EE441 Economics of Public Expenditure
Semester 1/2015

Fundamental Welfare Economics

Reading:

Rosen, Ch. 3 (Tools of Normative Analysis)

or

Stiglitz, Ch. 3 (Market Efficiency)

Market Efficiency

- Market Efficiency
  - Equilibrium in Competitive Market
  - Pareto Efficiency (Pareto Optimality)
  - The Fundamental Theorems of Welfare Economics
    - The First Theorem of Welfare Economics (FTWE)
    - The Second Theorem of Welfare Economics (STWE)
  - Analysing Economic Efficiency
- Efficiency and Equity

Efficiency

- **Efficiency requires avoiding waste of resources**
- **Resources are use efficiently if and only if it is impossible by using them differently to make a person better off w/o making at least another person worse off (i.e., Pareto efficiency).**
- **Productive efficiency: MC = MR**
- **Social Efficiency: MSC = MSB**

Allocative Efficiency

- **Production and consumption take place where MSB = MSC.**
  - **Demand curve reflects MSB.**
  - **Supply curve reflects MSC.**
  - **Using price as "*a signal*" to consumers and producers**

Pareto Efficiency

Vilfredo pareto (1848-1923)

Pareto Optimality: ***Resource allocation that have the property that no one can be made better off without someone being made worse off.***

- **Pareto Efficiency occurs *when* it is impossible to reallocate resources to make someone better off without hurting anybody else**

- **If it is possible to do so, then the reallocation of resources is *a Pareto improvement*.**

Conditions for Pareto Efficiency

- Efficiency in *Exchange*
- Efficiency in *Production*
- Efficiency in *Product Mix (i.e., Overall Efficiency)*

Exchange

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$.
- E.g. $\omega^A = (6,4)$ and $\omega^B = (2,2)$.
- The total quantities available are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1 and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.

Exchange

- Edgeworth and Bowley devised a diagram, called an **Edgeworth box**, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

Starting an Edgeworth Box

Feasible Allocations

- **What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?**
- **How can all of the feasible allocations be depicted by the Edgeworth box diagram?**
- **One feasible allocation is the before-trade allocation; i.e. the endowment allocation.**

Starting an Edgeworth Box

Width = $\omega_1^{A} + \omega_1^{B} = 6 + 2 = 8$

Starting an Edgeworth Box

Height =

$\omega_2^A + \omega_2^B$

$= 4 + 2$

$= 6$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B$ $= 4 + 2$ $= 6$

The dimensions of the box are the quantities available of the goods.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?

Feasible Allocations

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The Endowment Allocation

Height = $\omega_2^A + \omega_2^B$ $= 4 + 2$ $= 6$

The endowment allocation is $\omega^A = (6,4)$ and $\omega^B = (2,2)$.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation

Height =

$\omega_2^A + \omega_2^B$

$= 4 + 2$

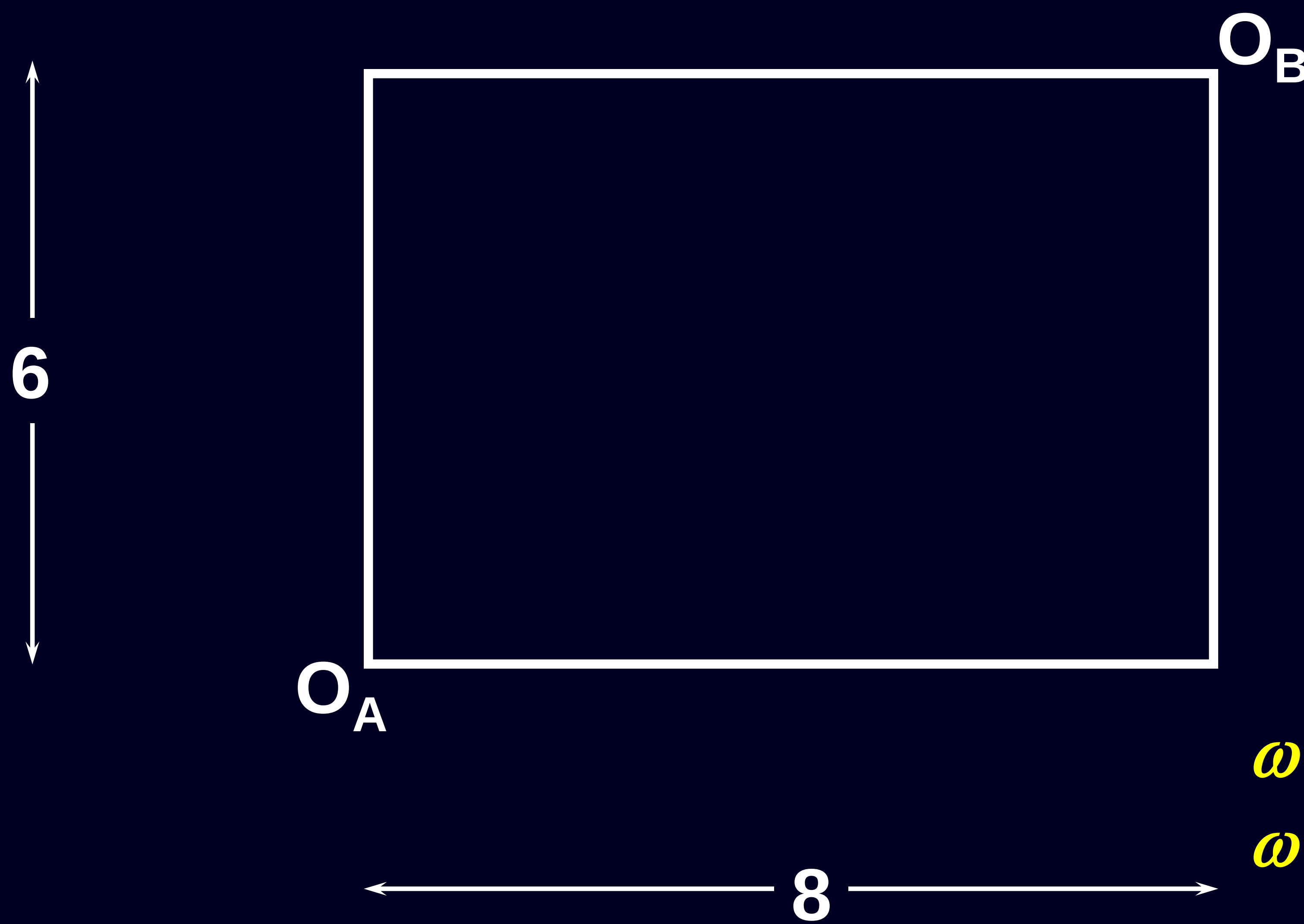
$= 6$

$\omega^A = (6,4)$

$\omega^B = (2,2)$

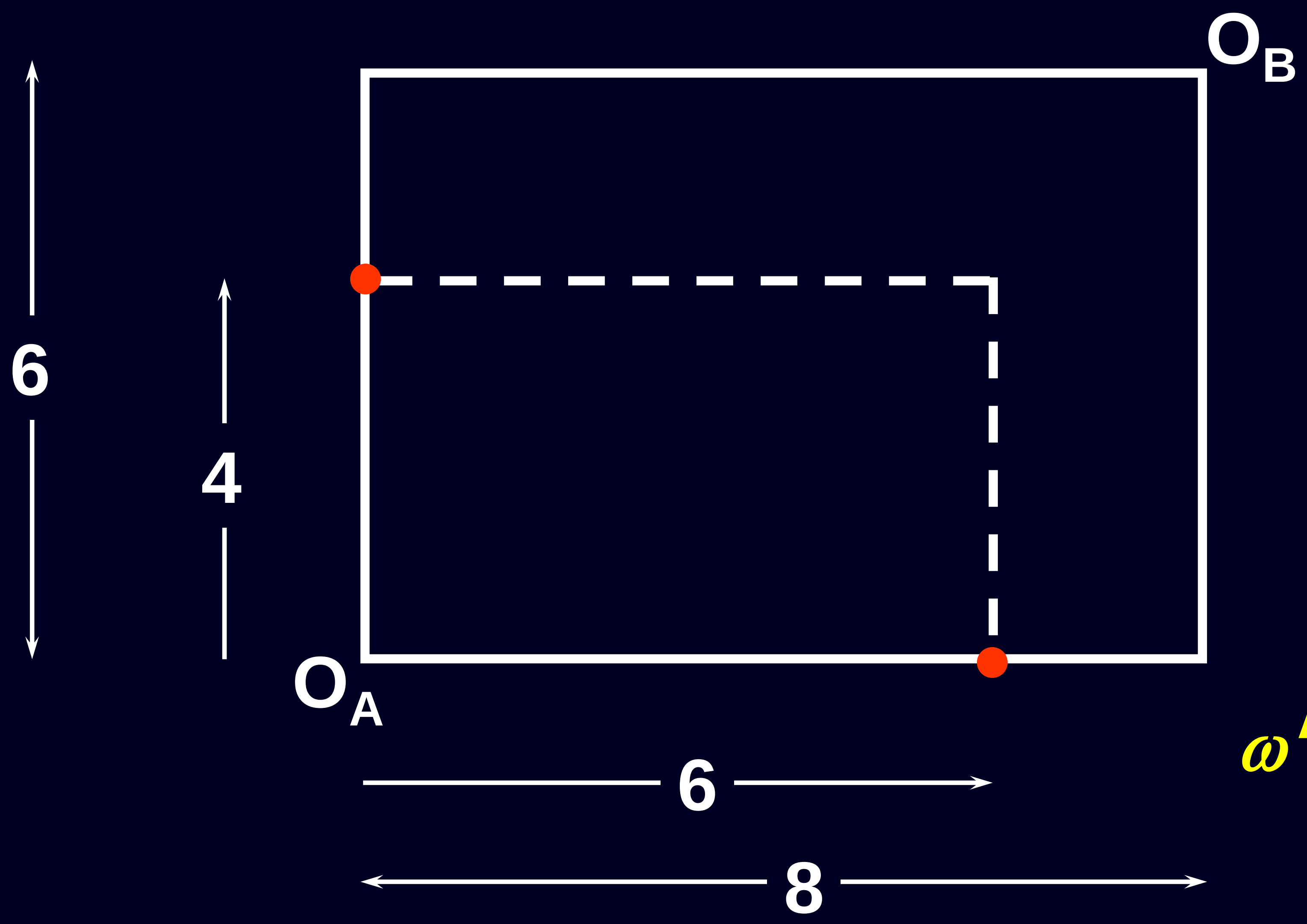
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation



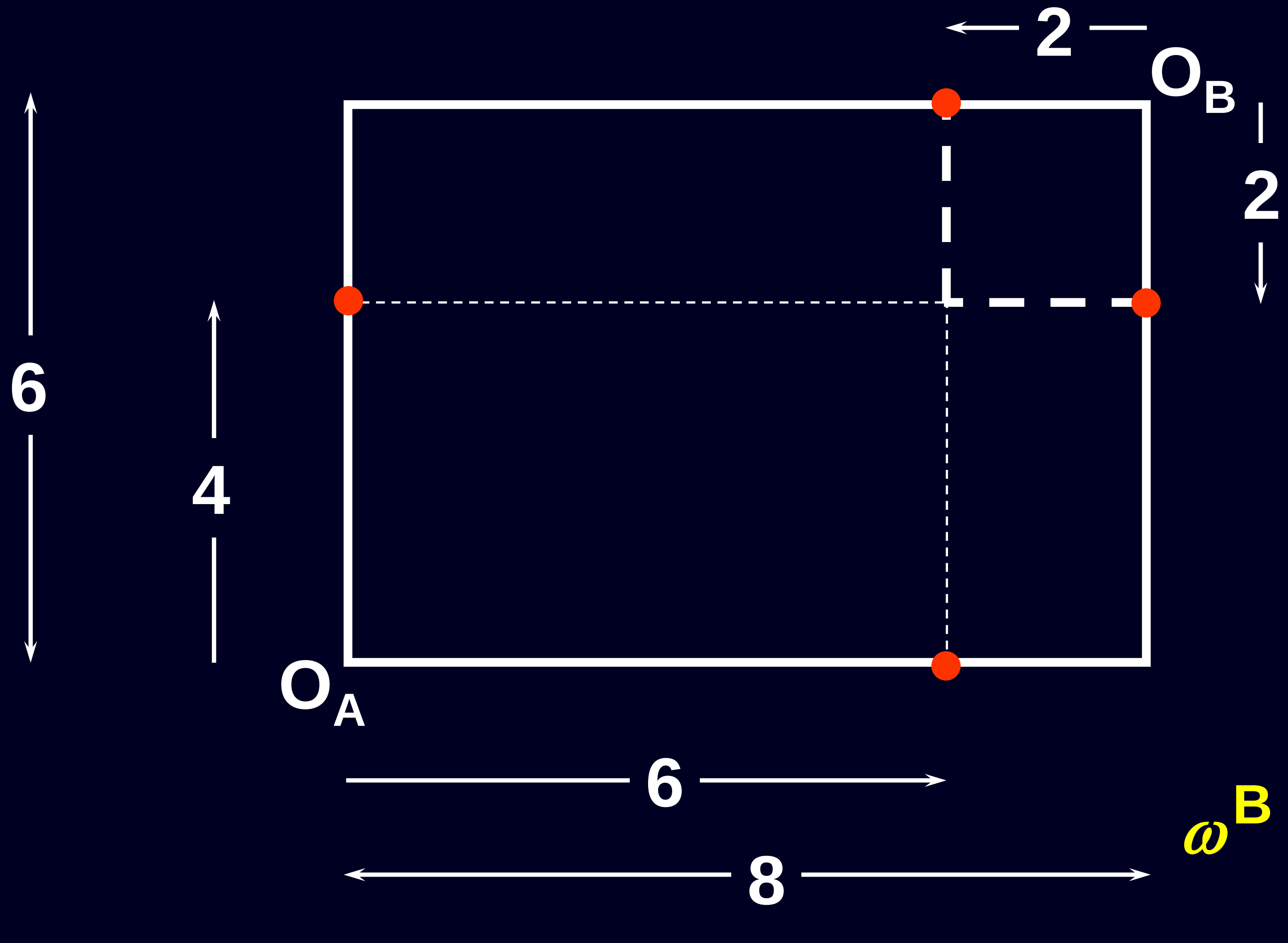
$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation



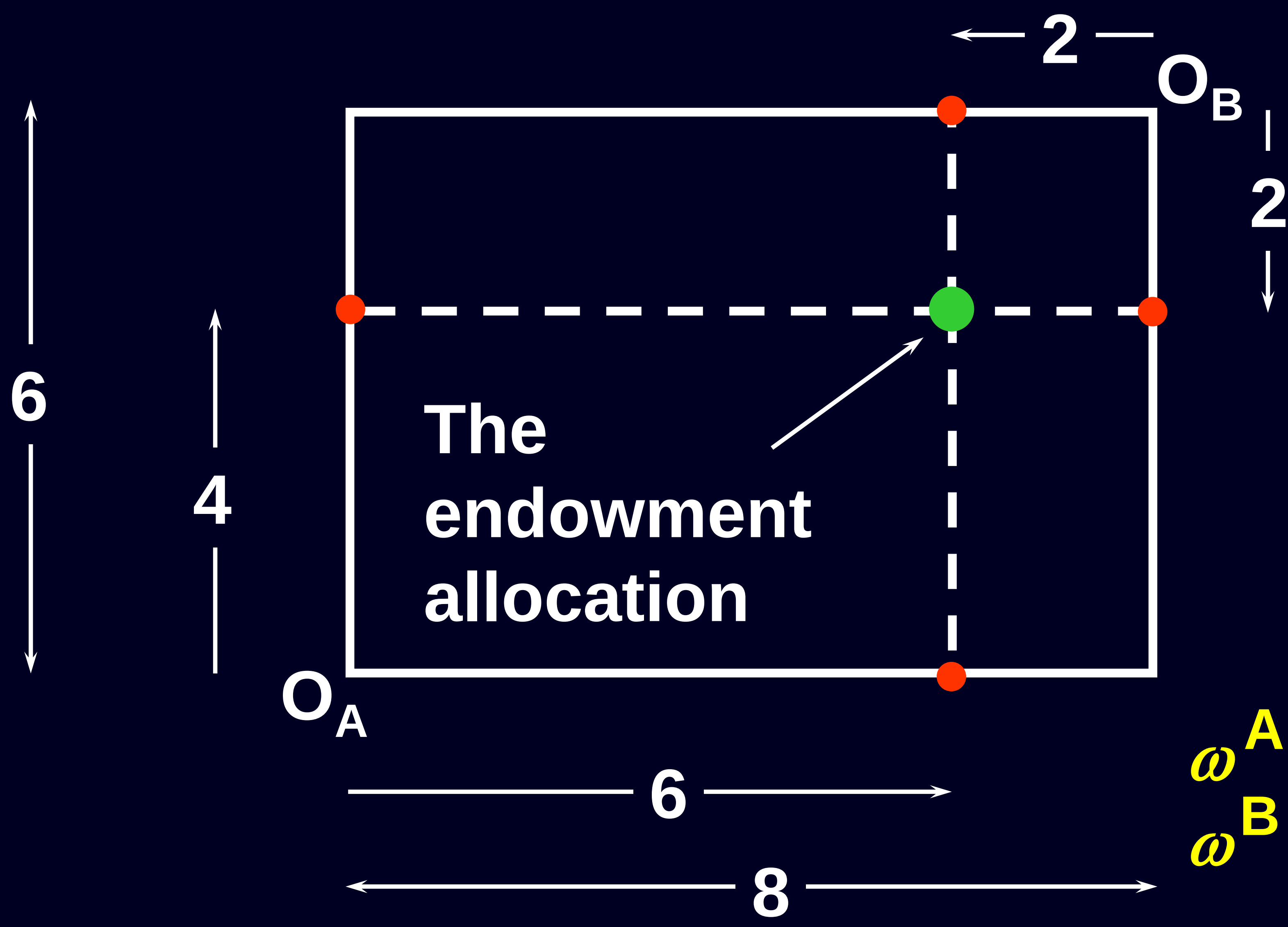
$$\omega^A = (6,4)$$

The Endowment Allocation



$\omega^B = (2,2)$

The Endowment Allocation



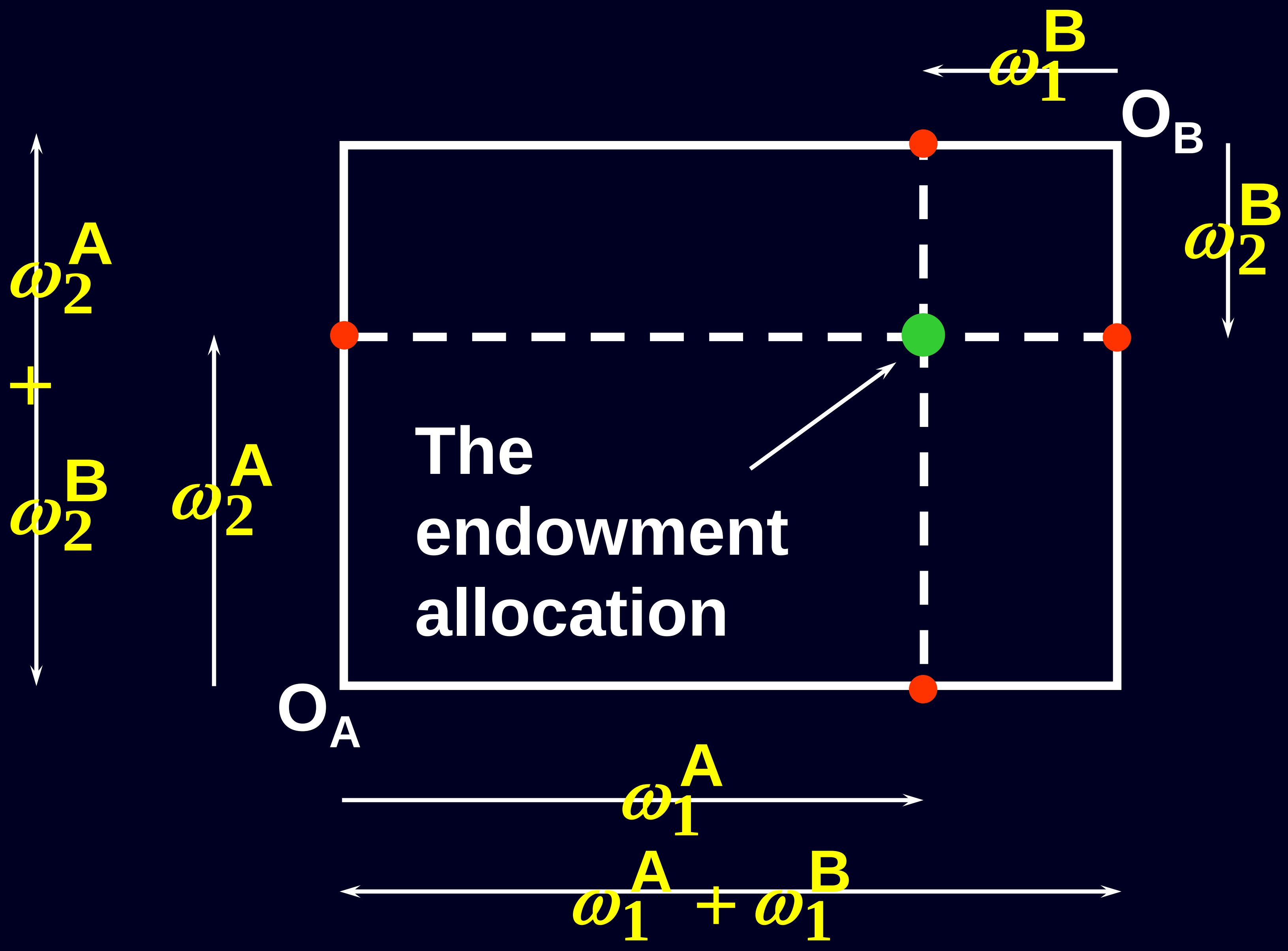
$\omega^A = (6,4)$

$\omega^B = (2,2)$

The Endowment Allocation

More generally, …

The Endowment Allocation



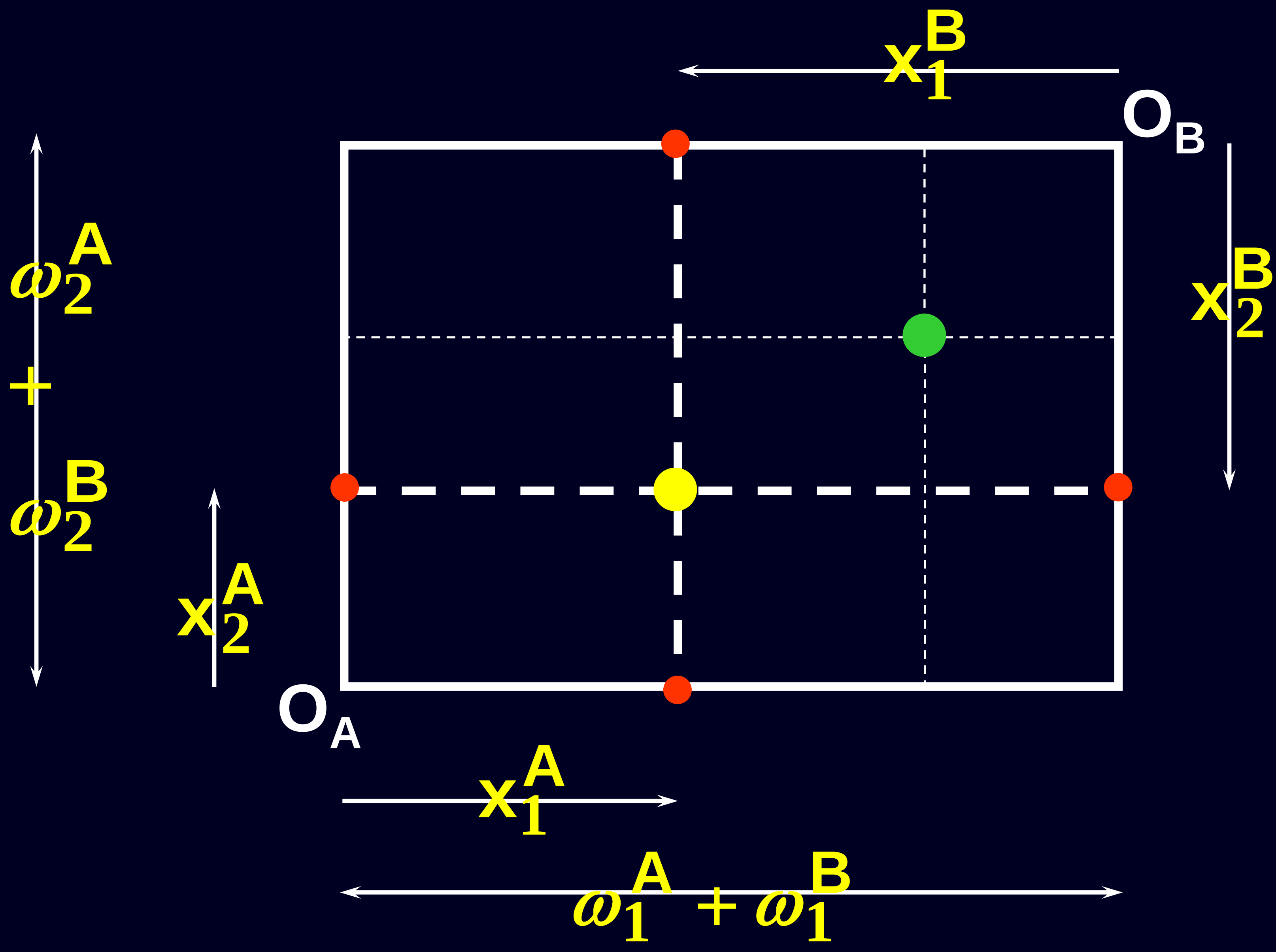
Other Feasible Allocations

- (x_1^A, x_2^A) denotes an allocation to consumer A.
- (x_1^B, x_2^B) denotes an allocation to consumer B.
- An allocation is **feasible** if and only if

