1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

$$Y = C + I + G$$

$$Y = a + b(Y - T) + I_0 + G_0$$

$$Y = a + bY - bT + I_0 + G_0$$

$$(1 - b)Y = a - bT + I_0 + G_0$$

$$Y^* = \frac{1}{1 - b}(a - bT + I_0 + G_0)$$

- Numerically, if a = 1, $T_0 = \$0$ $I_0 = \$1$, $G_0 = \$1$ and b = 0.5, this yields us,

$$Y^* = \frac{1}{1 - 0.5}(1 - 0.5(0) + 1 + 1)$$

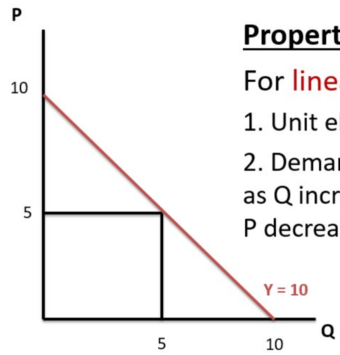
$$Y^* = 2 \times 3 = 6$$

$$C^* = 1 + 0.5(6 - 0)$$

$$C^* = 4$$

$$Y_d^* = 6 - 0 = 6$$

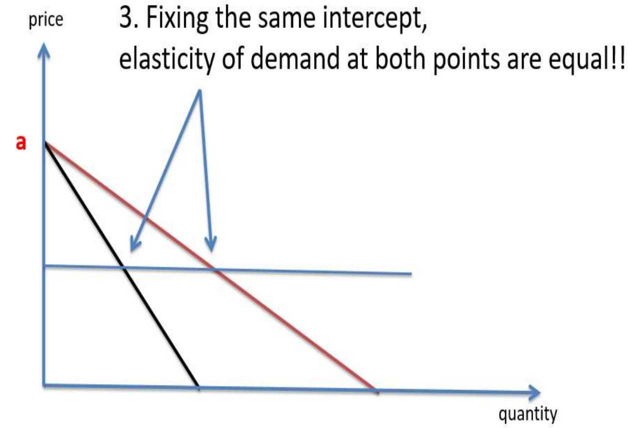
Linear function: slope v.s. elasticity



Property:

For linear demand curve:

1. Unit elasticity at the mid point.
2. Demand becomes more inelastic as Q increases, (correspondingly to P decreases)



Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

(i) elasticity of supply is always greater than 1 if $c > 0$,

(ii) elasticity of supply is always eqaul to 1 if $c = 0$,

(iii) elasticity of supply is always less to 1 if $c < 0$.

2.1 $PED = \frac{dQ}{dP} \times \frac{P}{Q}$

$= -\frac{1}{b} \times \left(\frac{a - bQ}{Q}\right)$

$= \frac{-a + bQ}{bQ}$

$= 1 - \frac{a}{bQ}$

2.2 $PES = \frac{1}{d} \times \left(\frac{c + dQ}{Q}\right)$

$= \frac{c + dQ}{dQ}$

$= 1 + \frac{c}{dQ}$

if $c > 0$, $PES = 1 + \frac{c}{dQ} > 1$

if $c = 0$, $PES = 1 + \frac{c}{dQ} = 1$

if $c > 0$, $PES = 1 + \frac{c}{dQ} < 1$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

revenue function

$TR(Q) = P(Q) \times Q$

$= (10 - Q) \times Q$

$= 10Q - Q^2$

slope $= \frac{dTR}{dQ} = 10 - 2Q$

max occurs when $\frac{dTR}{dQ} = 0$

$10 - 2Q = 0$

$Q = 5$

at $Q = 5$, TR is max $TR = 25$

- What is the break-even output?

$TR = TC$

$4Q = 10Q - Q^2$

$Q^2 - 6Q = 0$

$Q = 0, 6$

$\therefore Q = 6$

- What is the profit-maximizing level of output?

$\pi = TR - TC$

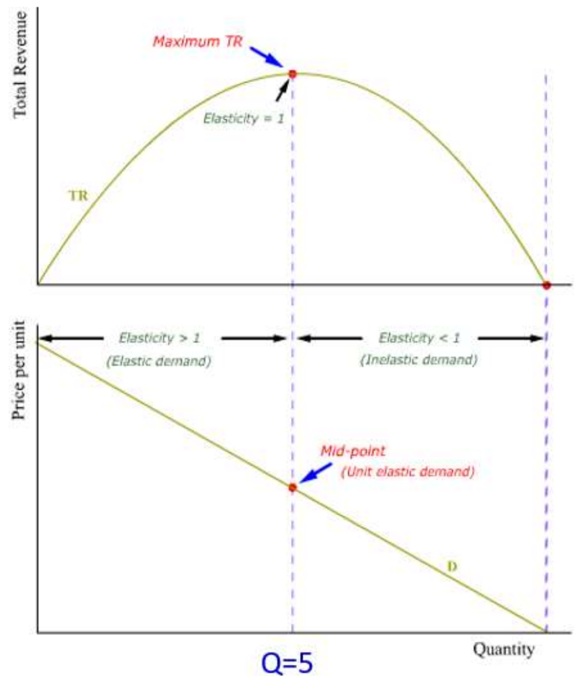
$= 10Q - Q^2 - 4Q$

$\frac{d\pi}{dQ} = 10 - 2Q - 4$

$6 - 2Q = 0$

$Q = 3$

Quadratic function: revenue function/break even analysis



$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \begin{array}{lll} >1 & ; & elastic \\ =1 & ; & unit\ elastic \\ <1 & ; & inelastic \end{array}$$

Revenue maximizing output: **unit** elasticity

Profit-maximizing output: under region with **elastic** demand

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Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?

a) $60 = 350t - 500t^2$

$500t^2 - 350t + 60 = 0$

$t = \dfrac{350 \pm \sqrt{350^2 - 4(500)(60)}}{1000}$

$t = 0.4, 0.3$

b) $61.25 = 350t - 500t^2$

$500t^2 = 350t + 61.25$

$t = 350 \pm \sqrt{350^2 - 4(500)(61.25)}$

$t = 0.35$

c) $\dfrac{dR}{dt} = 350 - 1000t$

$350 - 1000t = 0$

$t = 35\%$