)$$

$$Q^* = 10.9 \text{ units} \ \#$$



∴ As price decreases, demand curve flattened out.

f. Suppose that the government provides a subsidy of $5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

Ans: Subsidy ; $P^s - 5 = P^d$

Q^d market $= 400 - 10P^d \Rightarrow P^d = \frac{400 - Q}{10}$

Q^s market $= \frac{3P - 95}{2} \Rightarrow P^s = \frac{2Q + 95}{3}$

$P^s - 5 = P^d$

$\frac{2Q + 95}{3} - 5 = P^d$

$\frac{2Q + 95 - 15}{3} = P^d$

$\frac{2Q + 80}{3} = P^d$

Equate the two equations,

Eq$^{\underline{n}}$ $Q^d = Q^s$; $\frac{400 - Q}{10} = \frac{2Q + 80}{3}$

$1{,}200 - 3Q = 20Q + 800$

$1{,}200 - 800 = 23Q$

$17.39 = Q^*$

sub into P^d equation,

$P^{d'} = \frac{400 - 17.39}{10} = 38.26$

$P^{s'} = P^{d'} + 5$

$= 38.26 + 5$

$= 43.26$

After subsidy : $Q^* = 17.39$ units

$P^{d'} = 38.26$

$P^{s'} = 43.26$

Subsidy = (\$5 subsidy per unit) × (unit output sold)

= 5 × 17.39

= 86.95

	Before subsidy		After subsidy
Q	10.9	$\xrightarrow{+6.5}$	17.39
P^d	38.91	$\xrightarrow{-0.65}$	38.26
P^s	38.91	$\xrightarrow{+4.4}$	43.26

Summary

- Consumers receive : $(38.91 - 38.26) \times 17.39 = 11.30$
- Producers receive : $(43.26 - 38.26) \times 17.39 = 75.65$

$\oplus = 86.95$

$$Q_1^S = P - 5 - k_3 W = 43.26 - 35 = 8.26$$

$$Q_2^S = \frac{P - 20 - k_4 T}{2} = \frac{43.26 - 25}{2} = 9.13$$

- Producer 1 receives : $(43.26 - 38.91) \times 8.26 = 35.93$
- Producer 2 receives : $(43.28 - 38.91) \times 9.13 = 39.72$

benefit received (%)

consumers $= 11.30 / 86.95 \times 100 \approx 13\%$ #

producer 1 $= 35.93 / 86.95 \times 100 \approx 41.32\%$ #

producer 2 $= 39.72 / 86.95 \times 100 \approx 45.68\%$ #

100%

Question 3:

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.

Ans:

Y & r

$$IS: Y = C + I + G$$
$$Y = 0.3Y_d - k_1 r + 0.5Y - k_2 r + G_0$$
$$Y = 0.3(Y - T) - k_1 r + 0.5Y - k_2 r + G_0$$
$$Y = 0.3Y - 0.3T - k_1 r + 0.5Y - k_2 r + G_0$$
$$0.2Y = -0.3T - k_1 r - k_2 r + G_0$$
$$Y = (-0.3T + G_0 - k_1 r - k_2 r) \times \frac{1}{0.2} \; 5$$
$$Y = -1.5T + 5G_0 - 5r(k_1 + k_2) \; \#$$

b. Solve for the equilibrium solution (Y^*, r^*)

Ans:

r

$$\frac{M^S}{P} = L^d$$
$$M^S = M^d$$
$$M_0 = k_3 Y - k_4 r$$
$$r^* = \frac{k_3 Y - M_0}{k_4} \; \#$$

Equilibrium :

$$Y = -1.5T + 5G_0 - 5 \cdot \underbrace{\left(\frac{k_3 Y - M_0}{k_4}\right)}_{r^*} \cdot (k_1 + k_2)$$
$$Y = -1.5T + 5G_0 - 5k_1\left(\frac{k_3 Y - M_0}{k_4}\right) - 5k_2\left(\frac{k_3 Y - M_0}{k_4}\right)$$
$$Y = -1.5T + 5G_0 - \frac{5k_1 k_3 Y + 5k_1 M_0}{k_4} - \frac{5k_2 k_3 Y + 5k_2 M_0}{k_4}$$
$$Y = -1.5T + 5G_0 - \frac{5k_1 k_3 Y}{k_4} + \frac{5k_1 M_0}{k_4} - \frac{5k_2 k_3 Y}{k_4} + \frac{5k_2 M_0}{k_4}$$
$$Y + \frac{5k_1 k_3 Y}{k_4} + \frac{5k_2 k_3 Y}{k_4} = -1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}$$
$$Y\left(1 + \frac{5k_1 k_3}{k_4} + \frac{5k_2 k_3}{k_4}\right) = -1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}$$