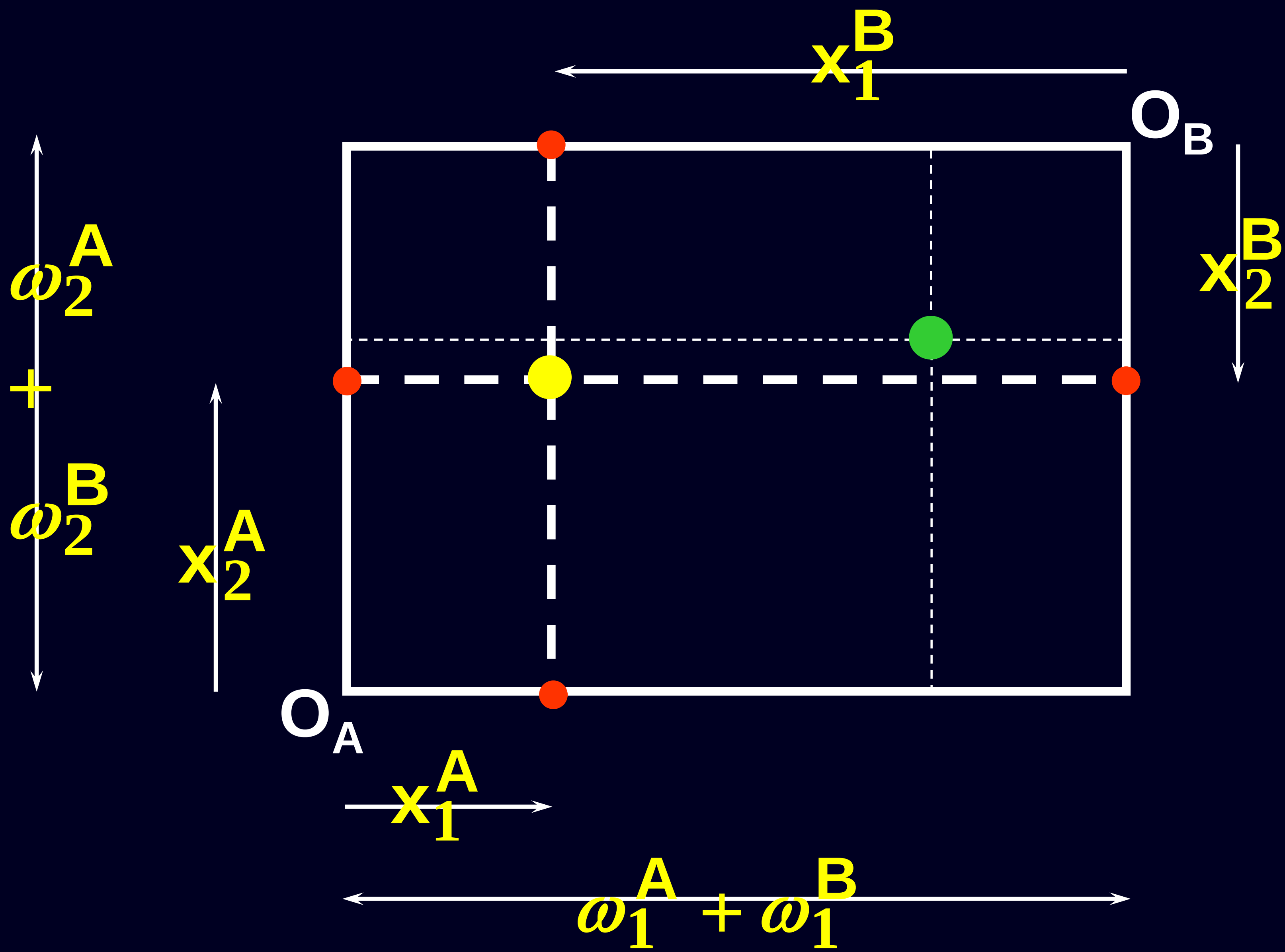
$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$

and $$x_2^A + x_2^B \le \omega_2^A + \omega_2^B.$$

Feasible Reallocations



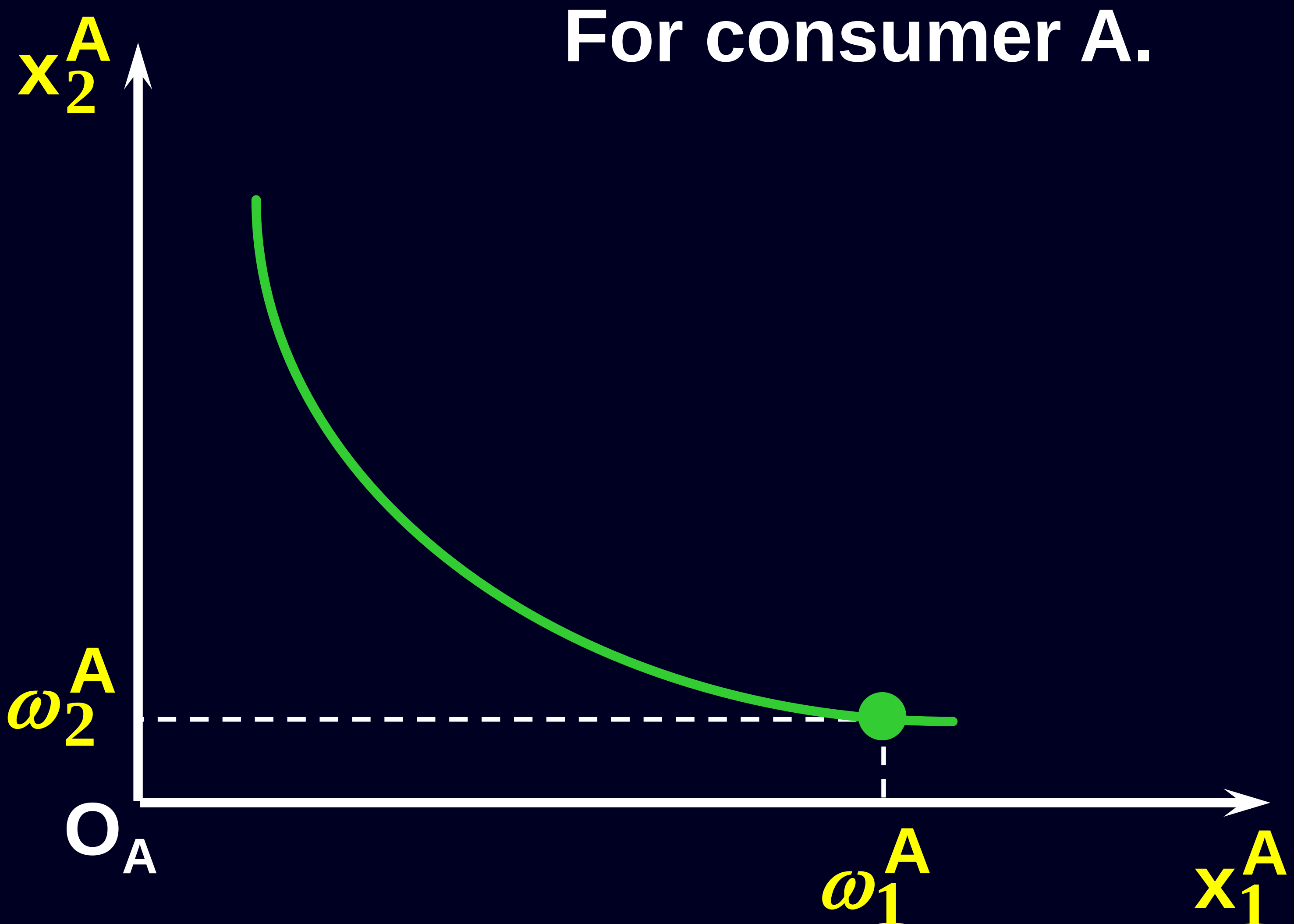
Feasible Reallocations



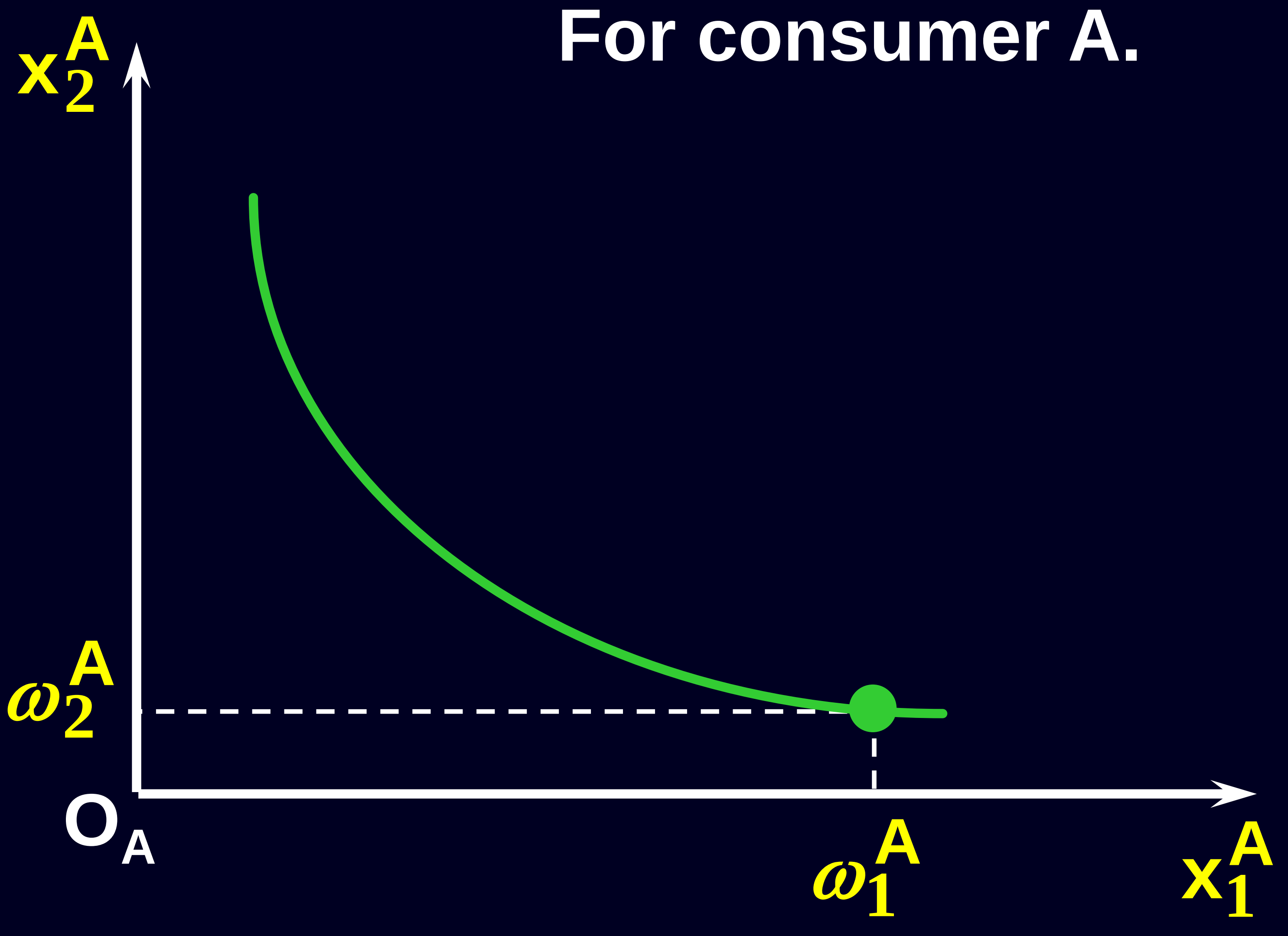
Feasible Reallocations

- **All points in the box, including the boundary, represent feasible allocations of the combined endowments.**

Adding Preferences to the Box

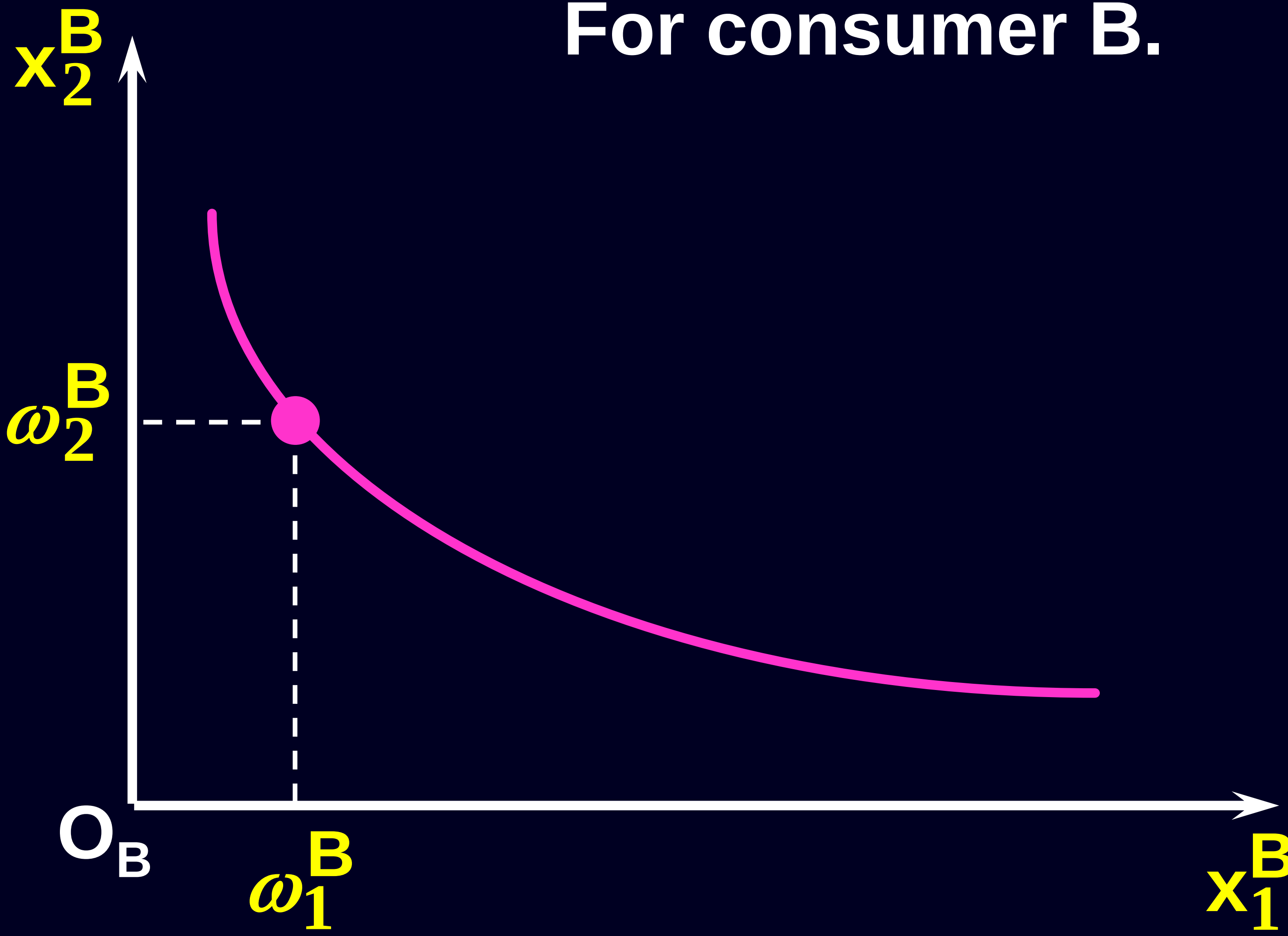


Adding Preferences to the Box



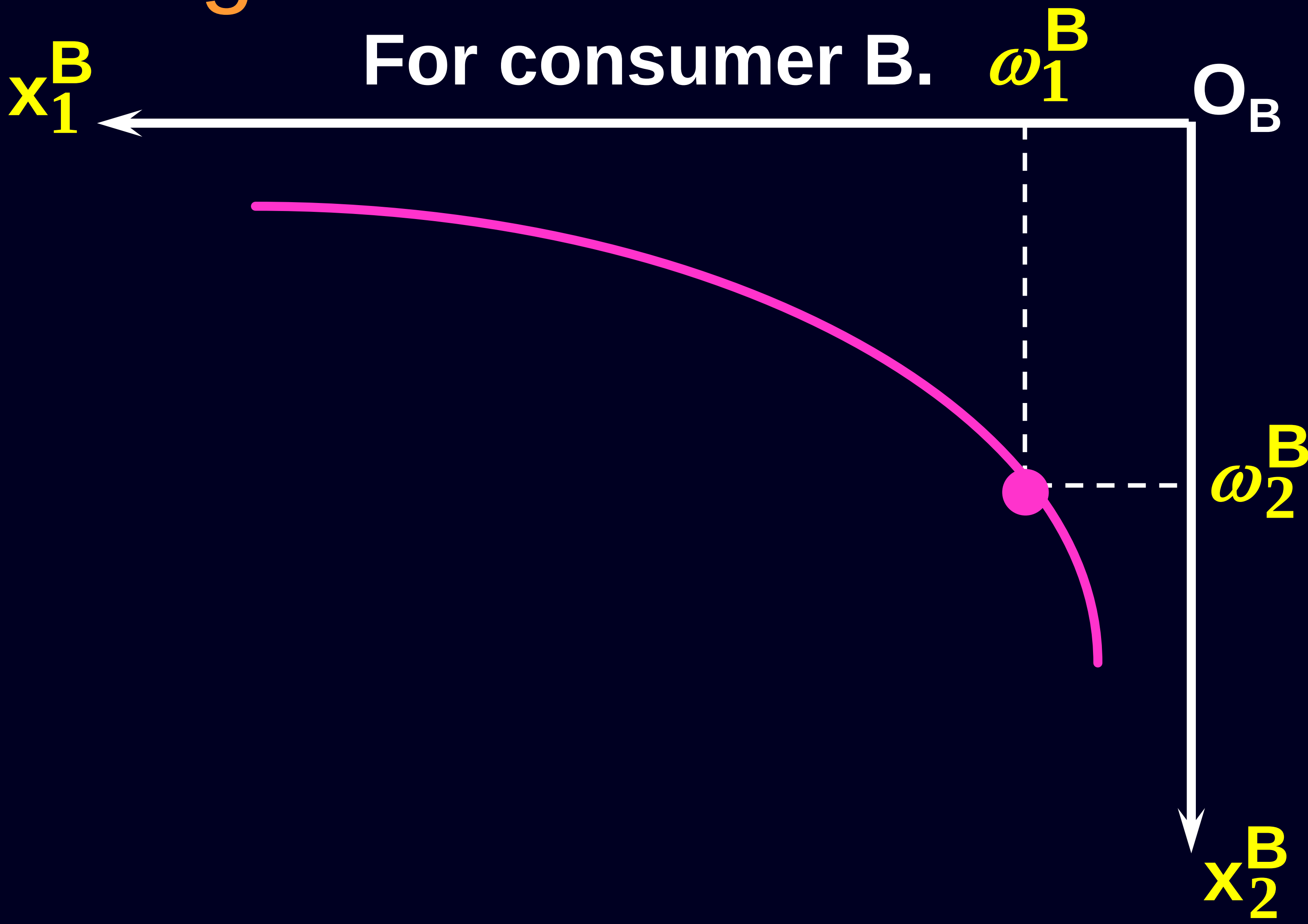
Adding Preferences to the Box

For consumer B.



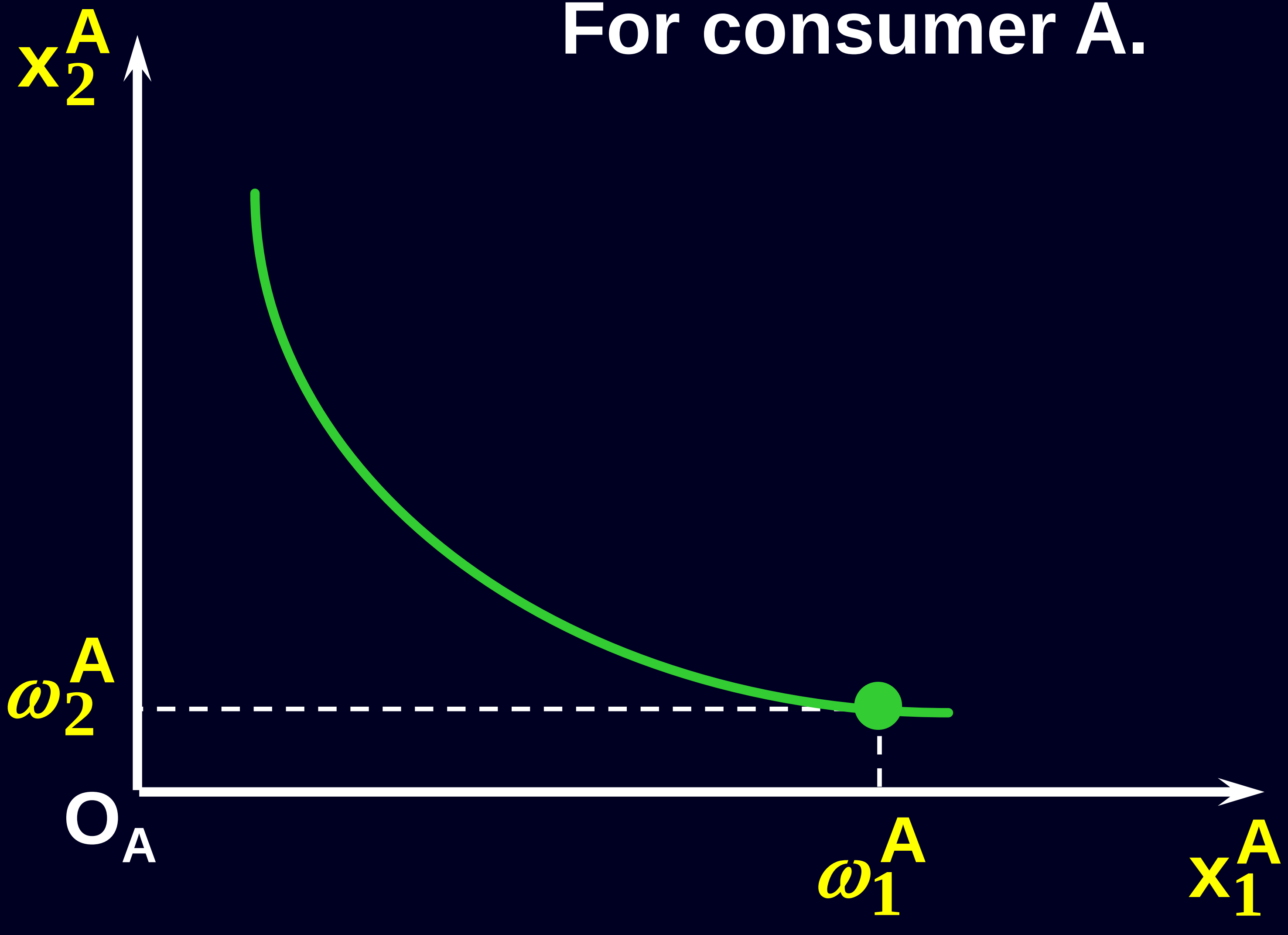
Adding Preferences to the Box

For consumer B.

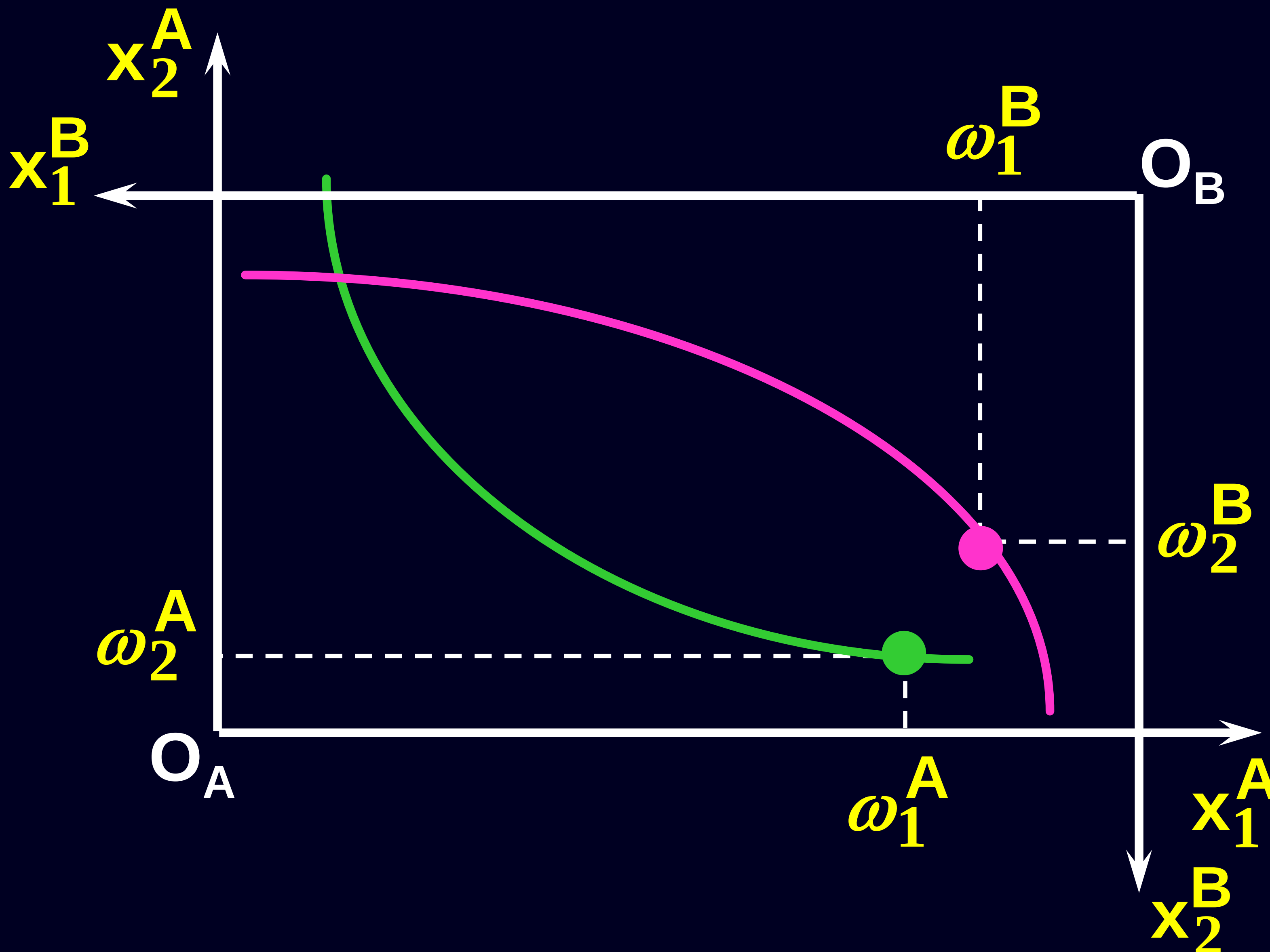


Adding Preferences to the Box

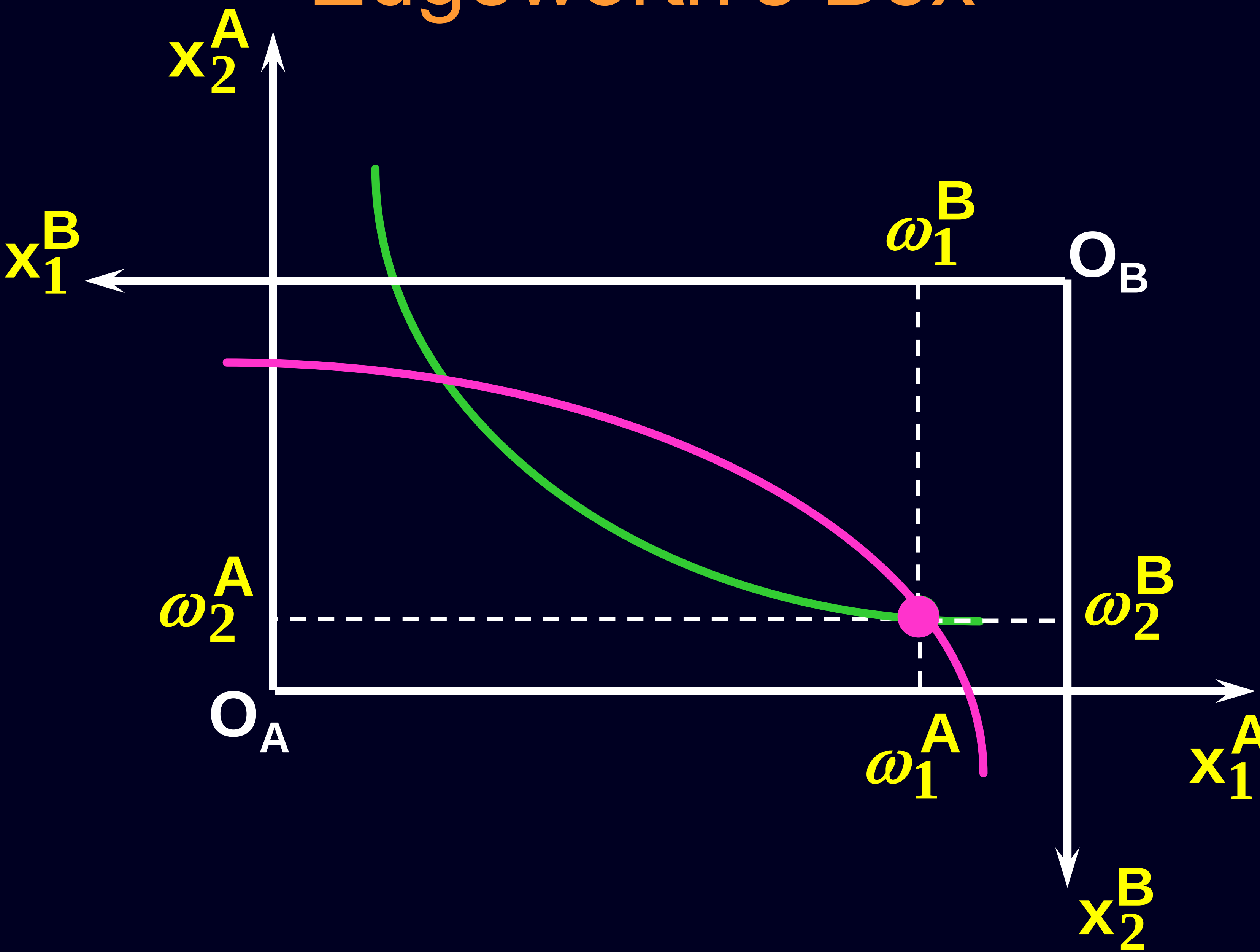
For consumer A.



Adding Preferences to the Box



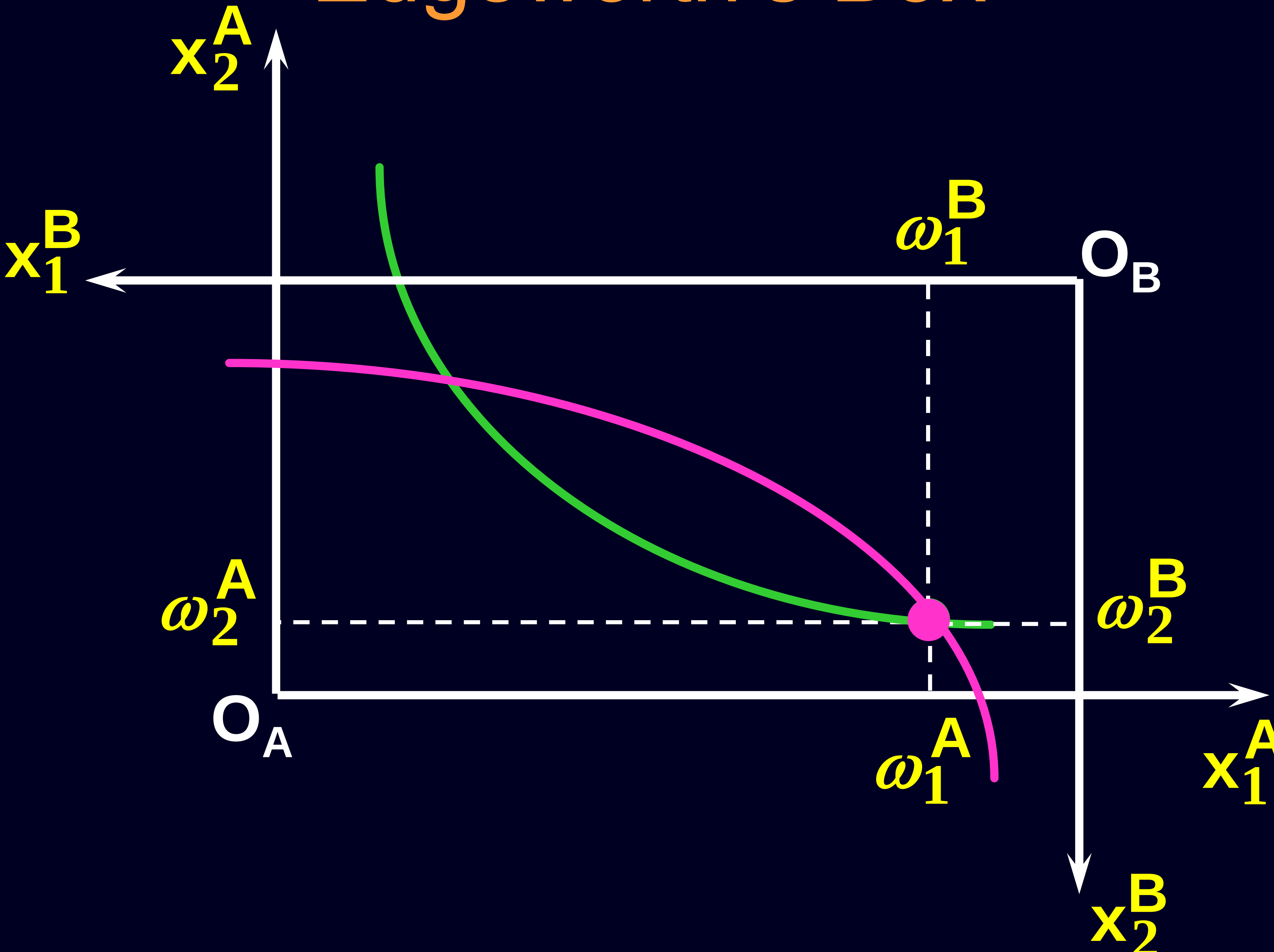
Edgeworth's Box



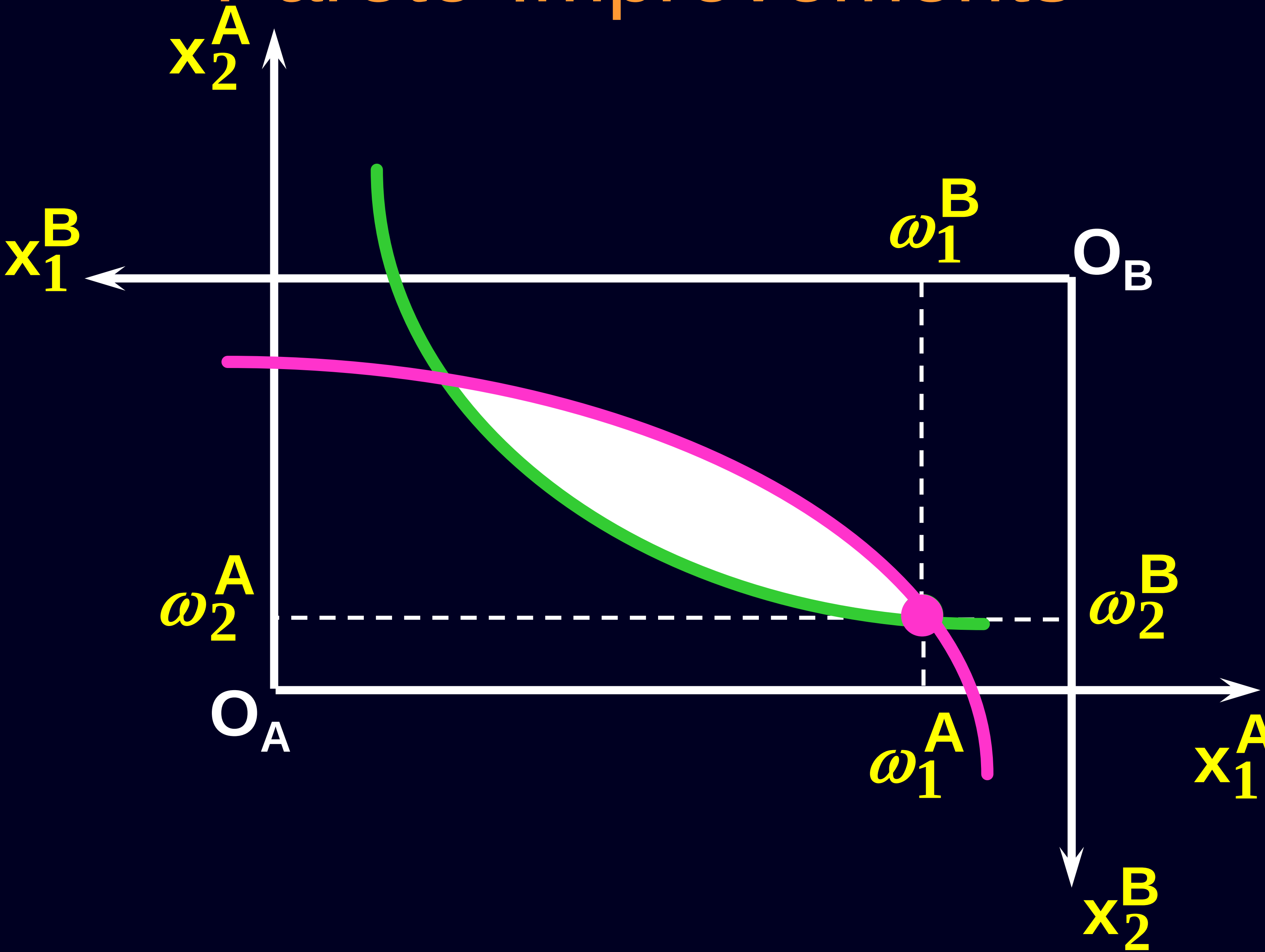
Pareto-Improvement

- **An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a ……………………………… ……………………….**
- **Where are the Pareto-improving allocations?**

Edgeworth's Box



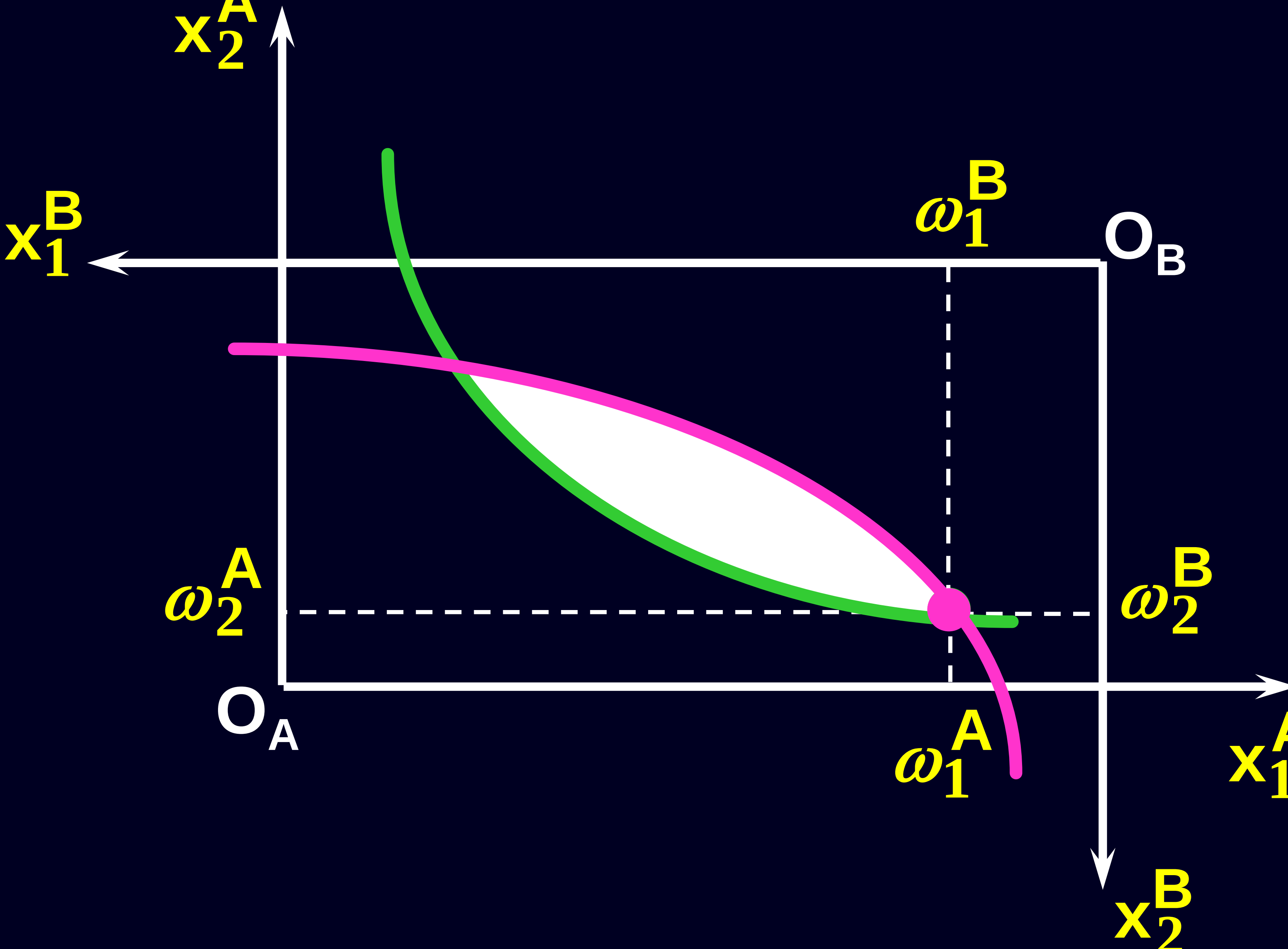
Pareto-Improvements



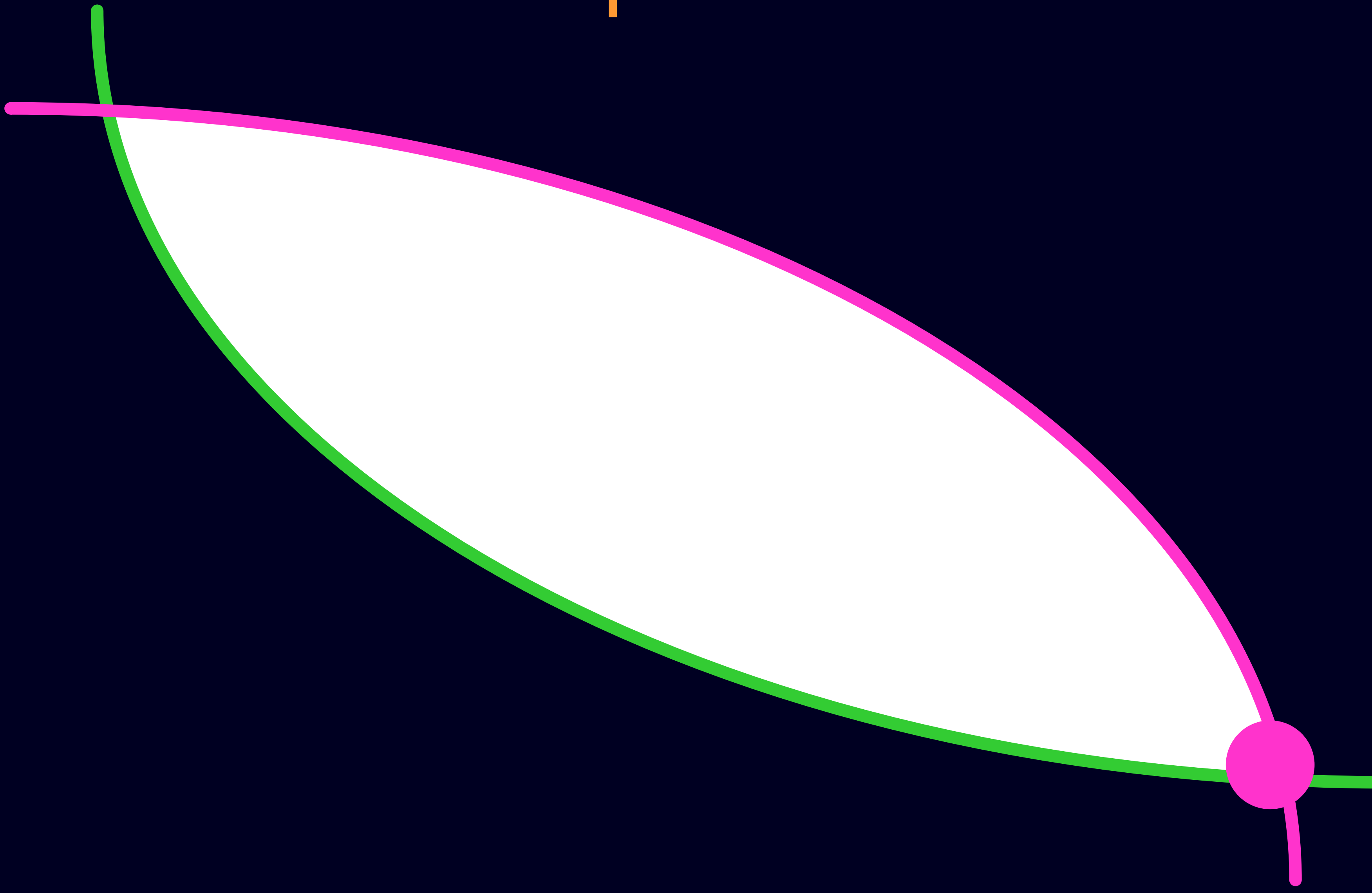
Pareto-Improvements

- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.
- But which particular Pareto-improving allocation will be the outcome of trade?

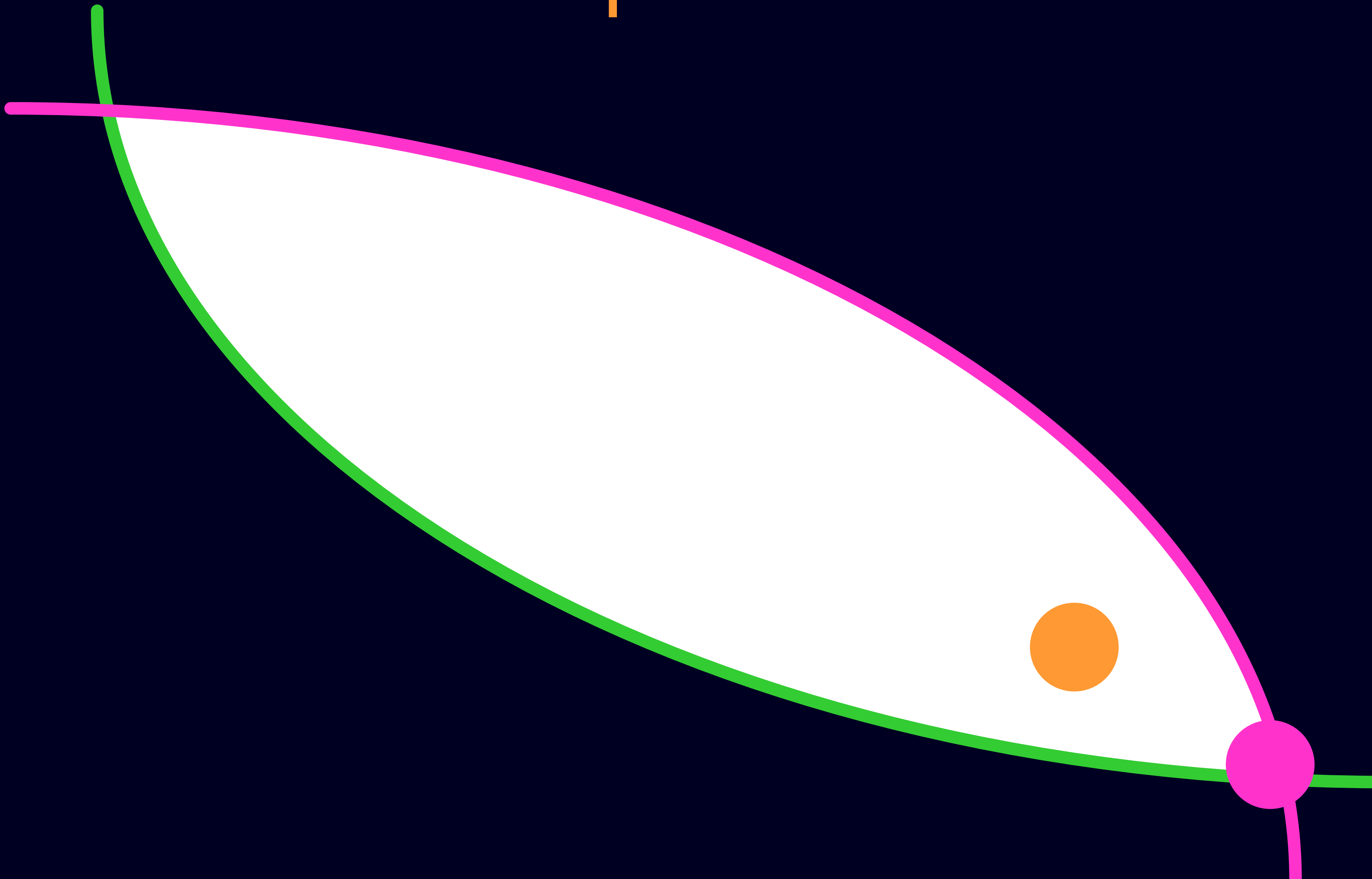
Pareto-Improvements



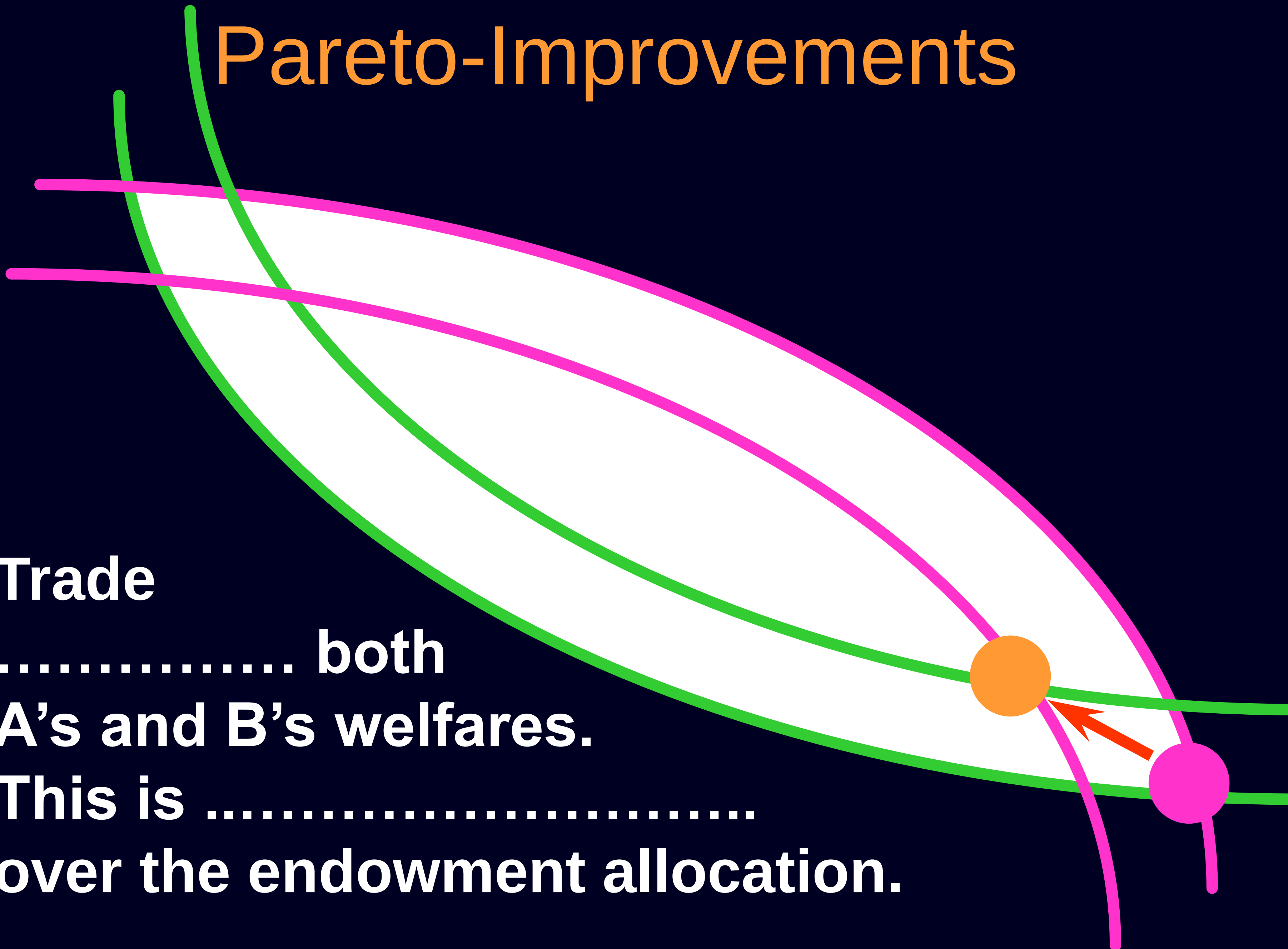
Pareto-Improvements



Pareto-Improvements



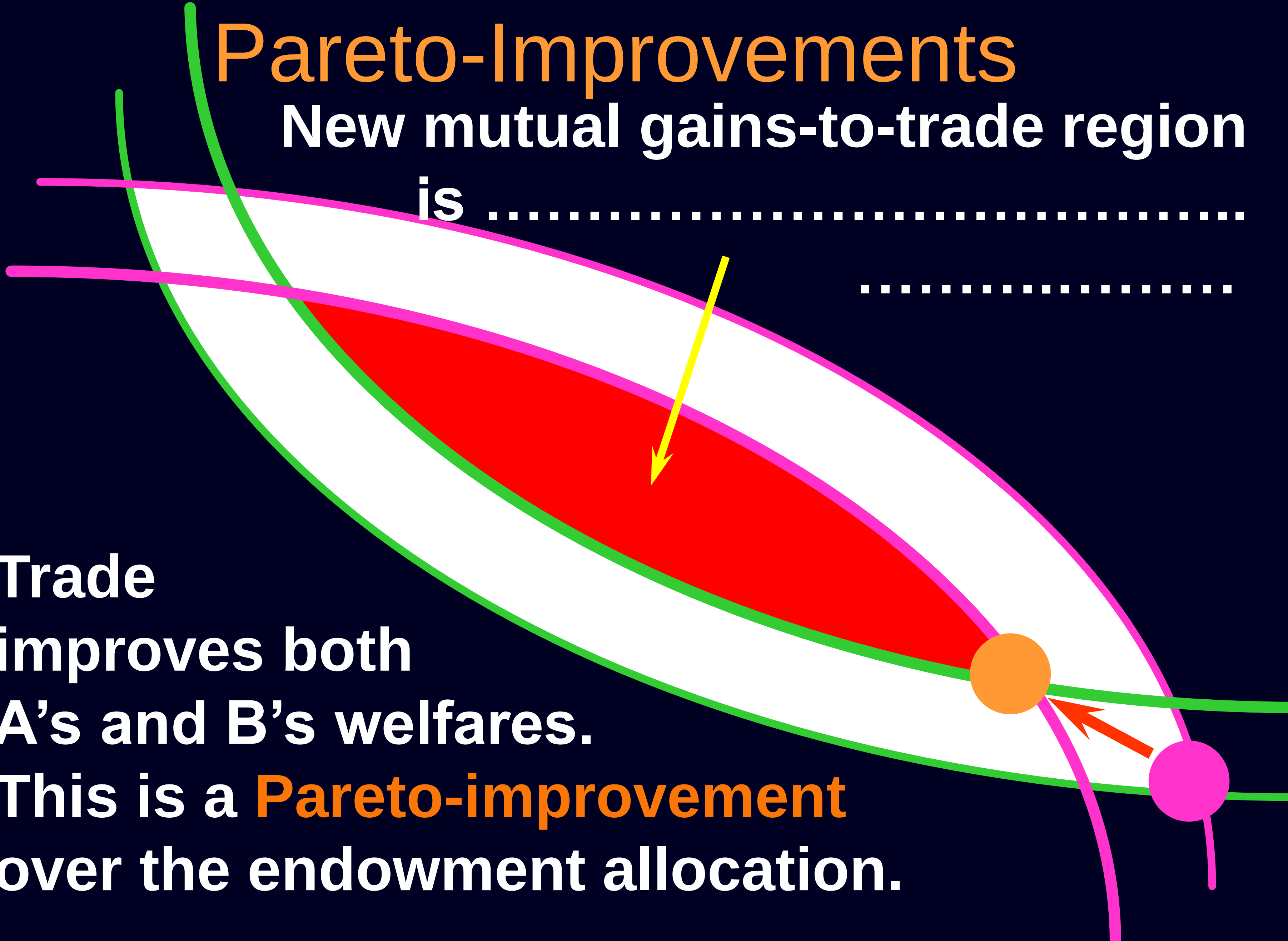
Pareto-Improvements



Trade
.... both
A’s and B’s welfares.
This is ....
over the endowment allocation.