$$y^* = \frac{-1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}}{1 + \frac{5k_1k_3}{k_4} + \frac{5k_2k_3}{k_4}}$$

$$y^* = \frac{\frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4}}{\frac{k_4 + 5k_1k_3 + 5k_2k_3}{k_4}}$$

$$y^* = \frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{\cancel{k_4}} \times \frac{\cancel{k_4}}{k_4 + 5k_1k_3 + 5k_2k_3}$$

$$y^* = \frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4 + 5k_1k_3 + 5k_2k_3} \quad \#$$

c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

Ans:

$$y^* = \frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4 + 5k_1k_3 + 5k_2k_3}$$

$$\frac{\Delta y^*}{\Delta G_0} = \frac{5k_4}{k_4 + 5k_1k_3 + 5k_2k_3}$$

$\therefore$ No it is not more effective because if k_1 and k_2 is getting larger, y^* (or GDP) will fall. #

Question 4:

a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero. $\therefore t=0$

Ans:

$$\text{Demand} = \text{Supply}$$
$$14-3Q = 4+2Q$$
$$10 = 5Q$$

pre-tax equilibrium: $Q^* = 2$ # $\longrightarrow$ sub into demand equation;

$$14-3(Q) = 14-3(2)$$
$$P^* = 8 \ \#$$

b. State the condition that links between consumer's and producer's price

Ans:

Demand; $P = 14-3Q$

$$Q^d = \frac{14-P}{3}$$

$$Q^d = \begin{cases} 0 & , P \geq 14 \\ \frac{14-P}{3} & , 0 \leq P < 14 \end{cases} \ \#$$

Supply; $P = 4+2Q^s$

$$Q^s = \frac{P-4}{2}$$

$$Q^s = \begin{cases} 0 & , 0 \leq P < 4 \\ \frac{P-4}{2} & , P > 4 \end{cases} \ \#$$

$\therefore$ The reservation price for consumer is \$14, while the reservation price for producer is \$4. Thus, an equilibrium exists.

c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)

Ans: Tax on consumer; $P^d = P^s + t$

Equilibrium after tax; $4+2Q+t = 14-3Q$

$$5Q+t = 10$$
$$t = 10-5Q$$

and $Q^{*\text{aft tax}} = \frac{10-t}{5}$ # $\longrightarrow$ ①

Q^* after tax: sub ① into demand function before tax,

$$14 - \left(\frac{10-t}{5} \times 3\right) = P^{d*}$$

$$14 - \frac{30+3t}{5} = P^{d*}$$

$$P^{d*\text{ a/f tax}} = \frac{40+3t}{5} = 8+\frac{3}{5}t \#$$

sub (1) into supply function before tax to find P^{s*} after tax,

$$P^{s*\text{ a/f tax}} = 4+2\cdot\left(\frac{10-t}{5}\right)$$
$$= 4+\frac{20-2t}{5}$$
$$= 8-\frac{2}{5}t \#$$

d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?

Ans: Tax burden $= P^{d\text{ a/f tax}} + P^{s\text{ a/f tax}} = \left(8+\frac{3}{5}t\right)+\left(8-\frac{2}{5}t\right) = 16+\frac{t}{5}$

% producer $= \frac{2}{5}\times 100 = 40\%$ #

% consumer $= \frac{3}{5}\times 100 = 60\%$.

∴ Consumer is paying more for the tax under the equilibrium #

e. Find the expression of the revenue that the government can collect from the market under the equilibrium.

Ans: Tax = \$t per unit

Output sold after tax $= \frac{10-t}{5}$ units

Tax revenue = (\$t per unit) × (unit output sold)

$$= t\times\left(\frac{10-t}{5}\right)$$
$$= \$\,\frac{10t-t^2}{5} \#$$

f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

Ans:
$$\left.\begin{aligned} P^d &= a-bQ^d = 14-3Q^d \\ P^s &= c+dQ^s = 4+2Q^s \end{aligned}\right\}$$

t^* maximizes $T(t)$;

$$t^* = -\frac{\left(\frac{a-c}{b+d}\right)}{2\left(\frac{-1}{b+d}\right)}$$
$$= -\frac{\left(\frac{14-4}{-3+2}\right)}{2\left(\frac{-1}{3+2}\right)}$$
$$= \frac{10}{2} = 5 \#$$