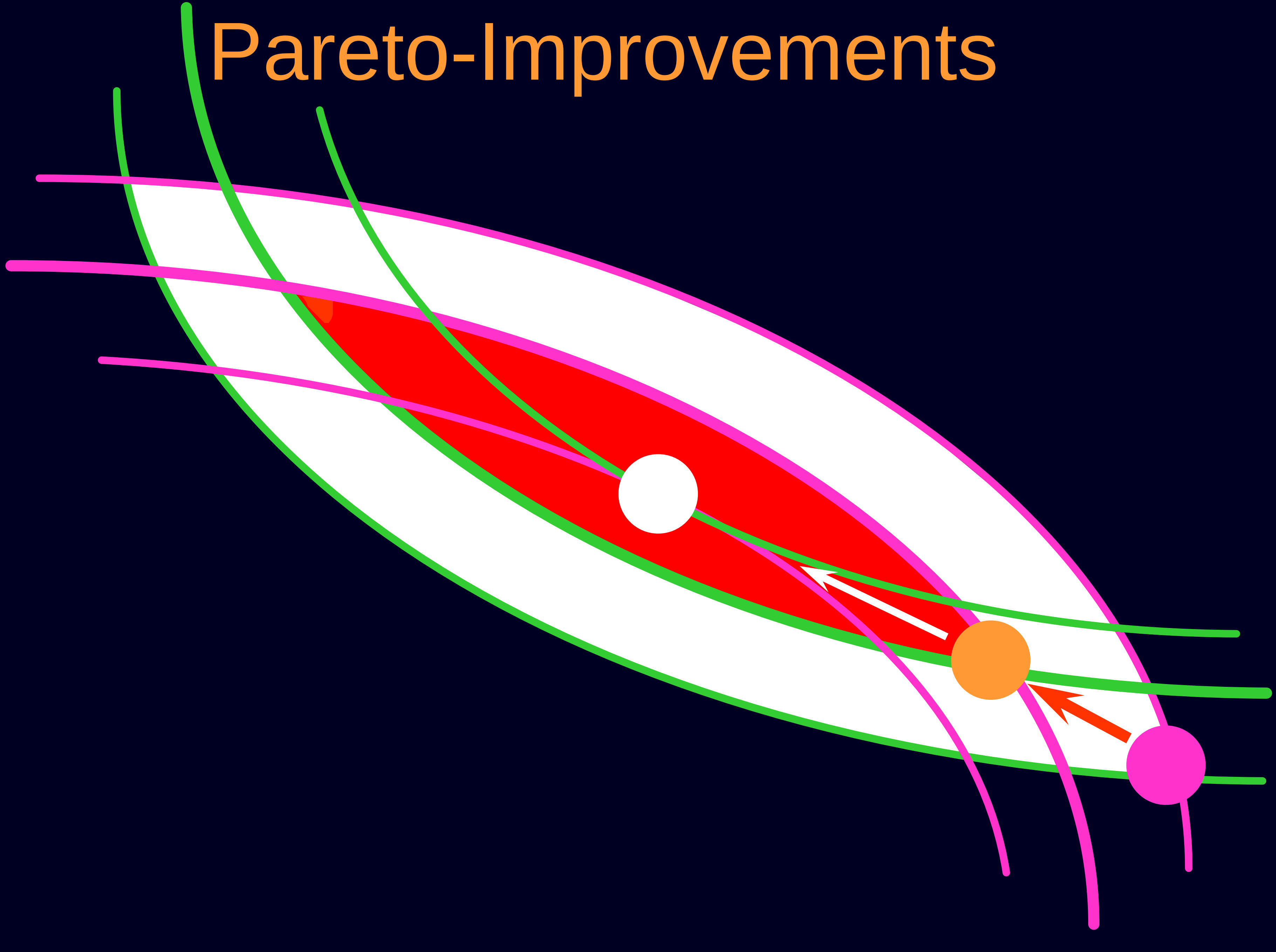
Pareto-Improvements

New mutual gains-to-trade region is ....

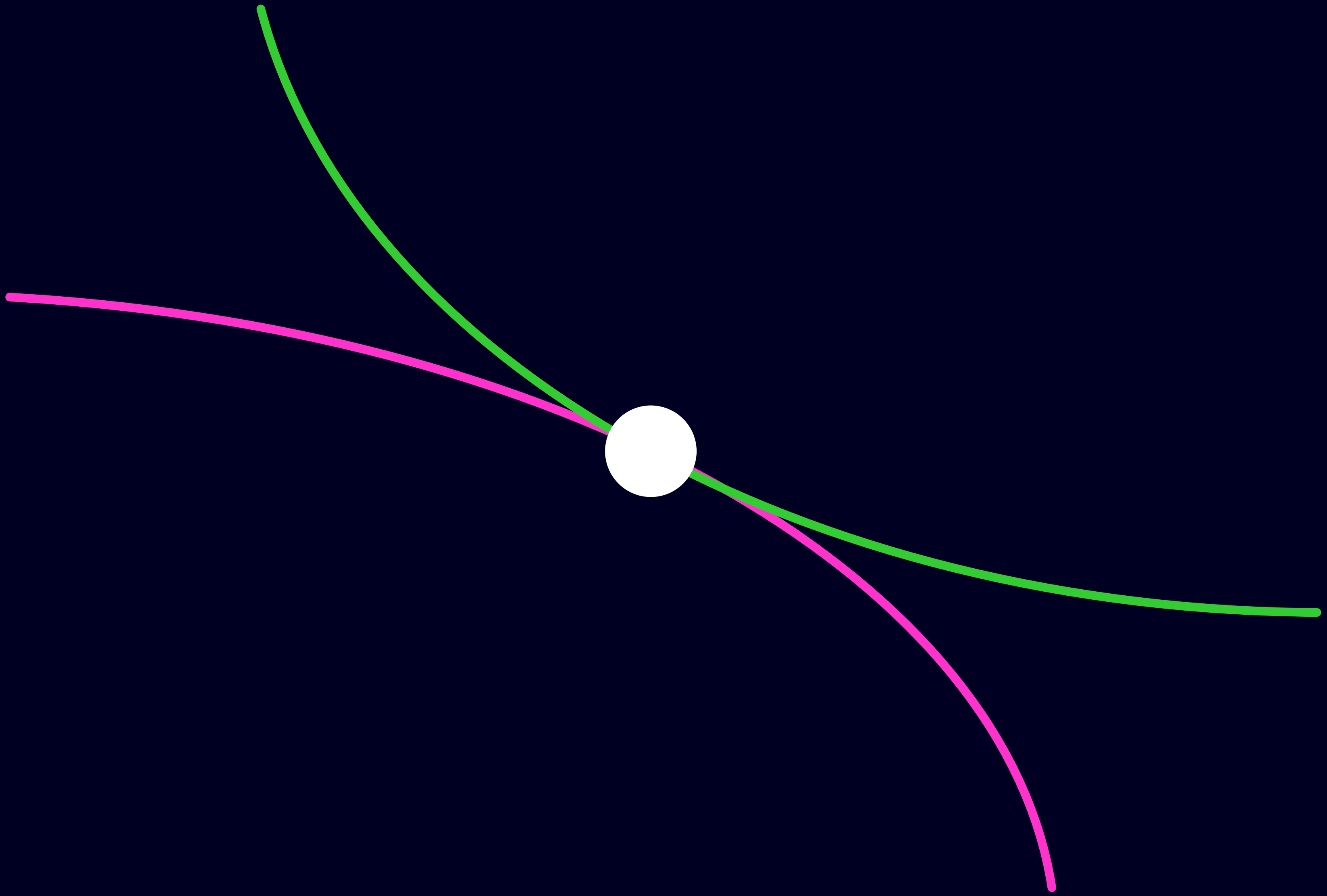


Trade improves both A's and B's welfares.
This is a Pareto-improvement over the endowment allocation.

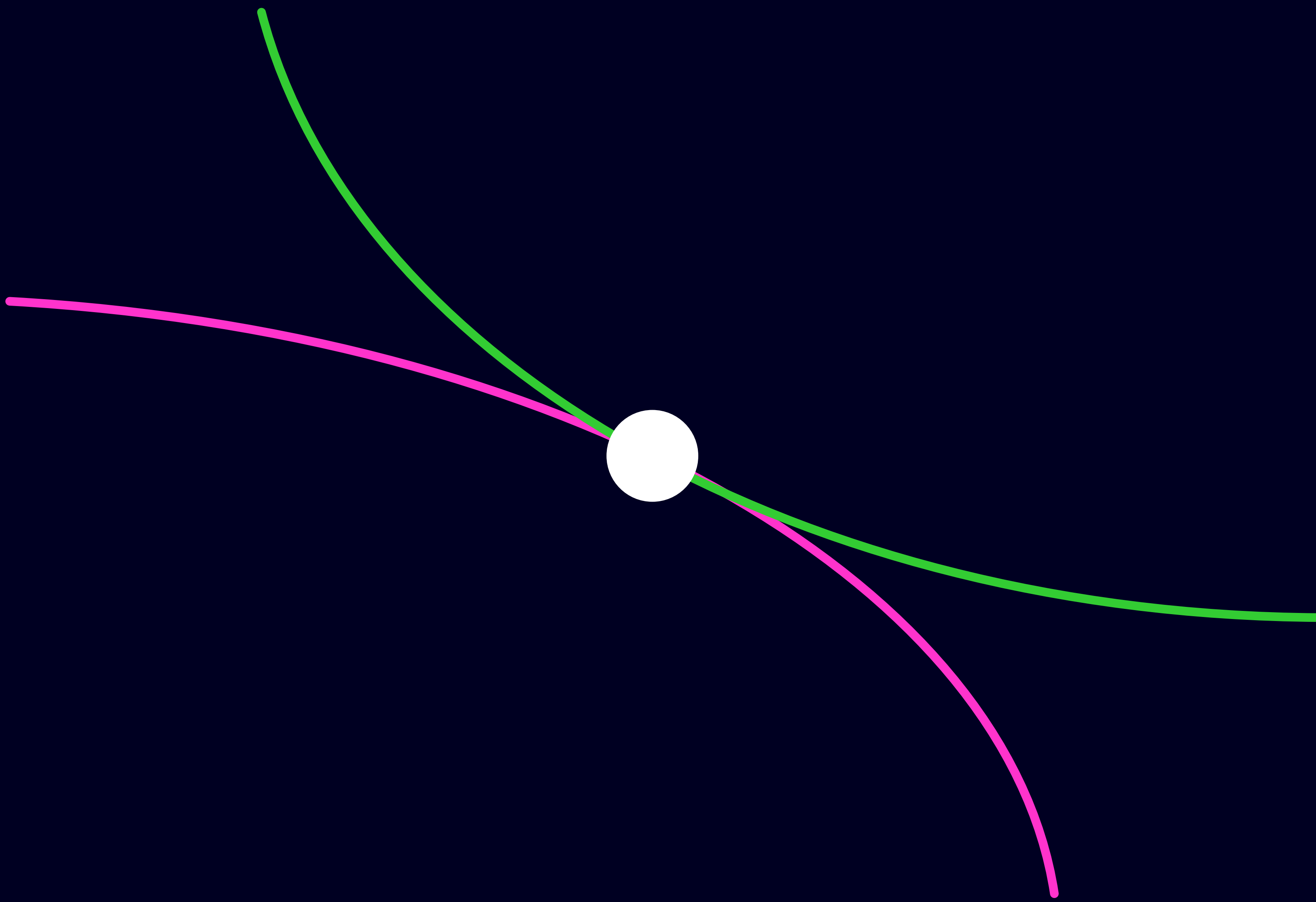
Pareto-Improvements



Pareto-Optimality



Pareto-Optimality



Pareto-Optimality

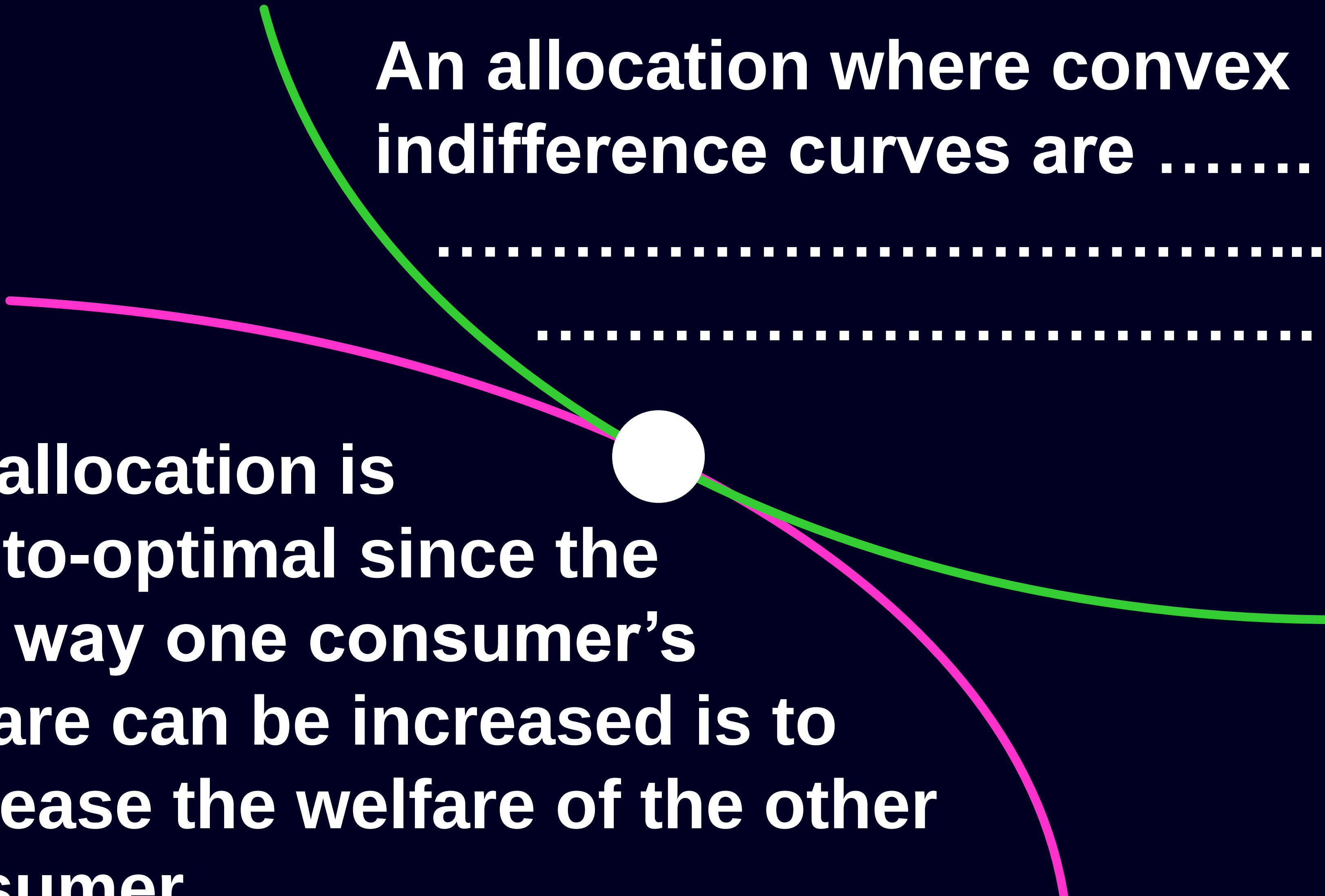
The allocation is .... since the only way one consumer’s welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

An allocation where convex indifference curves are …….

……………………………………………

……………………………………………

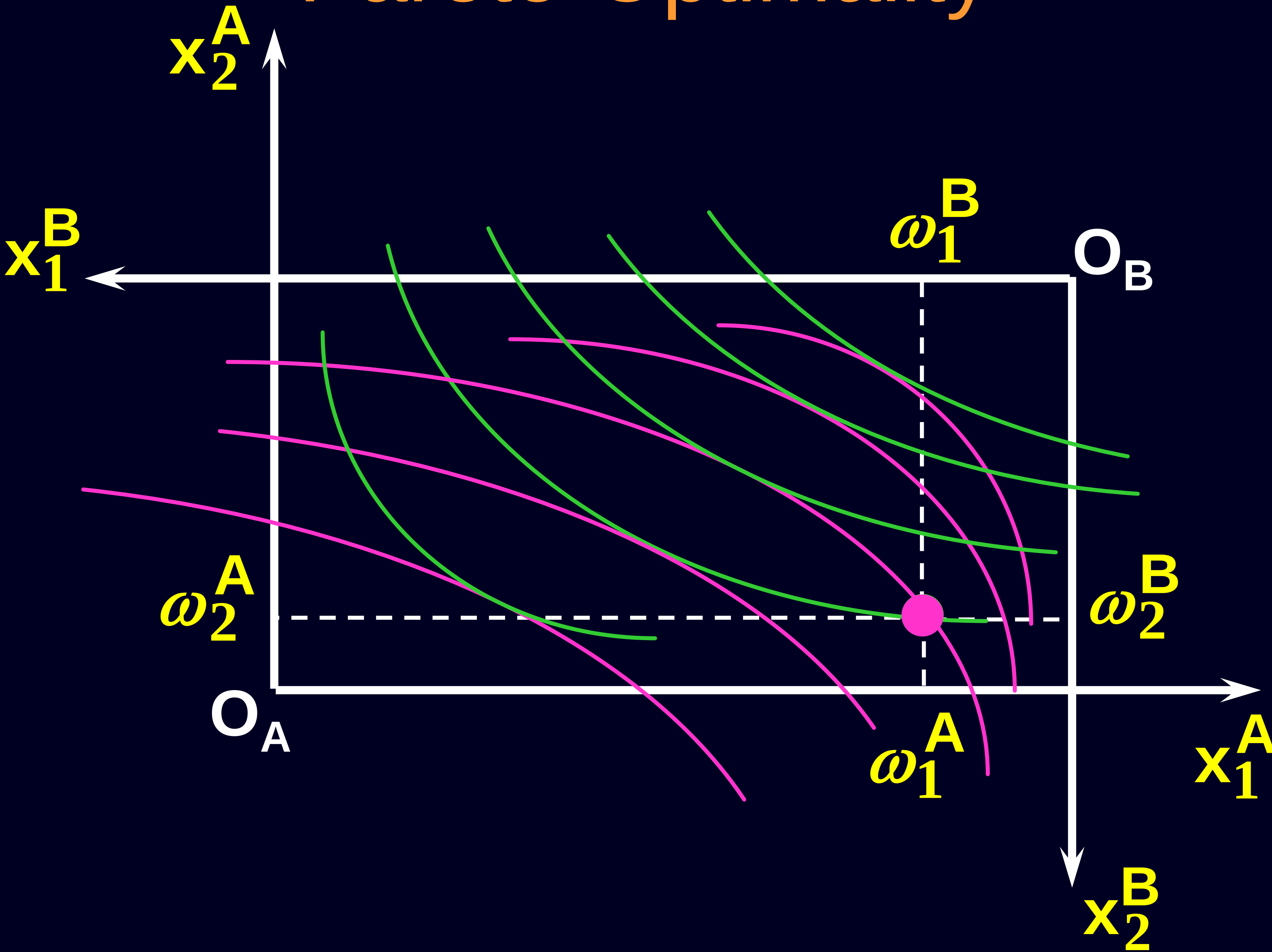


The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

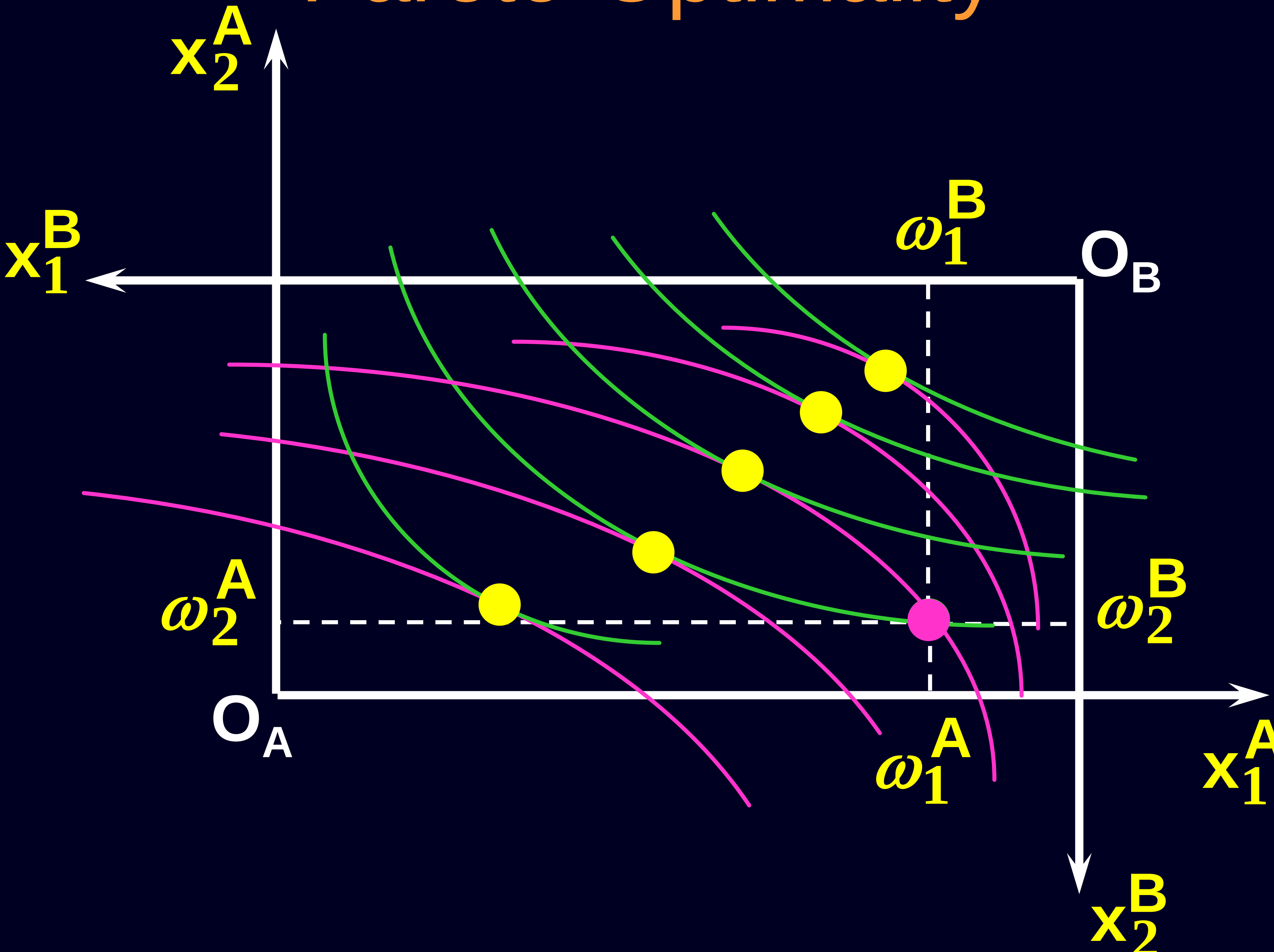
Pareto-Optimality

- **Where are all of the Pareto-optimal allocations of the endowment?**

Pareto-Optimality



Pareto-Optimality

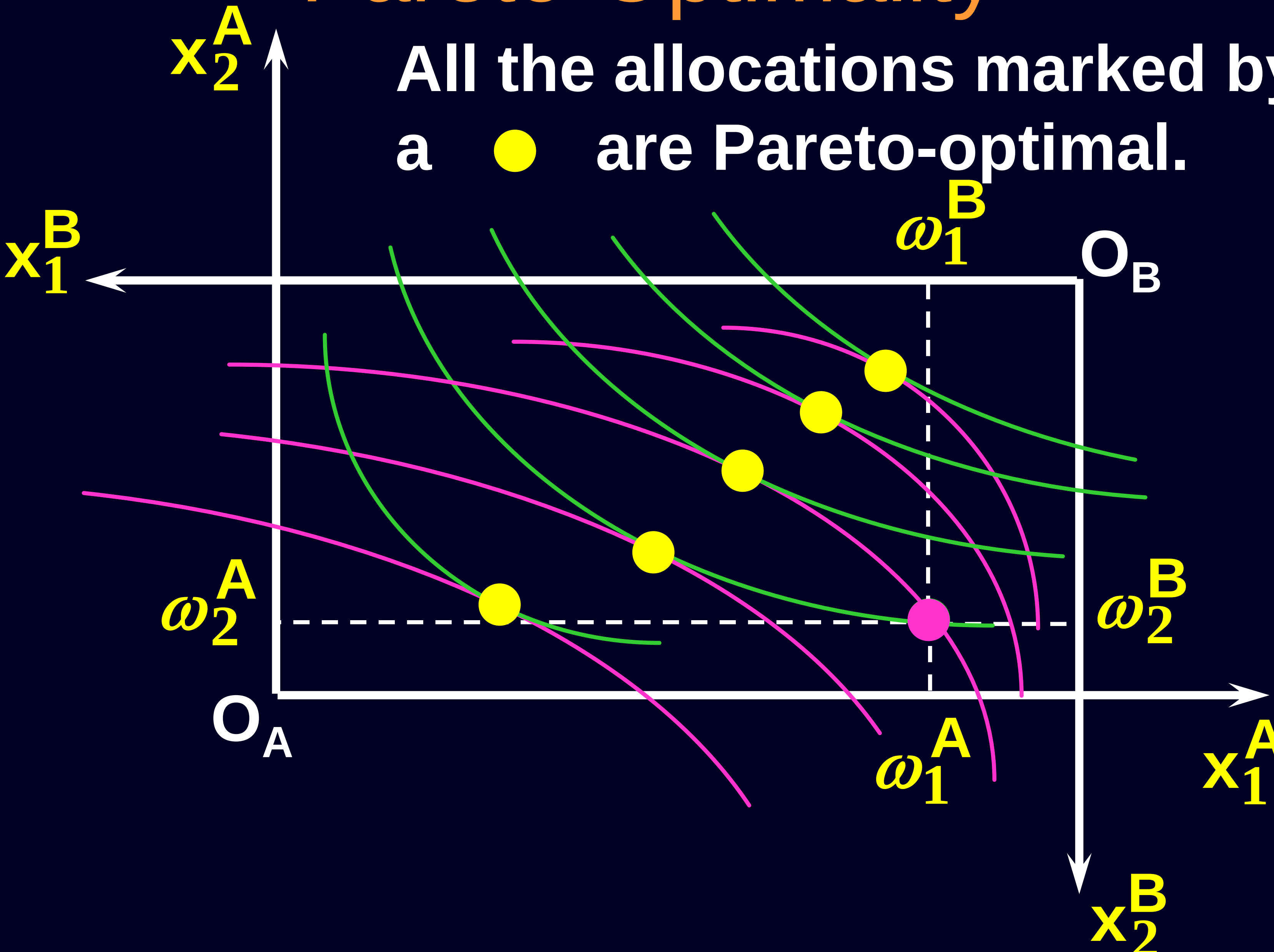


Pareto-Optimality

- **The contract curve is the set of all Pareto-optimal allocations**

Pareto-Optimality

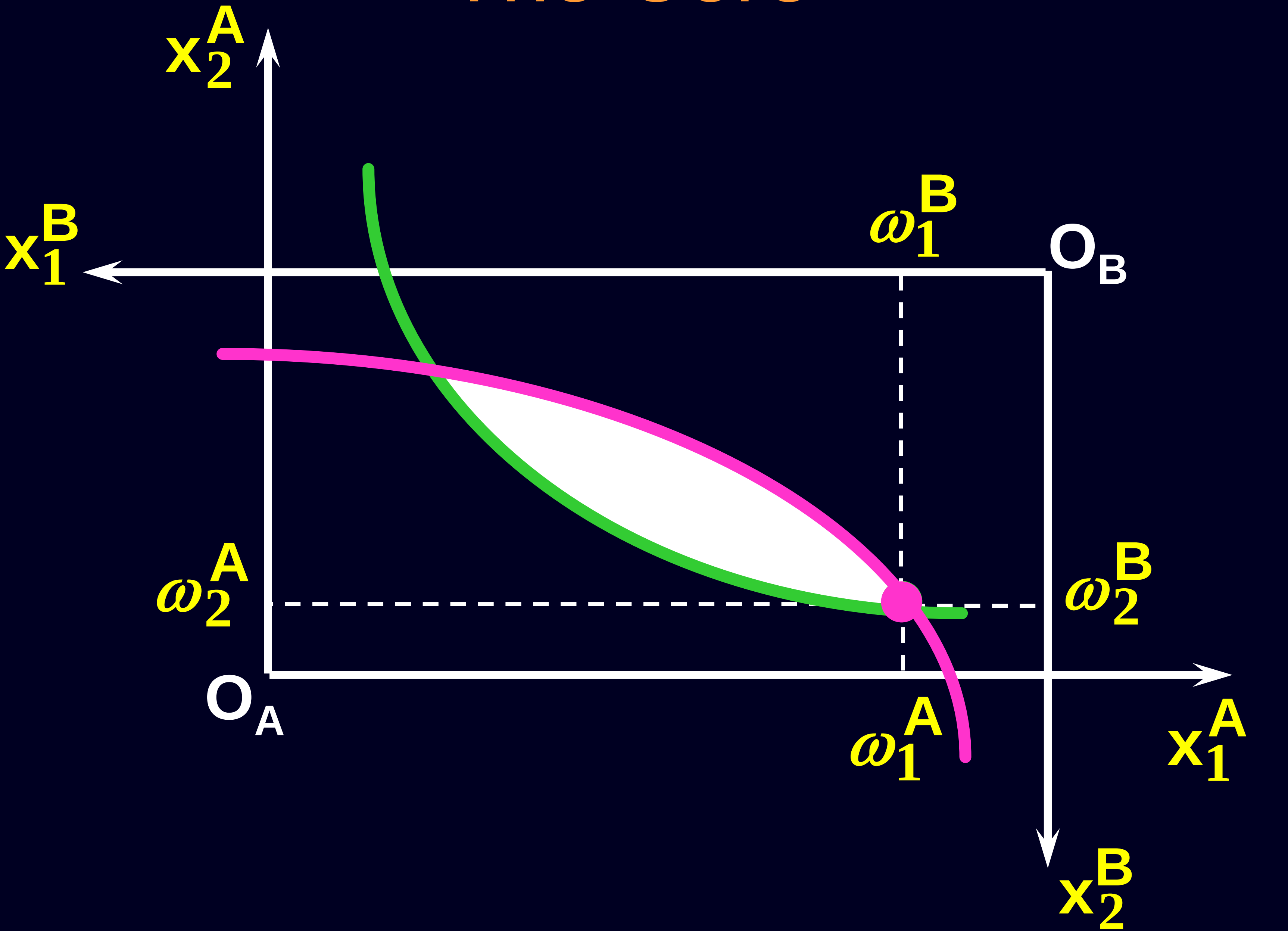
All the allocations marked by a ● are Pareto-optimal.



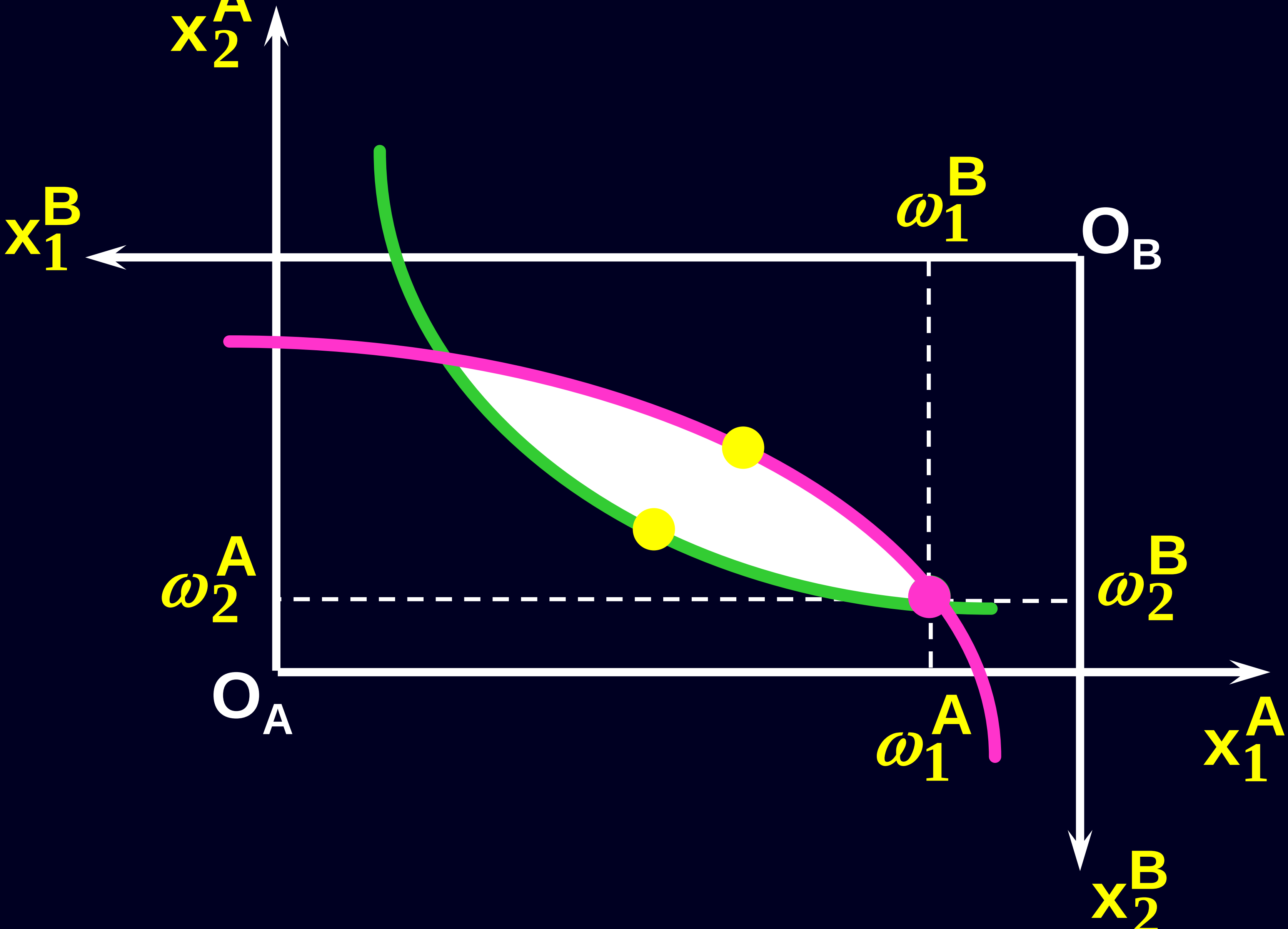
Pareto-Optimality

- **But to which of the many allocations on the contract curve will consumers trade?**
- **That depends upon how trade is conducted.**
- **In perfectly competitive markets? By one-on-one bargaining?**

The Core



The Core



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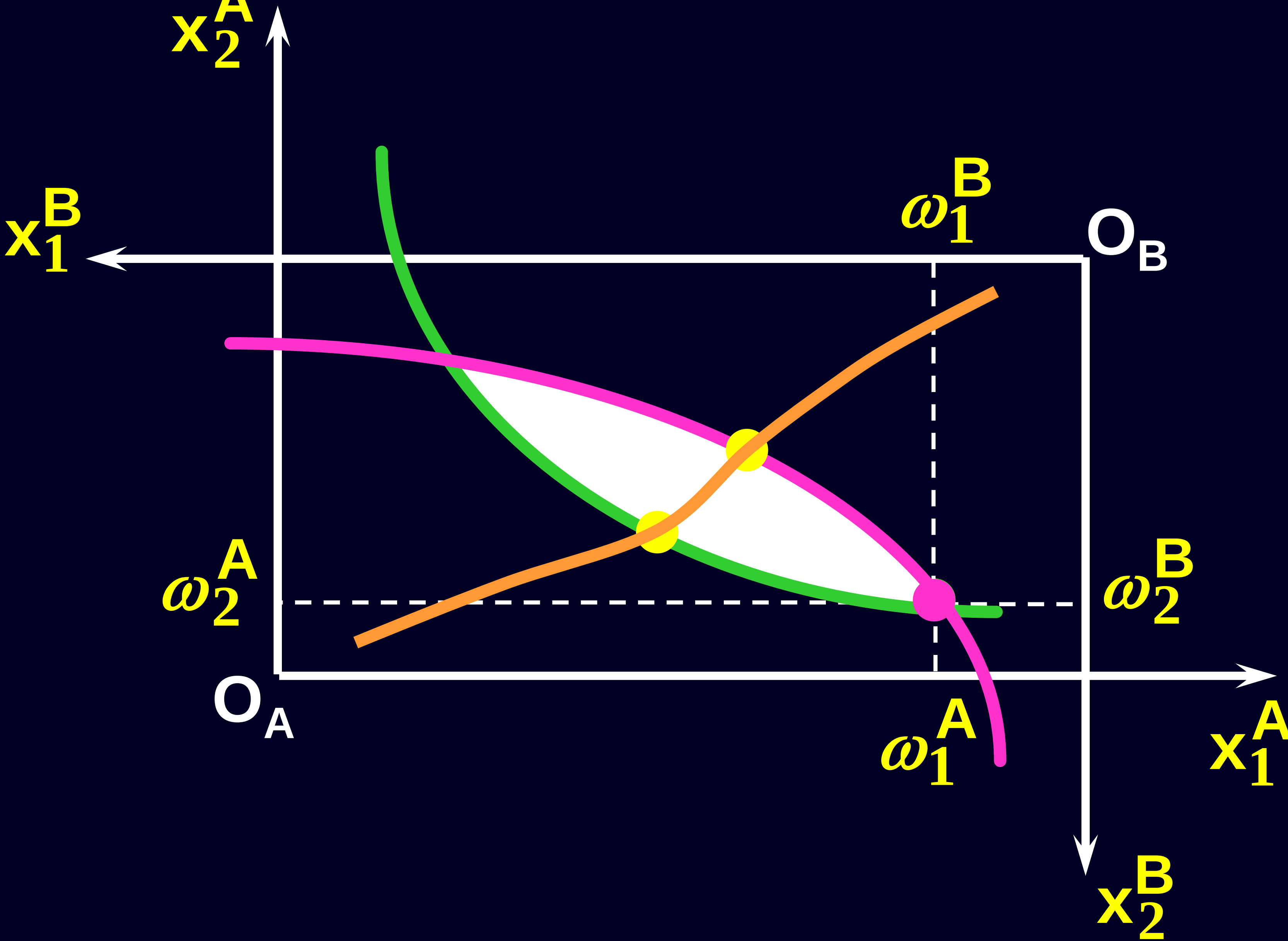

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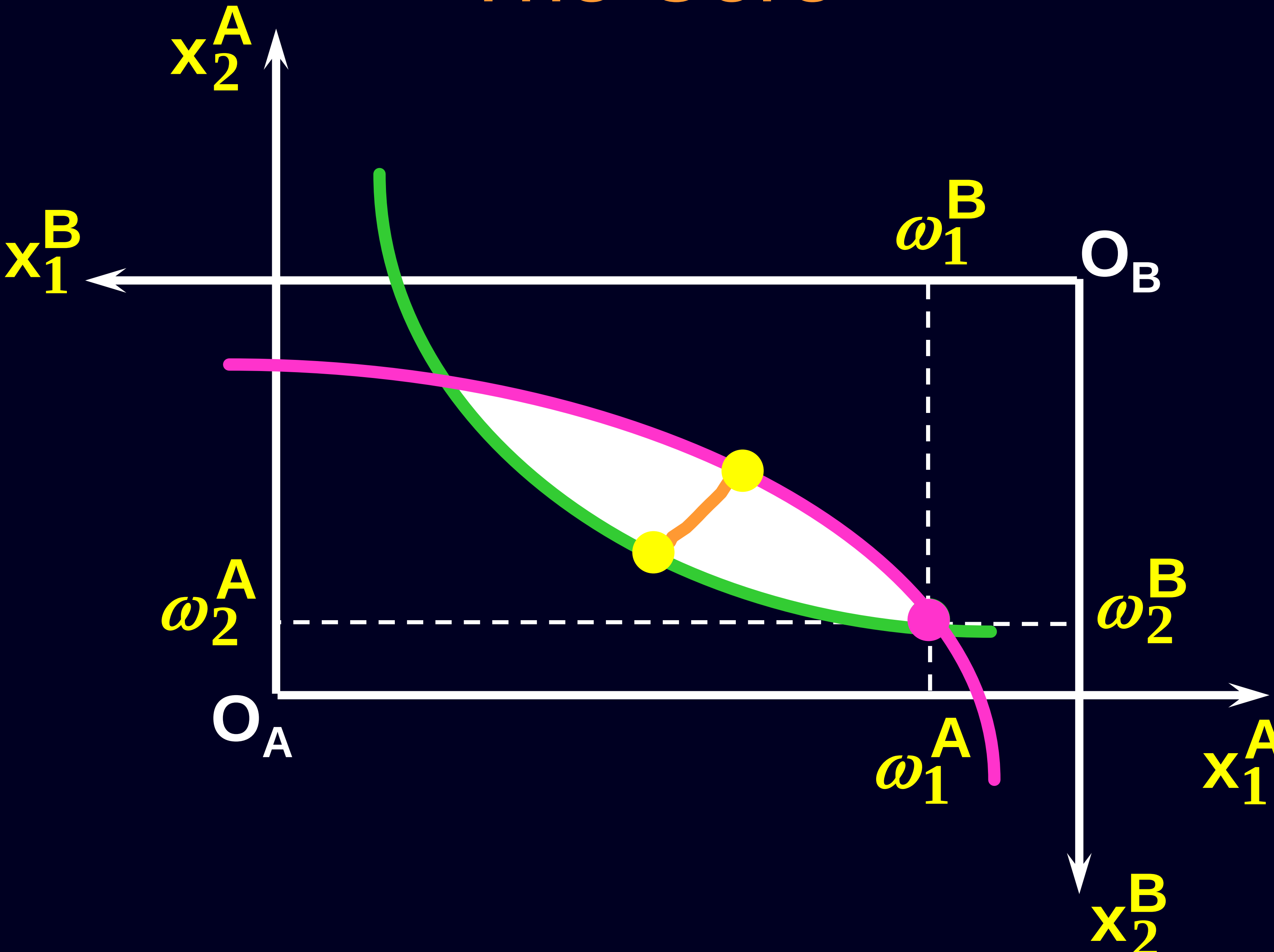

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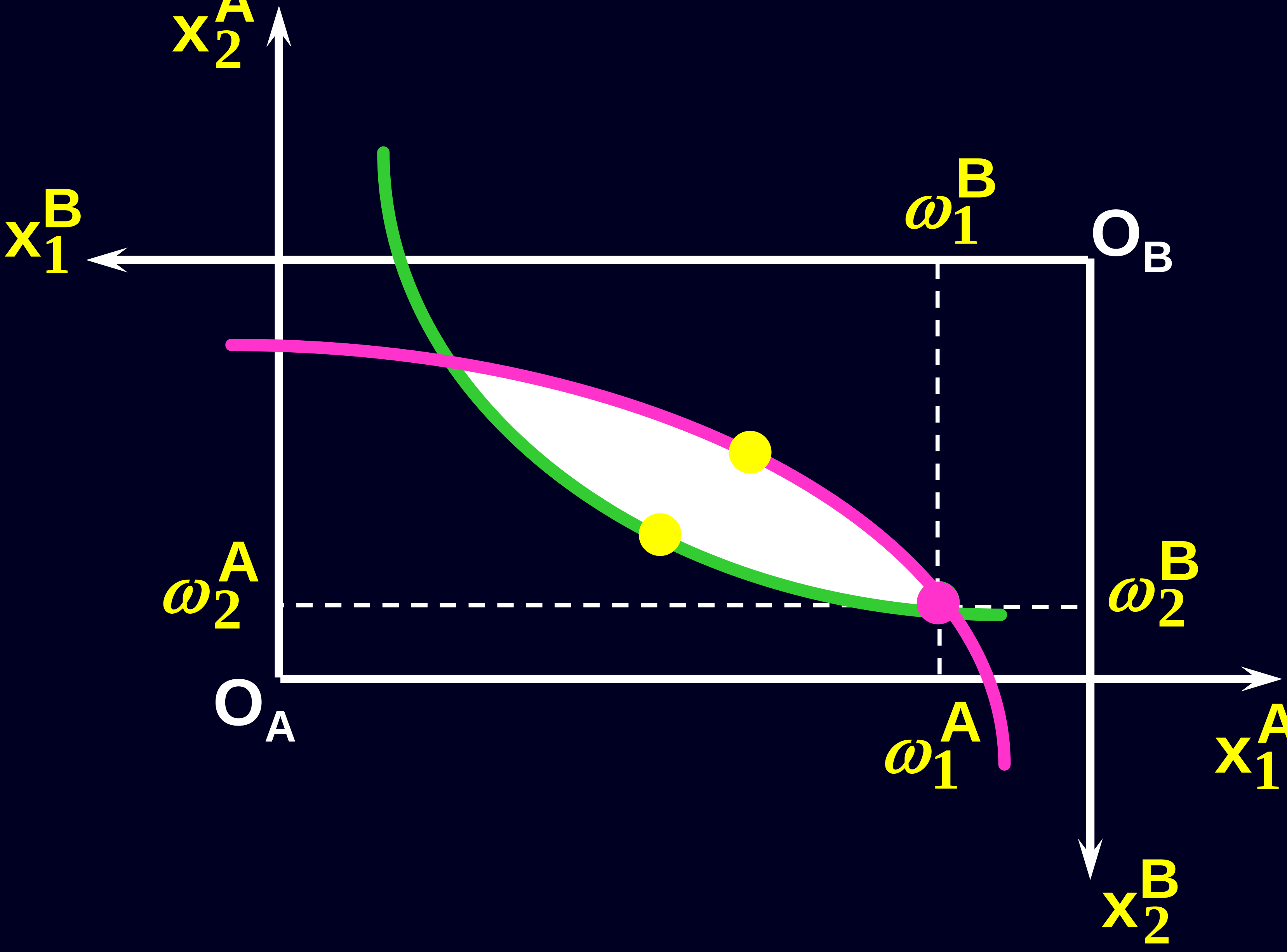


The Core

- The **core** is **the set of all Pareto-optimal allocations that are welfare-improving for both consumer relative to their own endowments.**
- Rational trade should achieve a core allocation.

The Core

- **But which core allocation?**
- **Again, that depends upon the manner in which trade is conducted.**

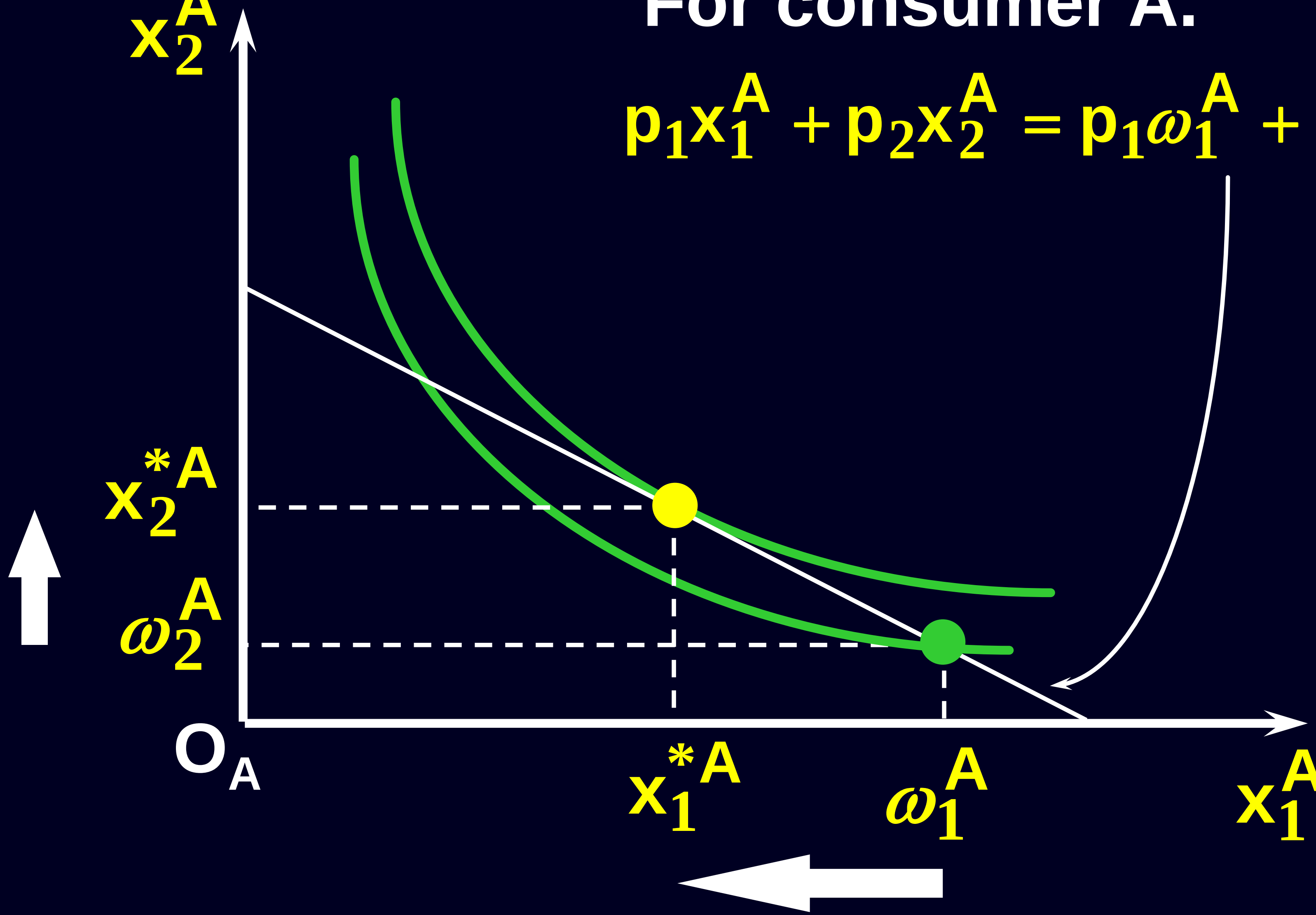
Trade in Competitive Markets

- **Consider trade in perfectly competitive markets.**
- **Each consumer is a price-taker trying to maximize her own utility given p_1, p_2 and her own endowment. That is, ...**

Trade in Competitive Markets

For consumer A.

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are

$$x_1^{*A} - \omega_1^A \quad \text{and} \quad x_2^{*A} - \omega_2^A.$$

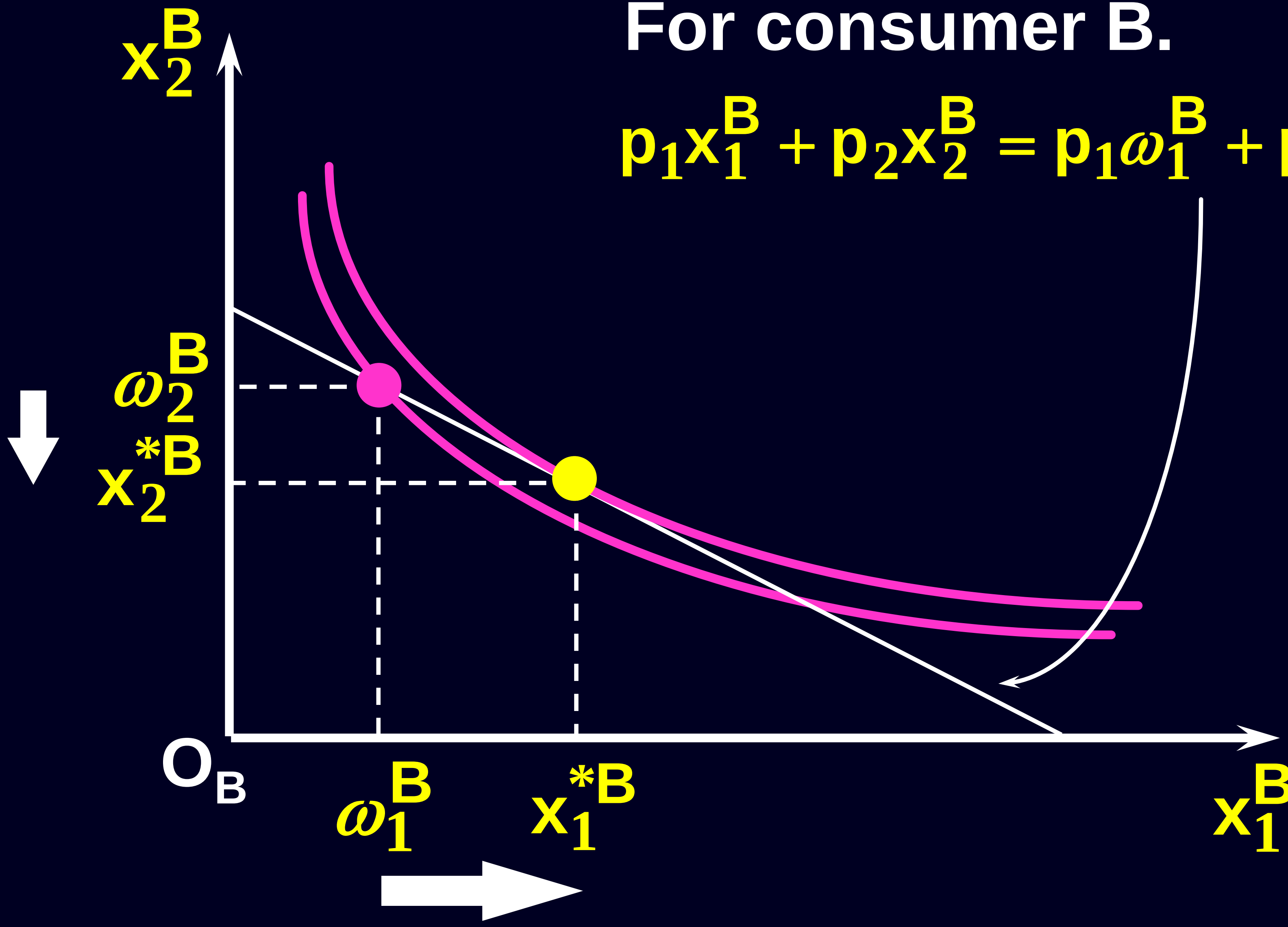
Trade in Competitive Markets

- **And, similarly, for consumer B …**

Trade in Competitive Markets

For consumer B.

$$p_1x_1^B + p_2x_2^B = p_1\omega_1^B + p_2\omega_2^B$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer B's net demands for commodities 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \text{and} \quad x_2^{*B} - \omega_2^B.$$

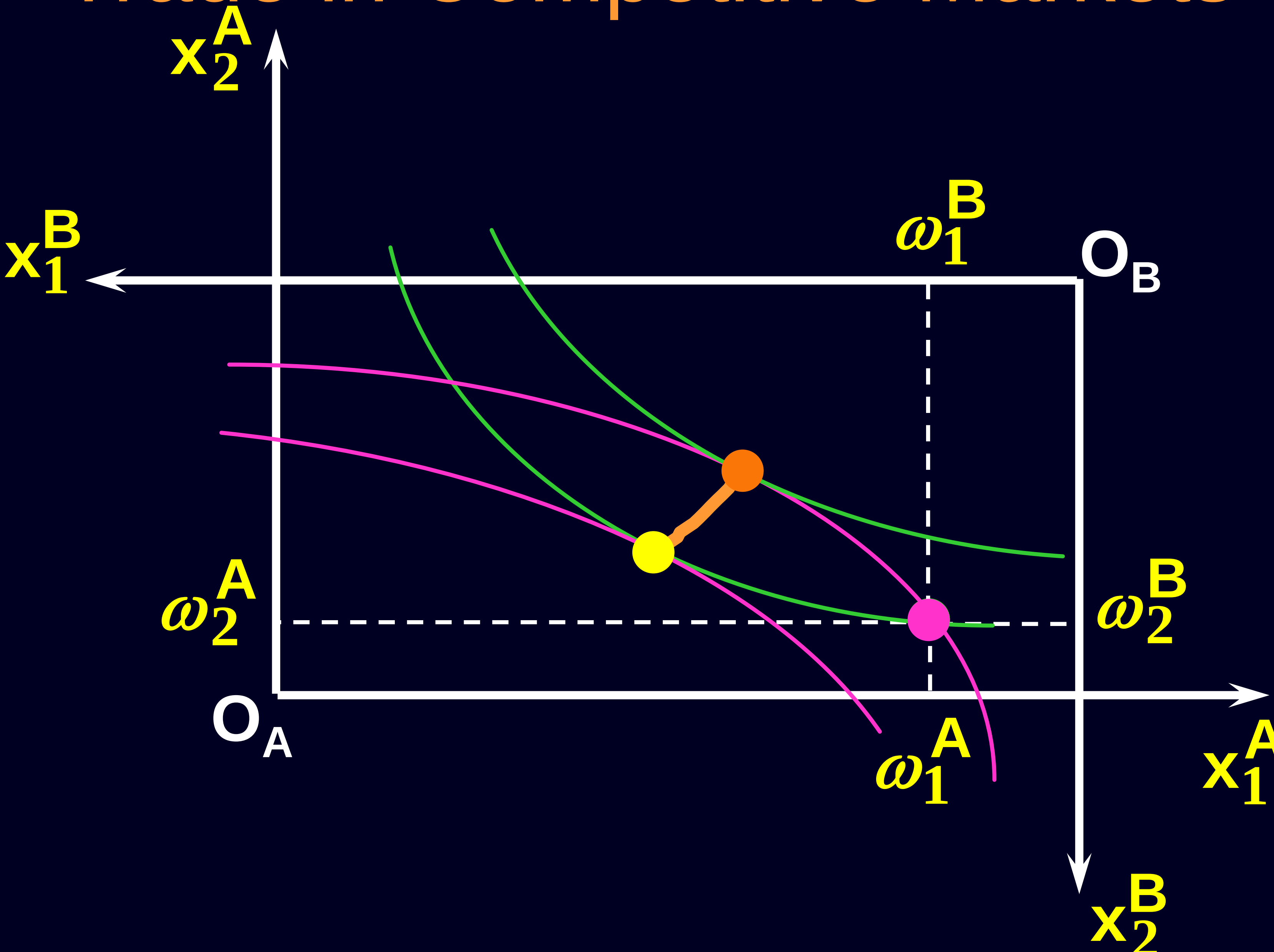
Trade in Competitive Markets

- A **general equilibrium** occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

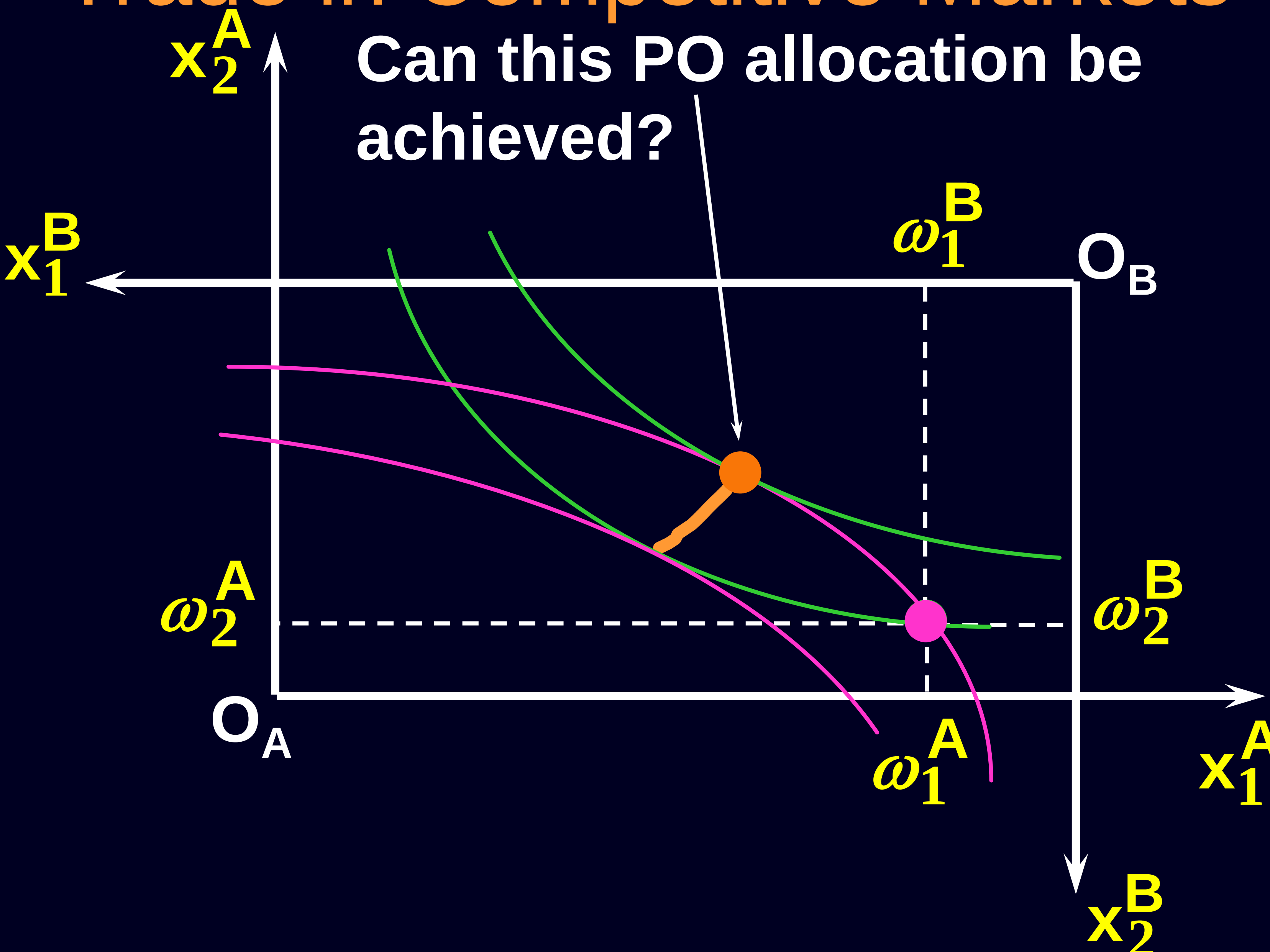
$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and $$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

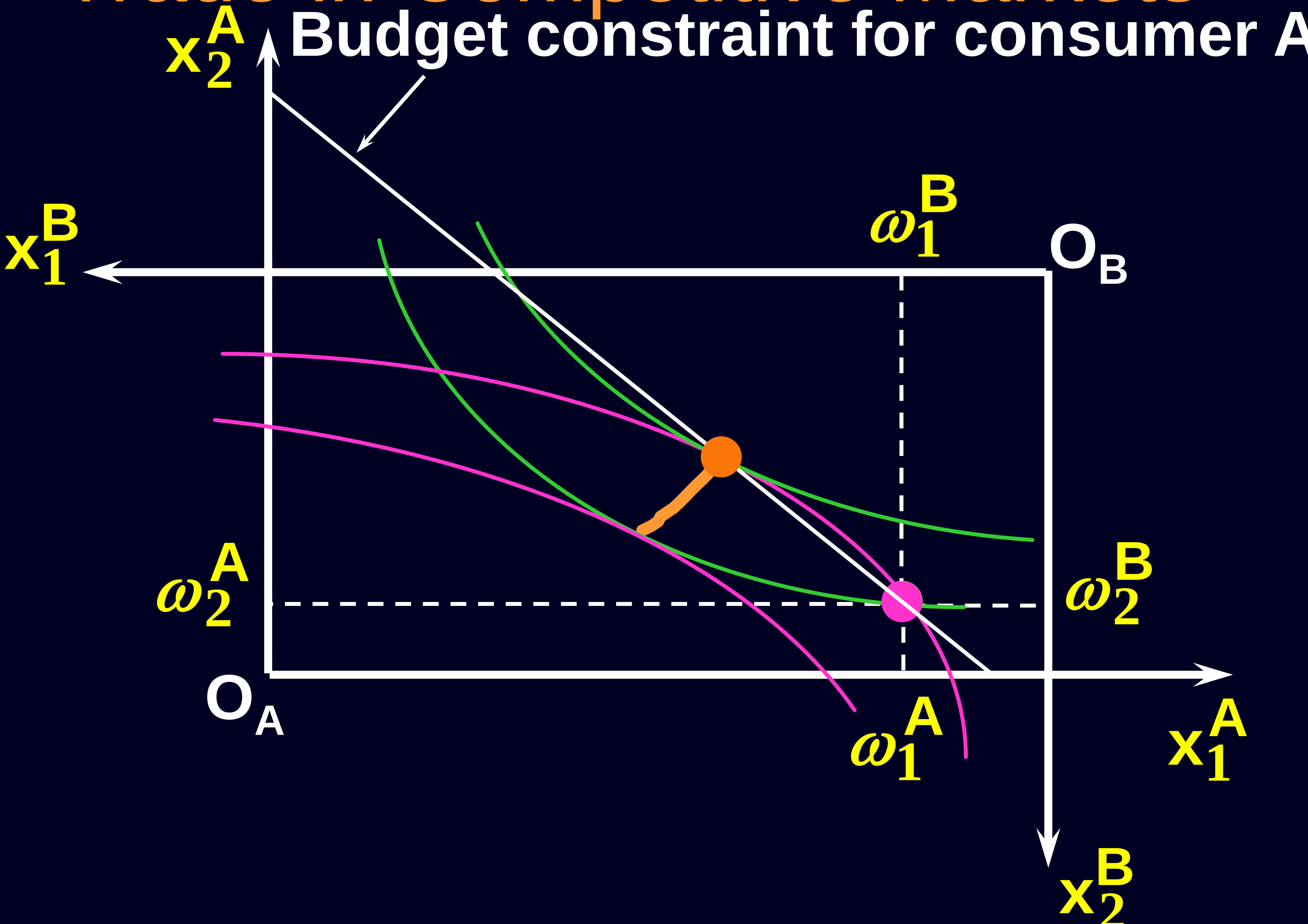
Trade in Competitive Markets



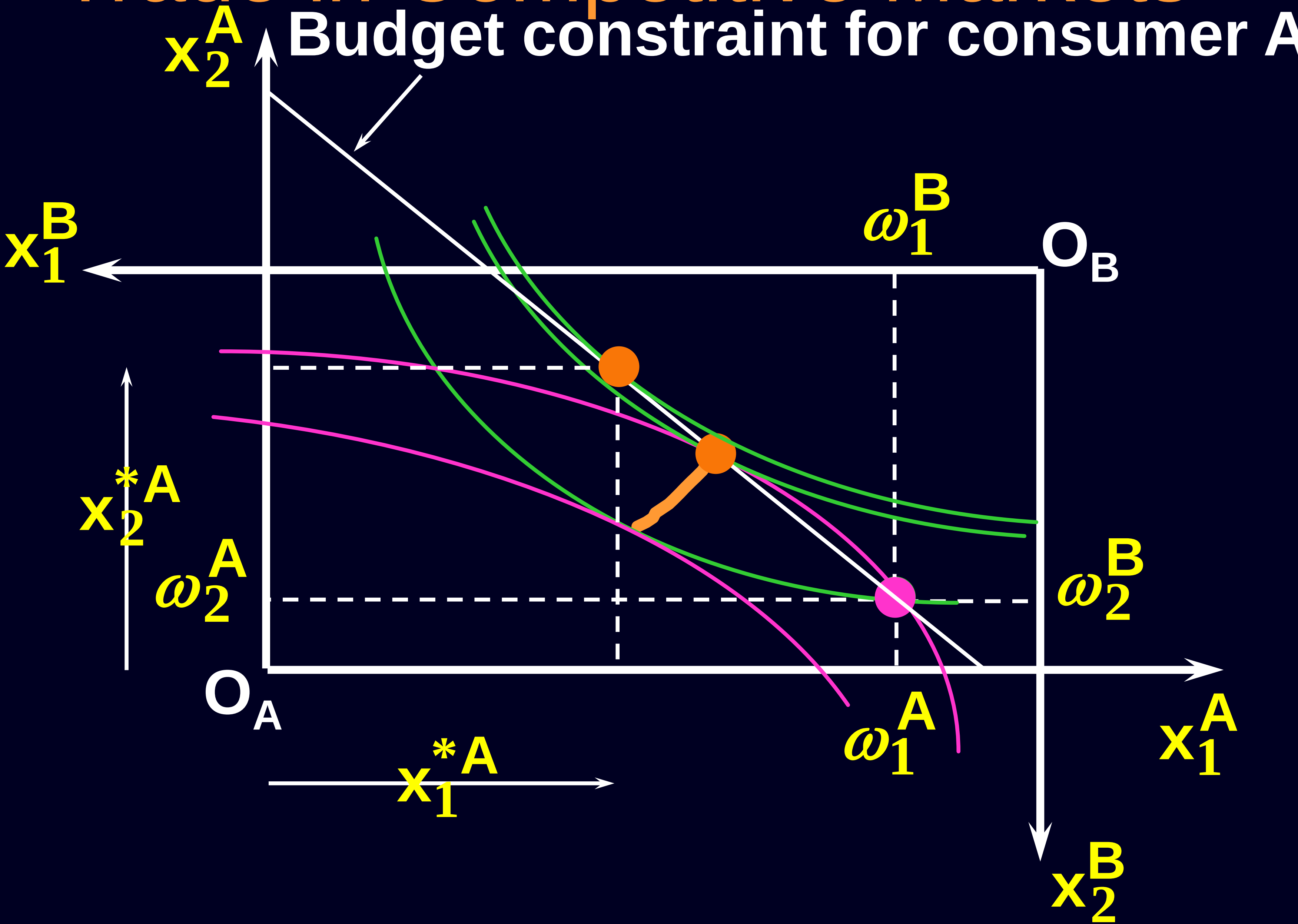
Trade in Competitive Markets



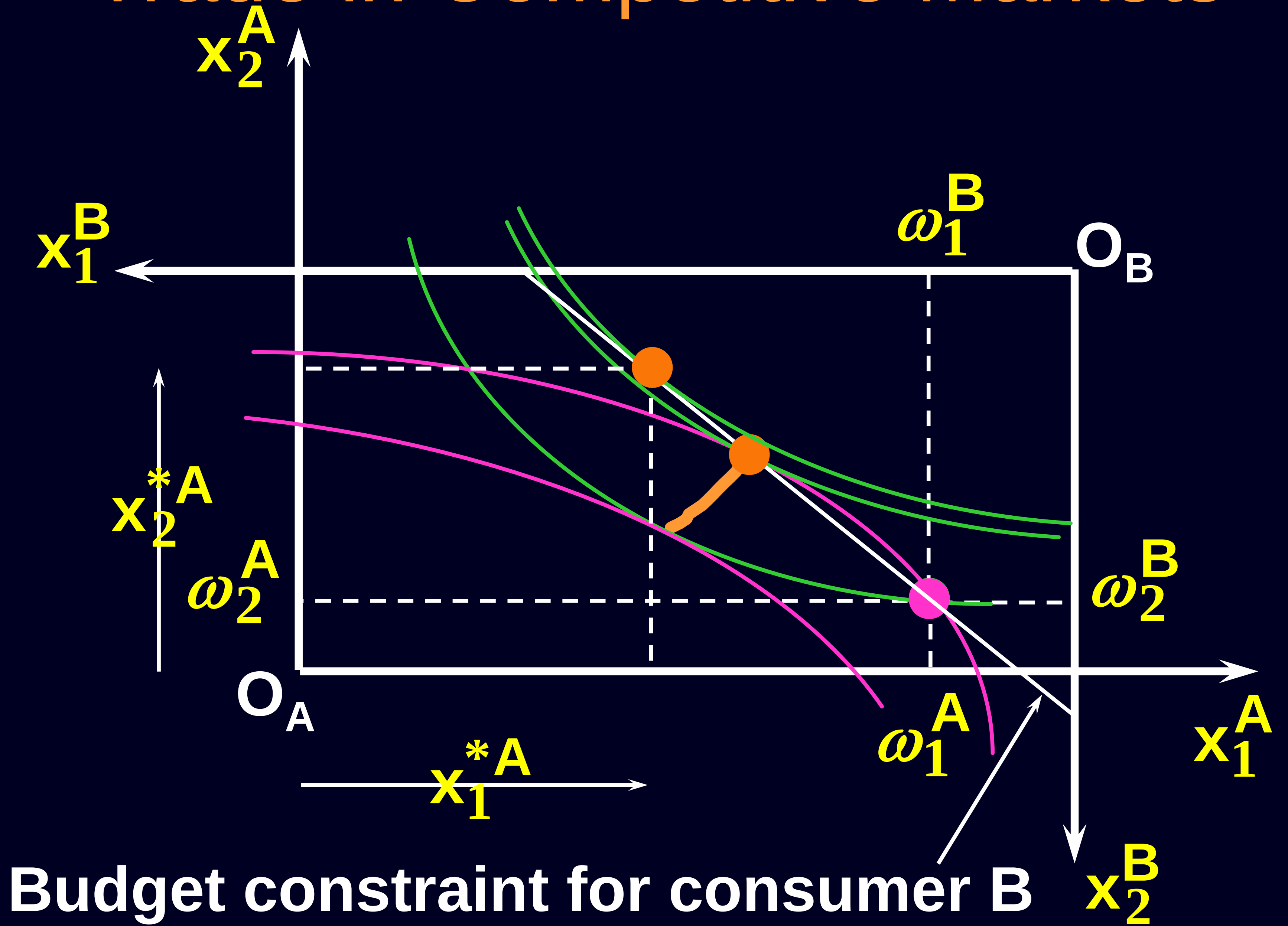
Trade in Competitive Markets



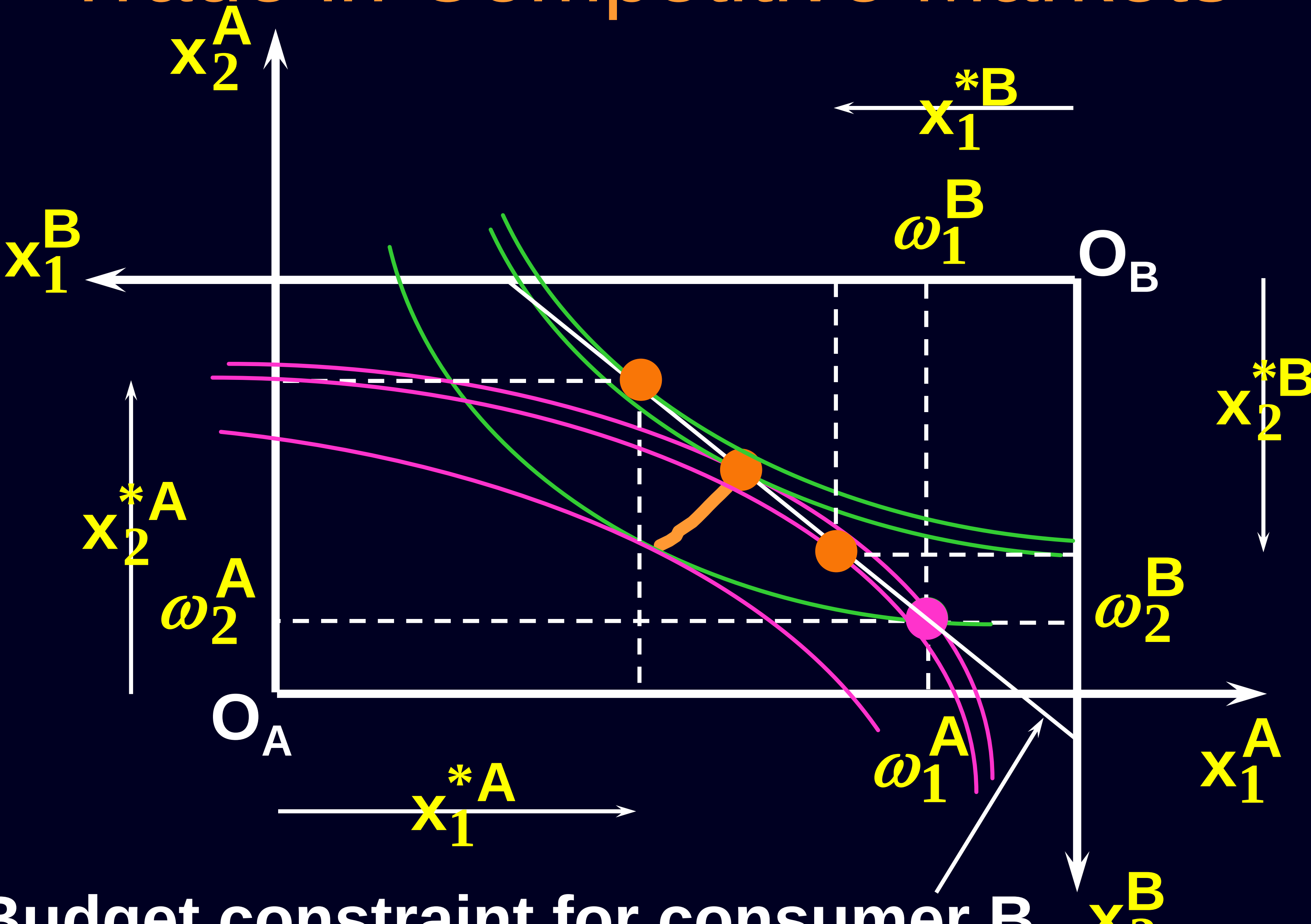
Trade in Competitive Markets



Trade in Competitive Markets

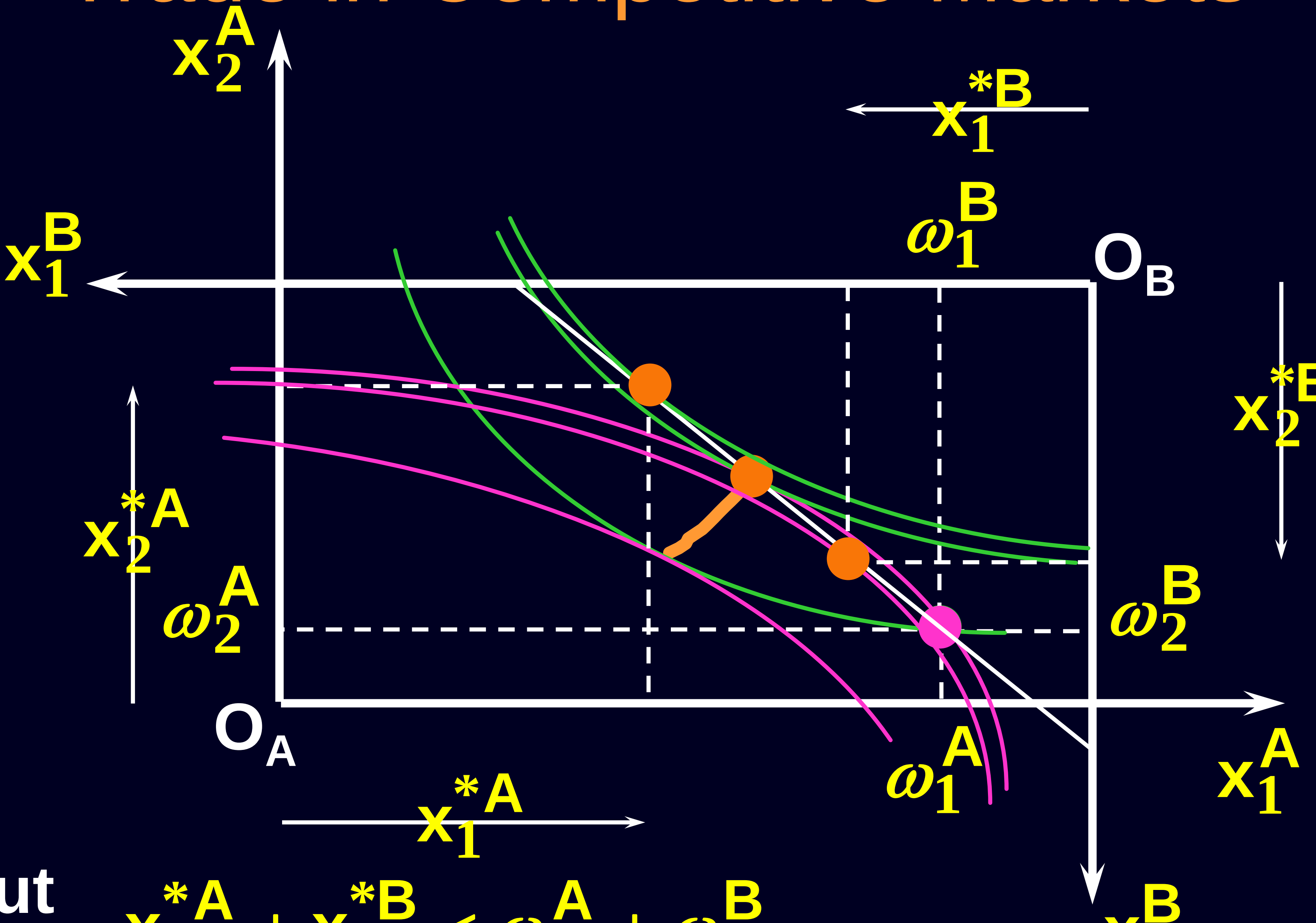


Trade in Competitive Markets



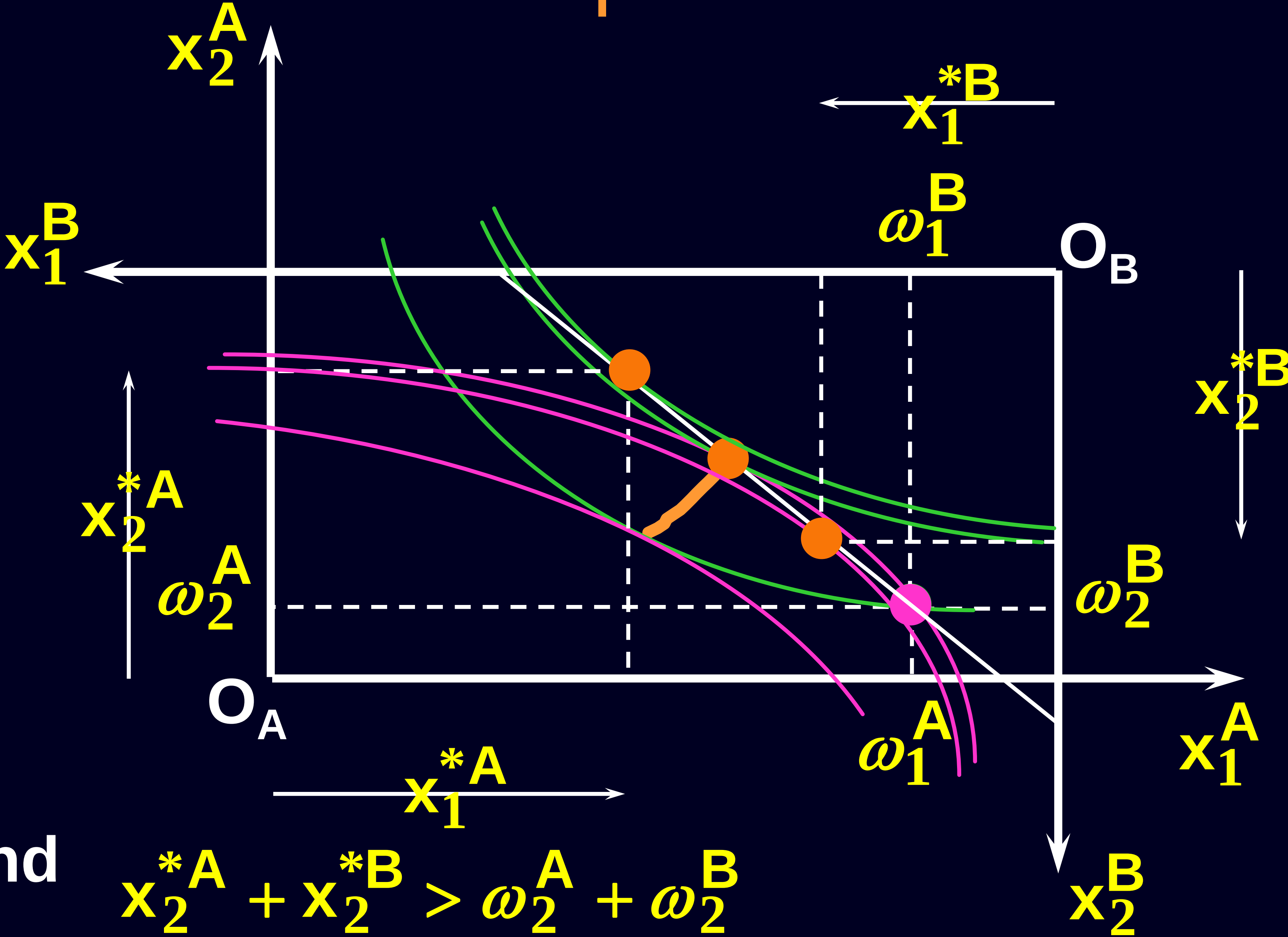
Budget constraint for consumer B

Trade in Competitive Markets



But $x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$

Trade in Competitive Markets

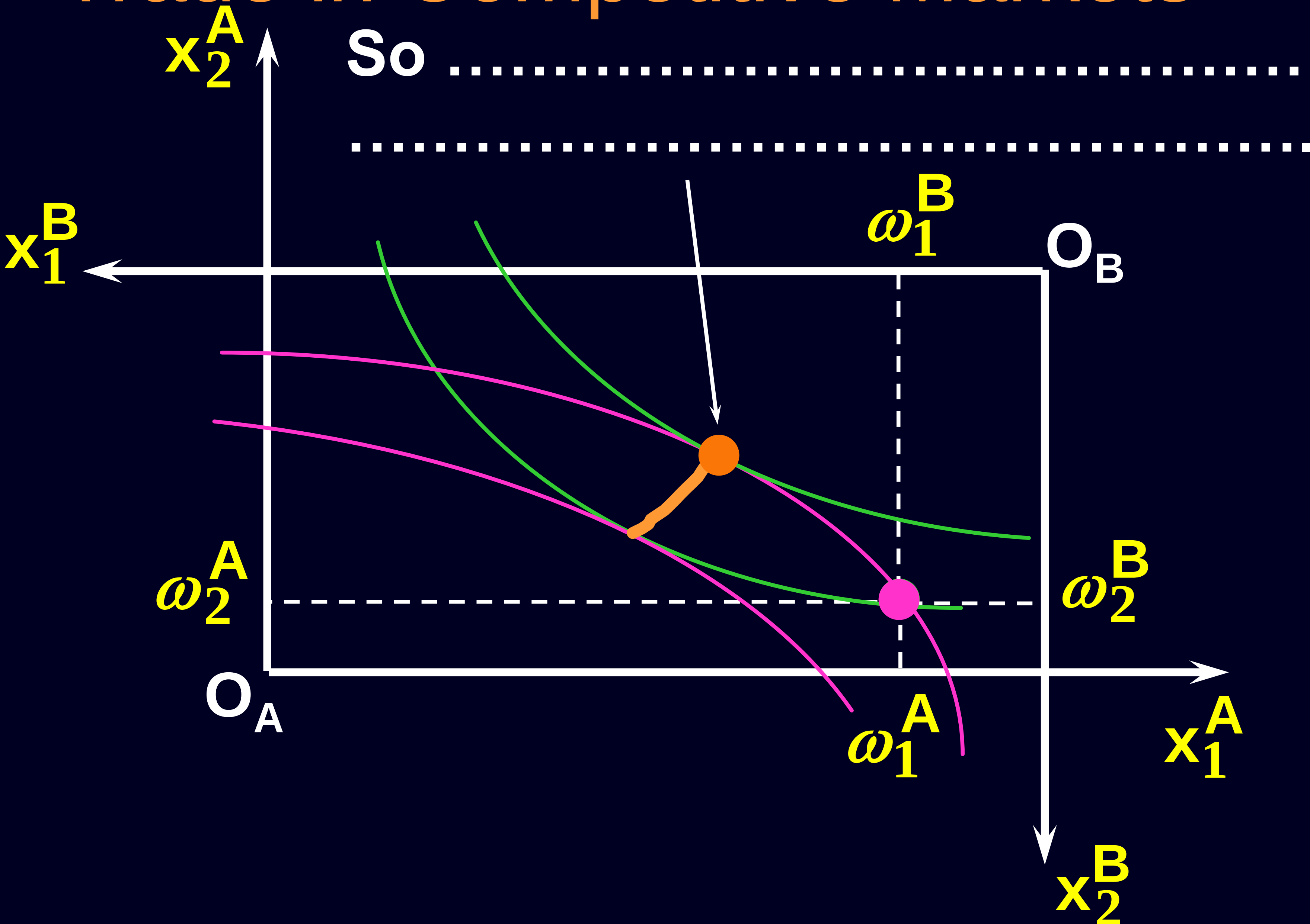


and $x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$

Trade in Competitive Markets

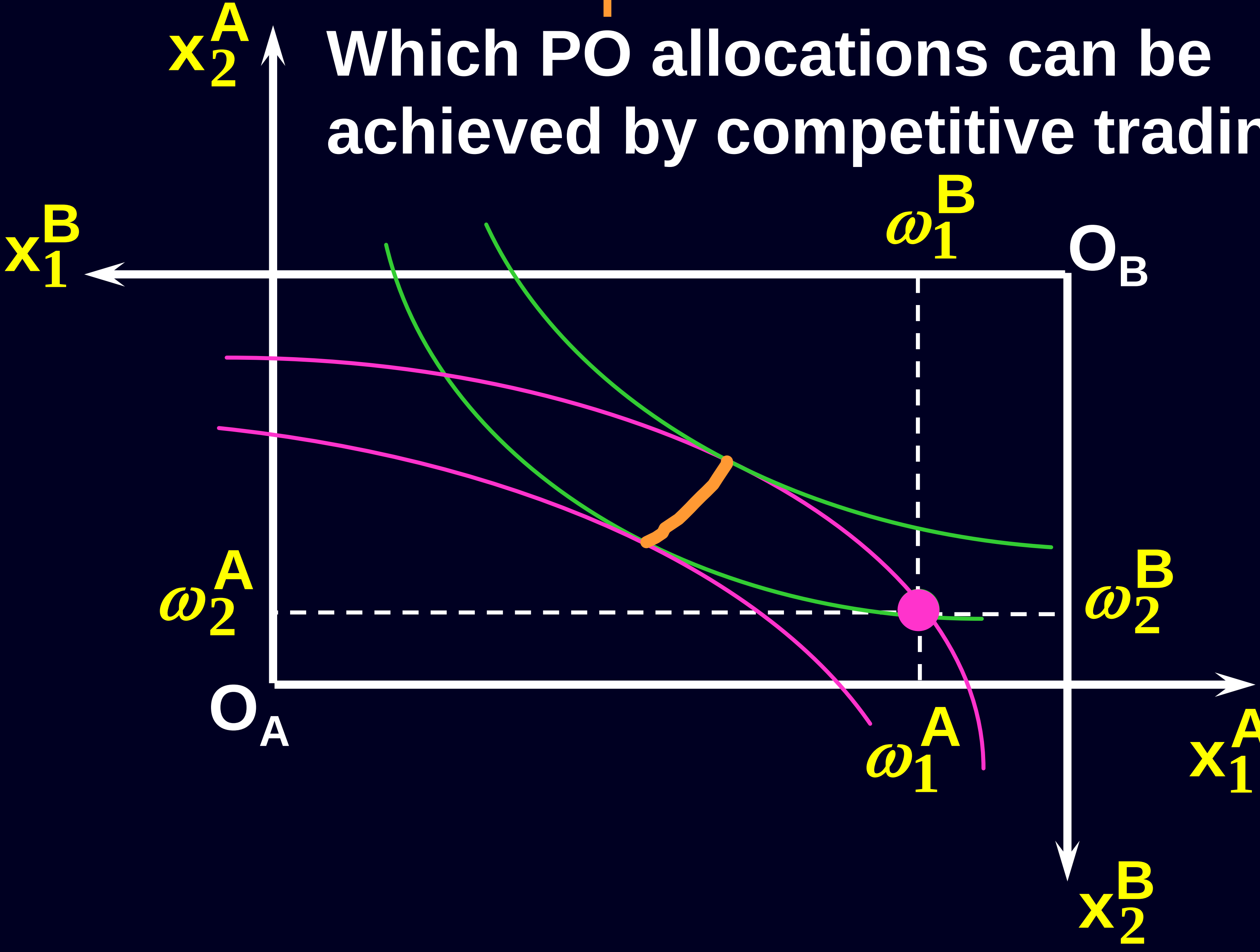
- **So at the given prices p_1 and p_2 there is an**
  - **excess supply of ……………………**
  - **excess demand for …………………..**
- **Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.**

Trade in Competitive Markets



Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?

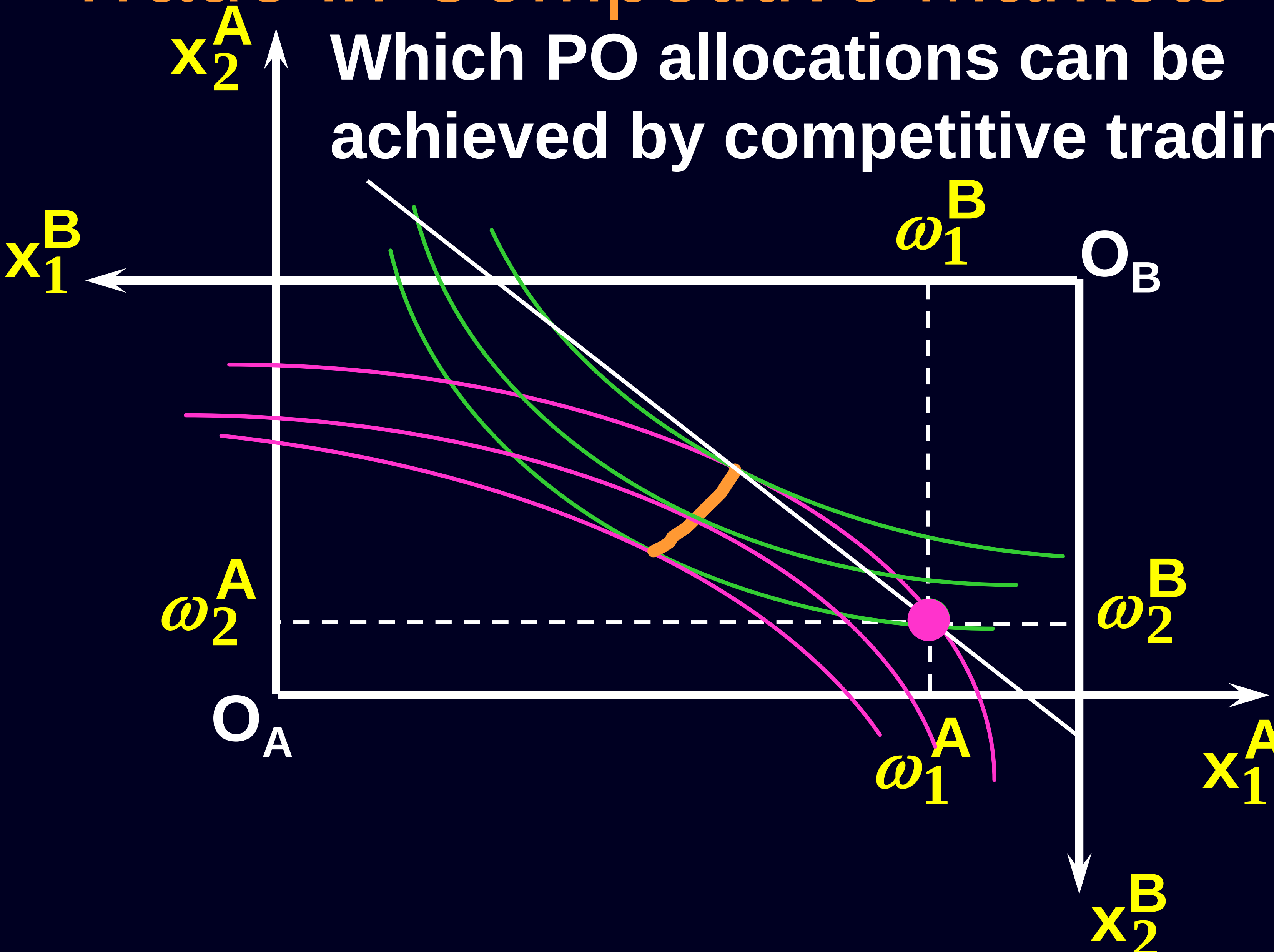


Trade in Competitive Markets

- **Since there is an excess demand for commodity 2, p_2 will…….**
- **Since there is an excess supply of commodity 1, p_1 will…….**
- **The slope of the budget constraints is ………. so the budget constraints will pivot about the endowment point and become ………………**

Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



Trade in Competitive Markets

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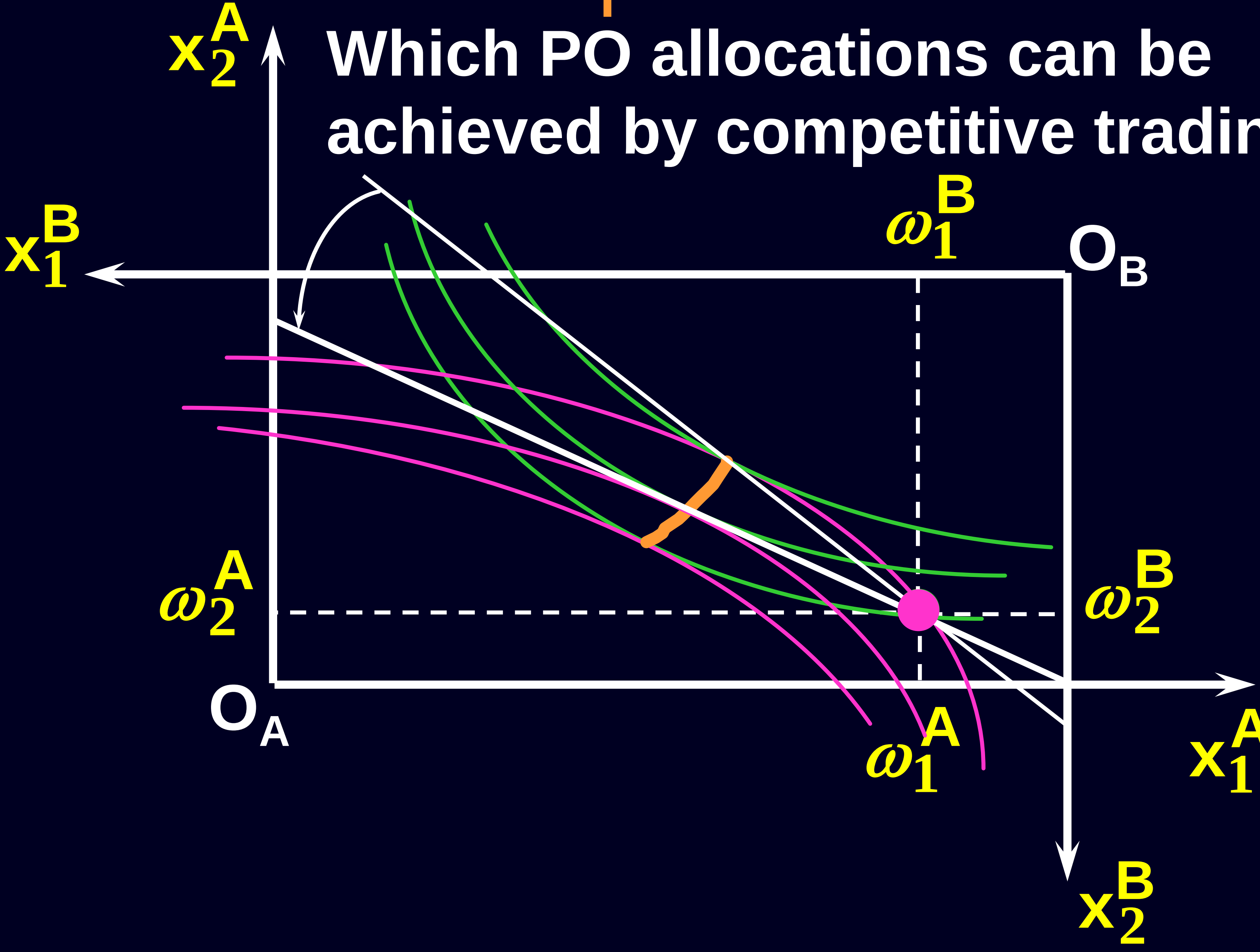
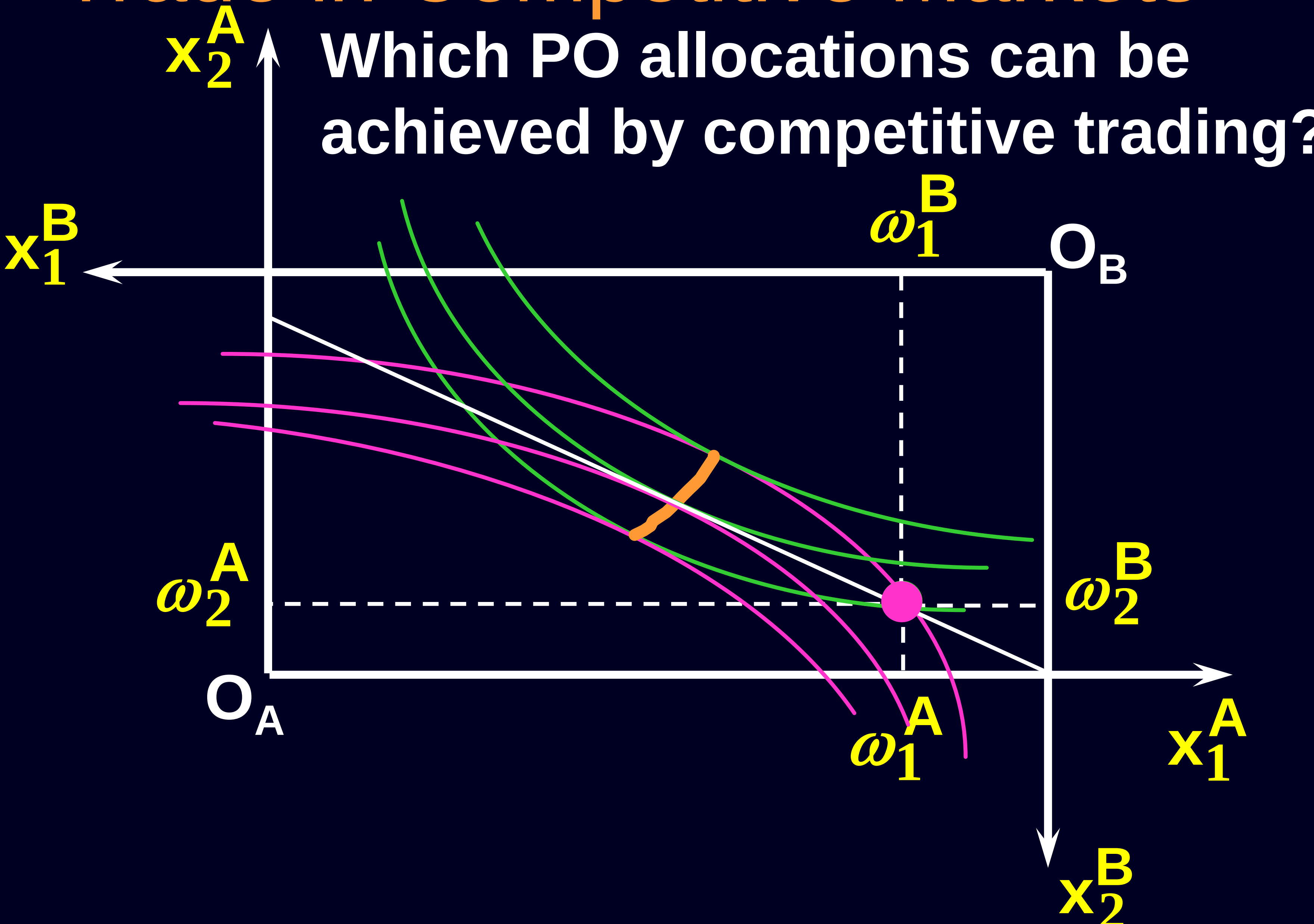
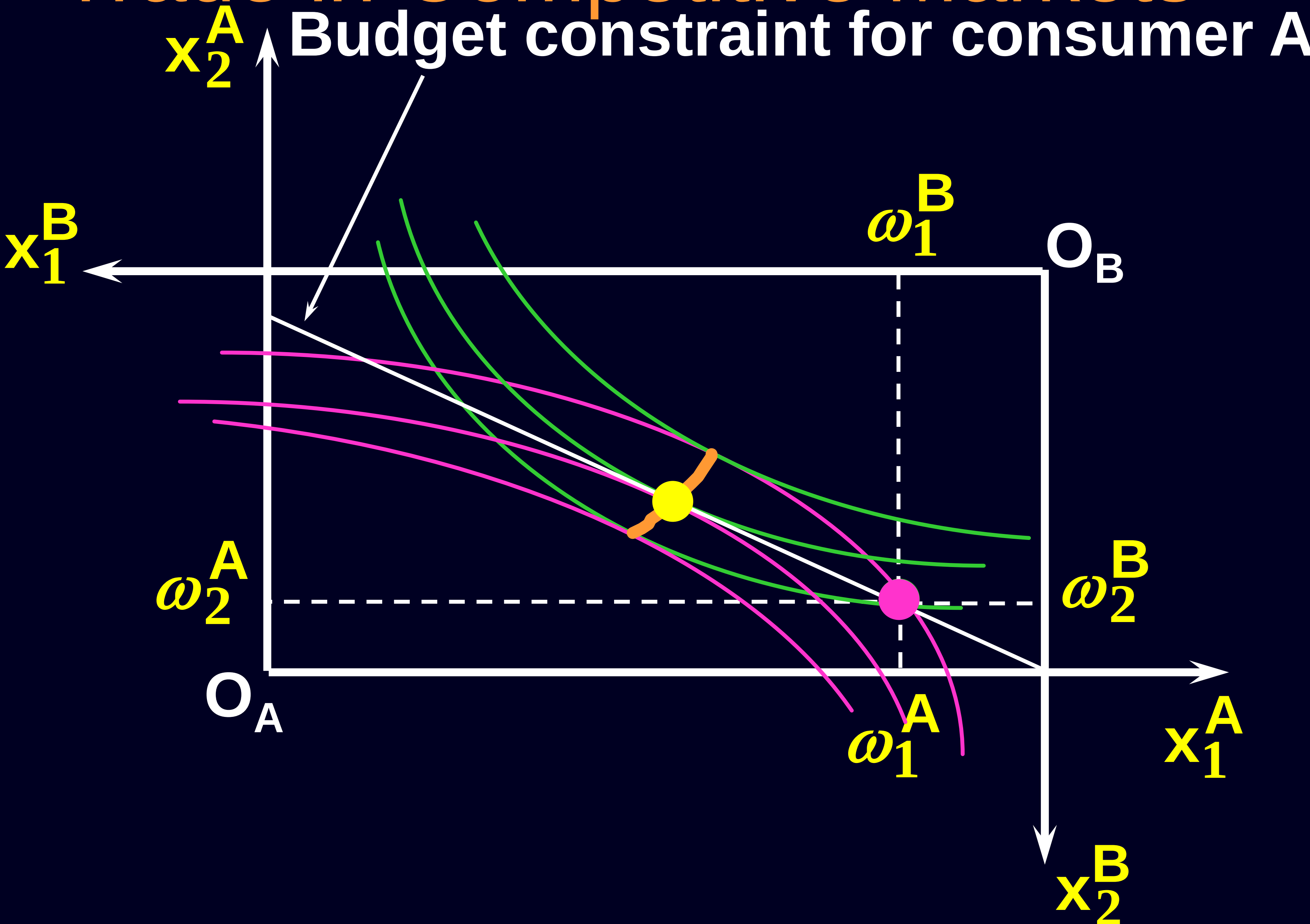


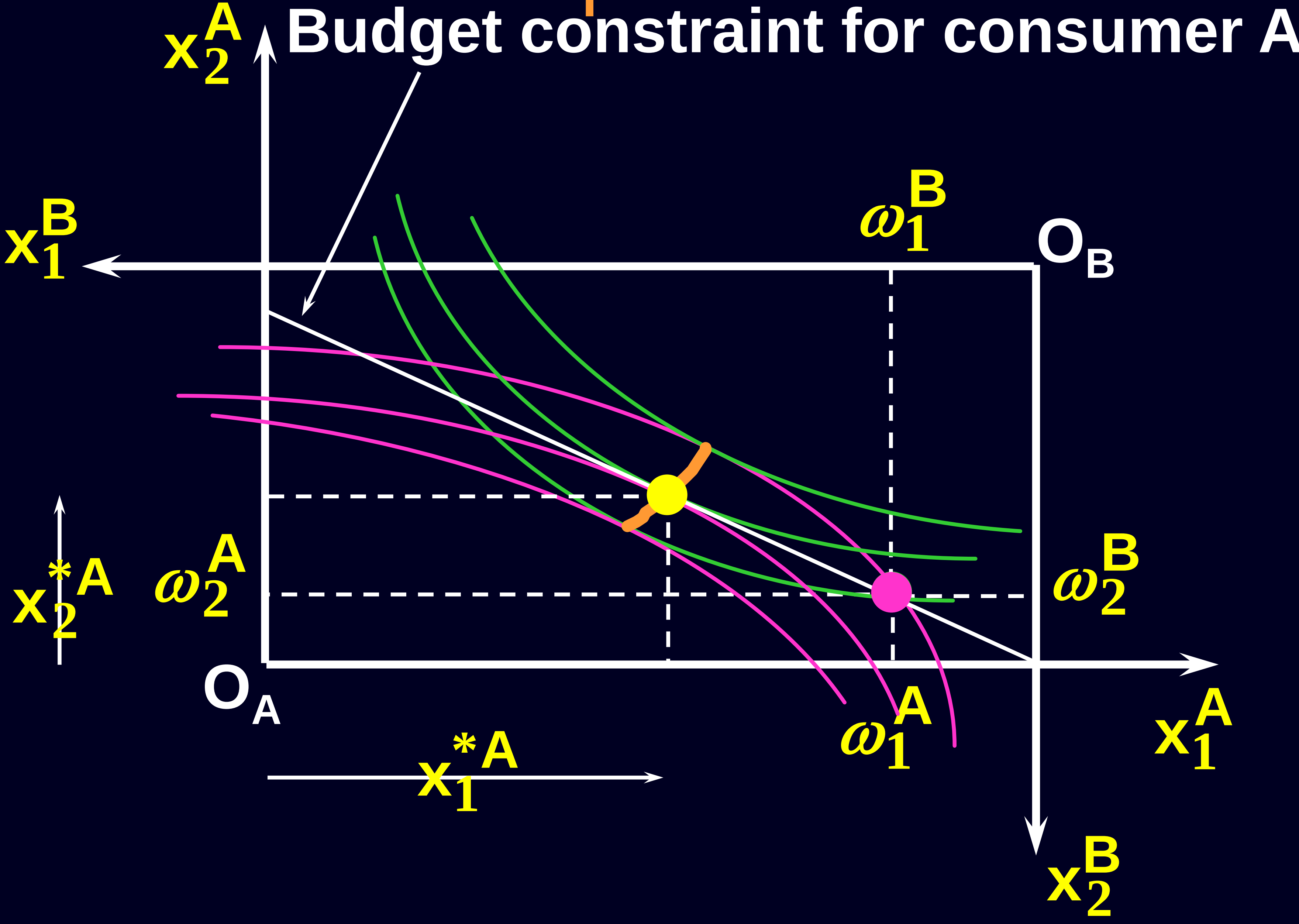
Trade in Competitive Markets



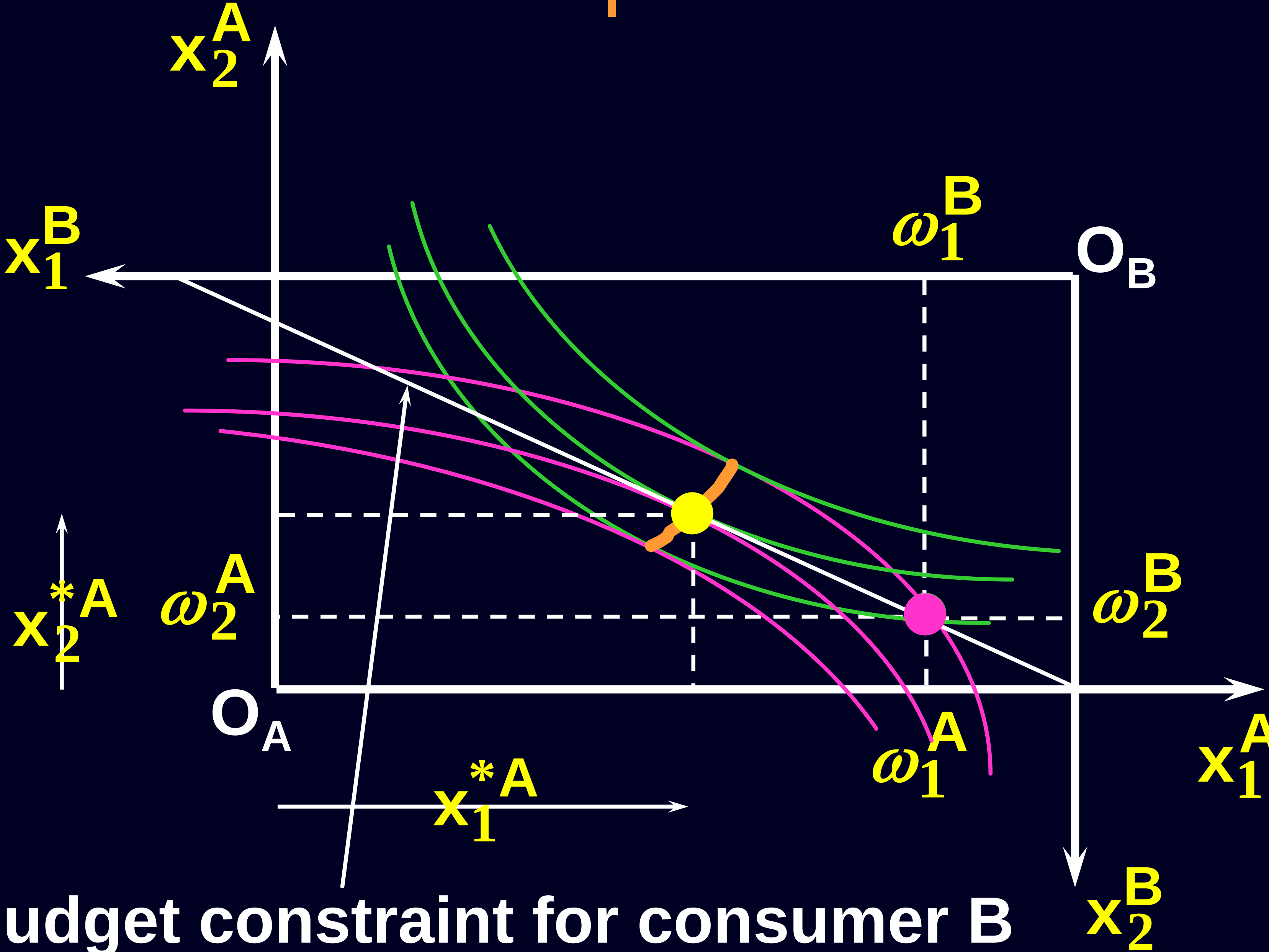
Trade in Competitive Markets



Trade in Competitive Markets

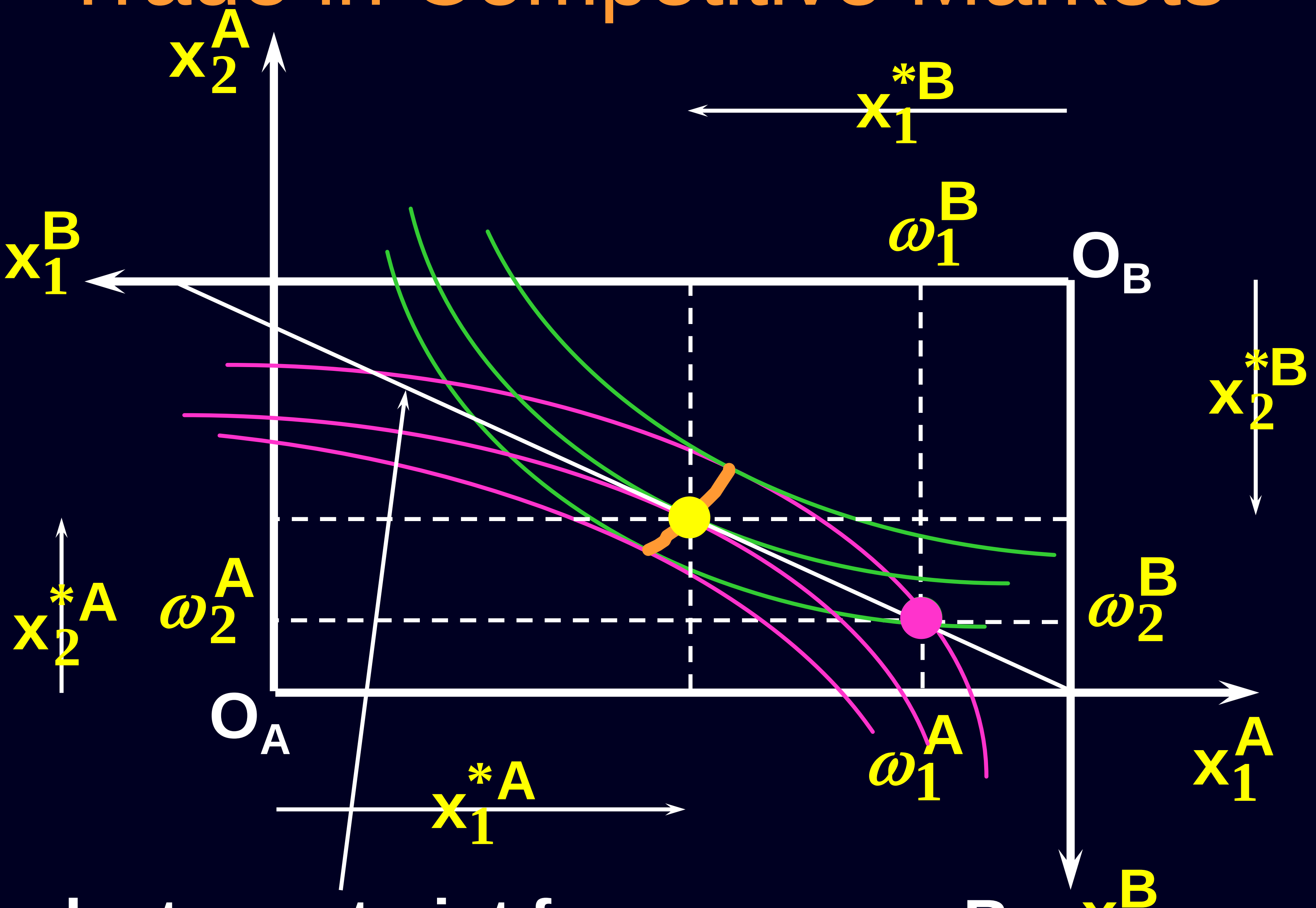


Trade in Competitive Markets



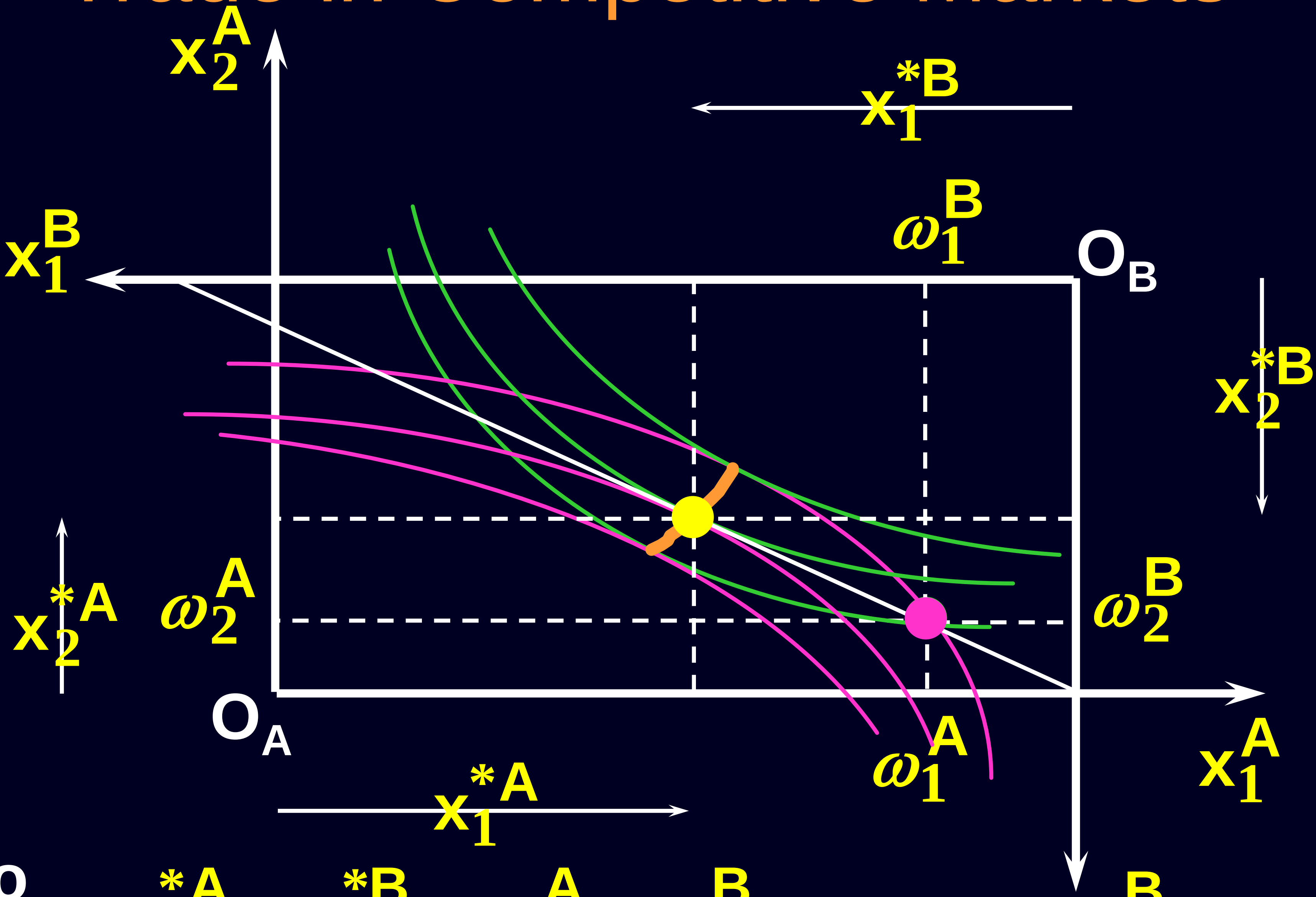
Budget constraint for consumer B

Trade in Competitive Markets



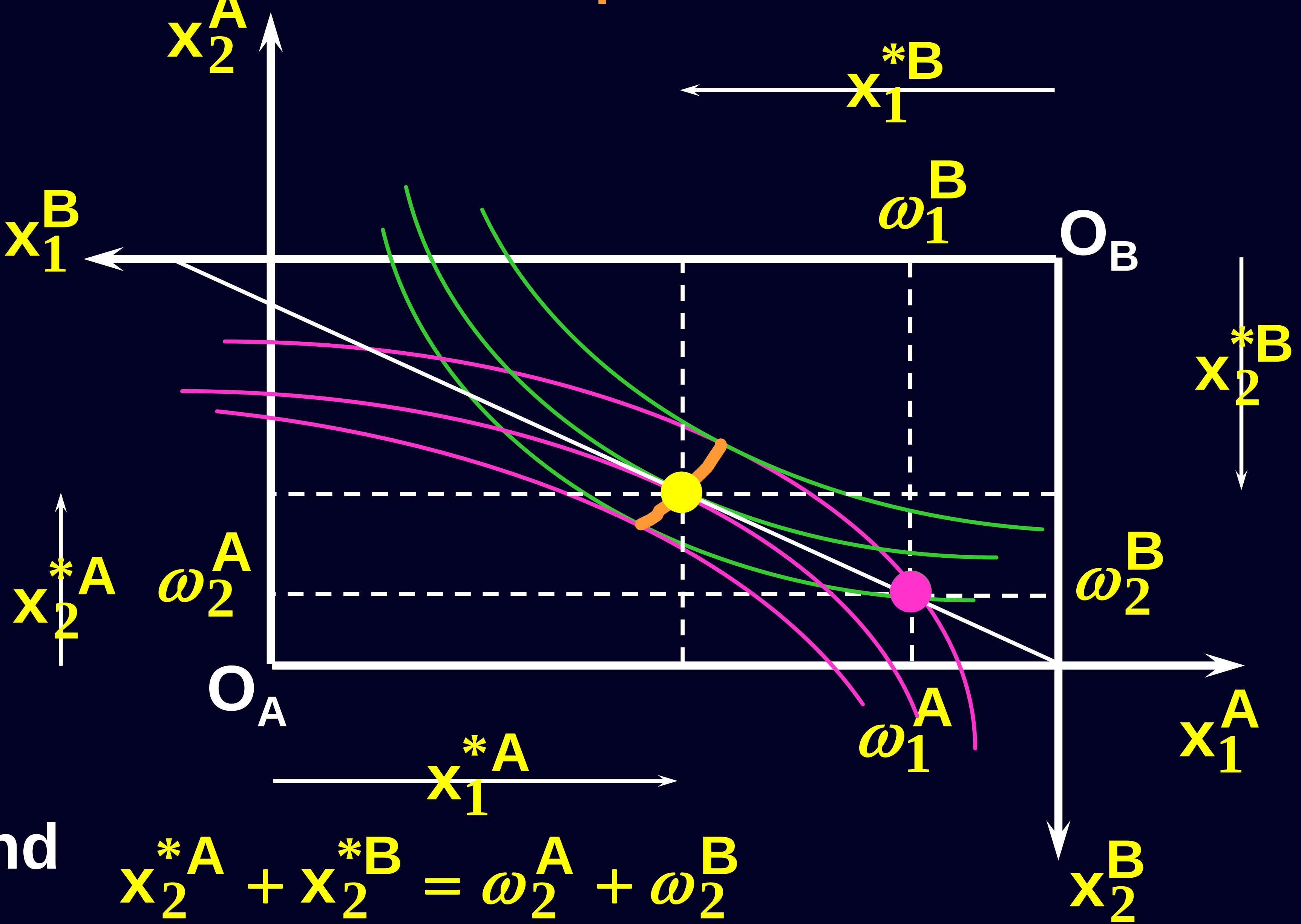
Budget constraint for consumer B

Trade in Competitive Markets



So $x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$

Trade in Competitive Markets



and $x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$

Trade in Competitive Markets

- At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.
- Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.
- This is an example of the **First Fundamental Theorem of Welfare Economics**.

First Fundamental Theorem of Welfare Economics

- **Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.**

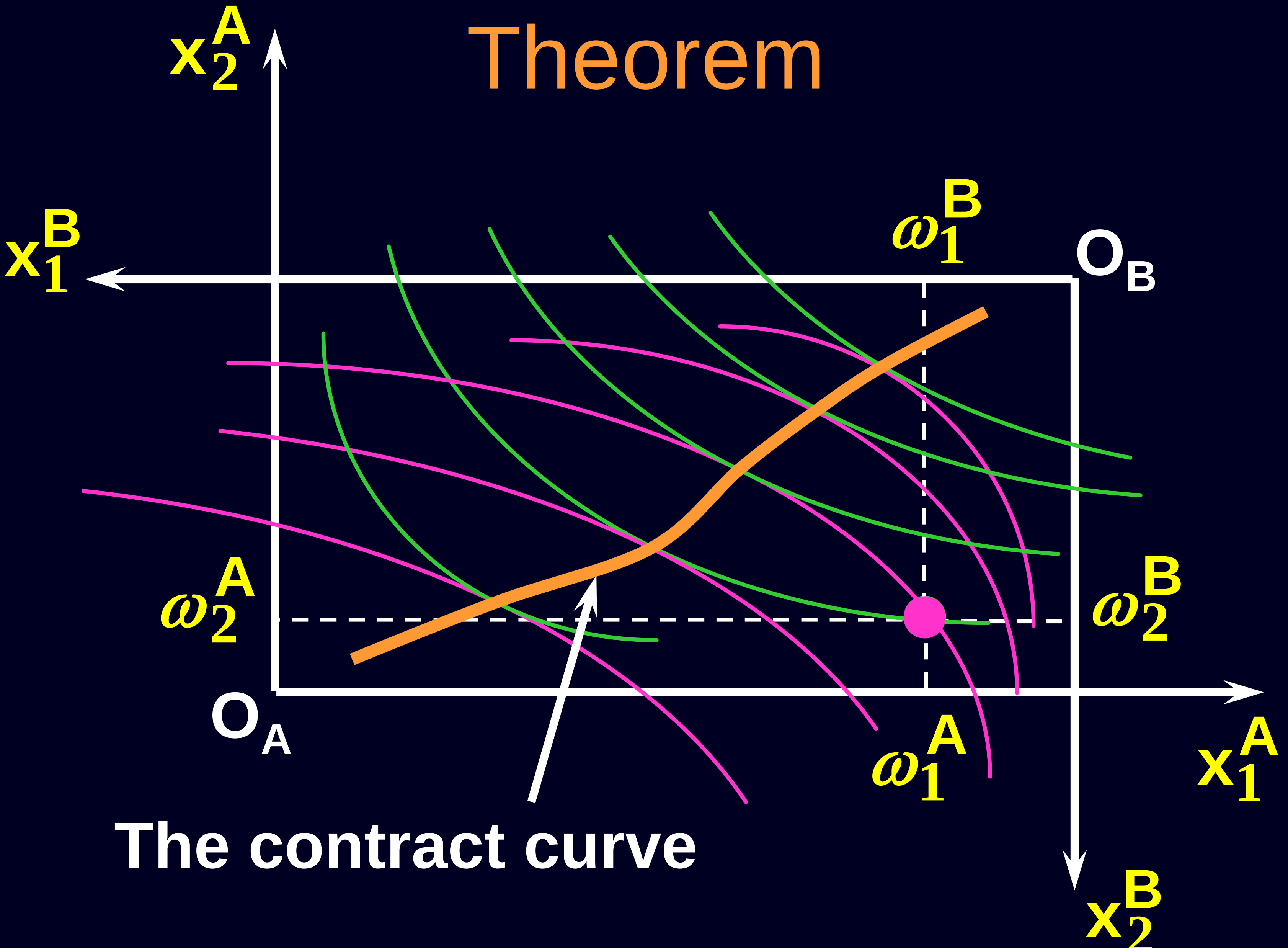
Second Fundamental Theorem of Welfare Economics

- **The First Theorem is followed by a second that states that any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.**

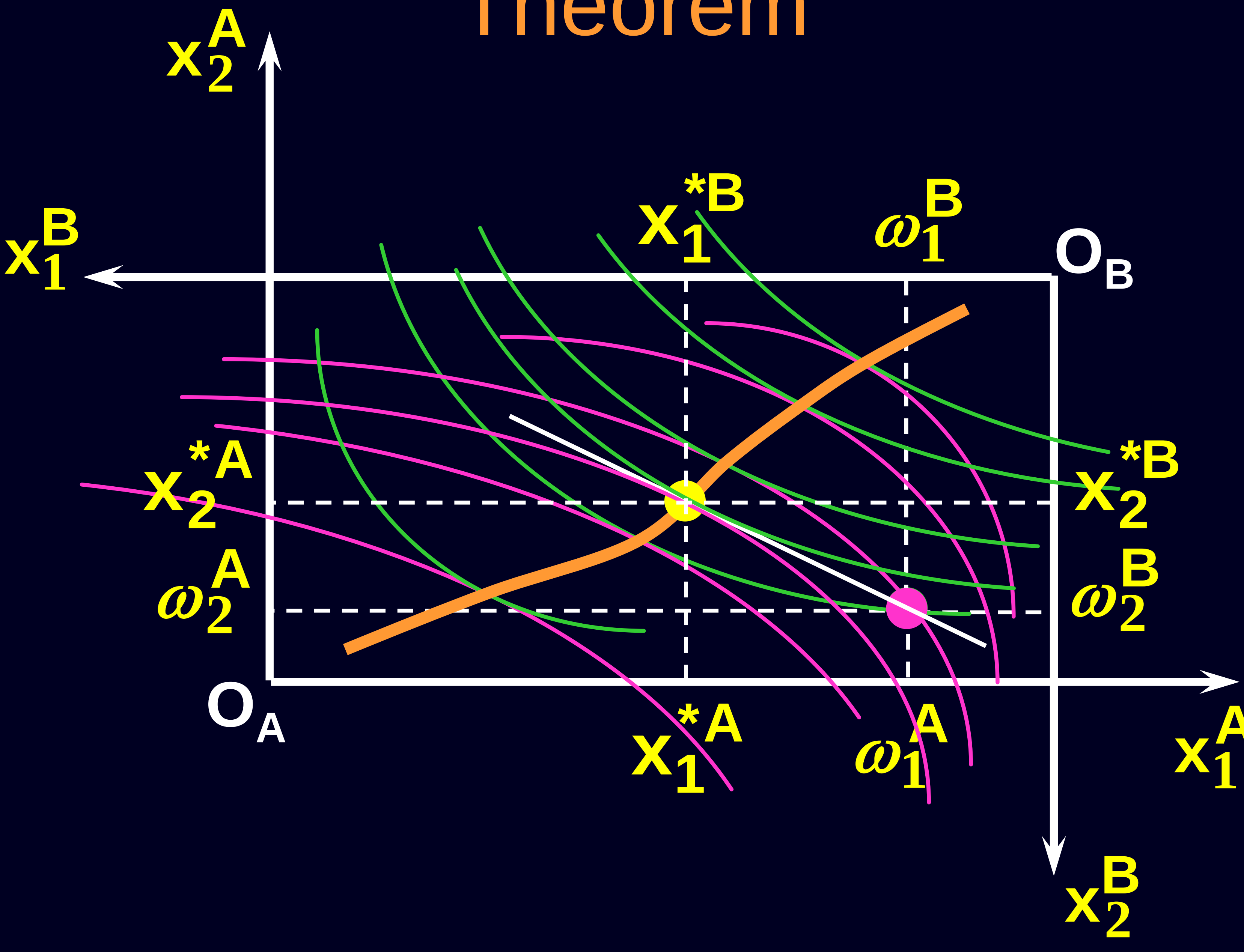
Second Fundamental Theorem of Welfare Economics

- **Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.**

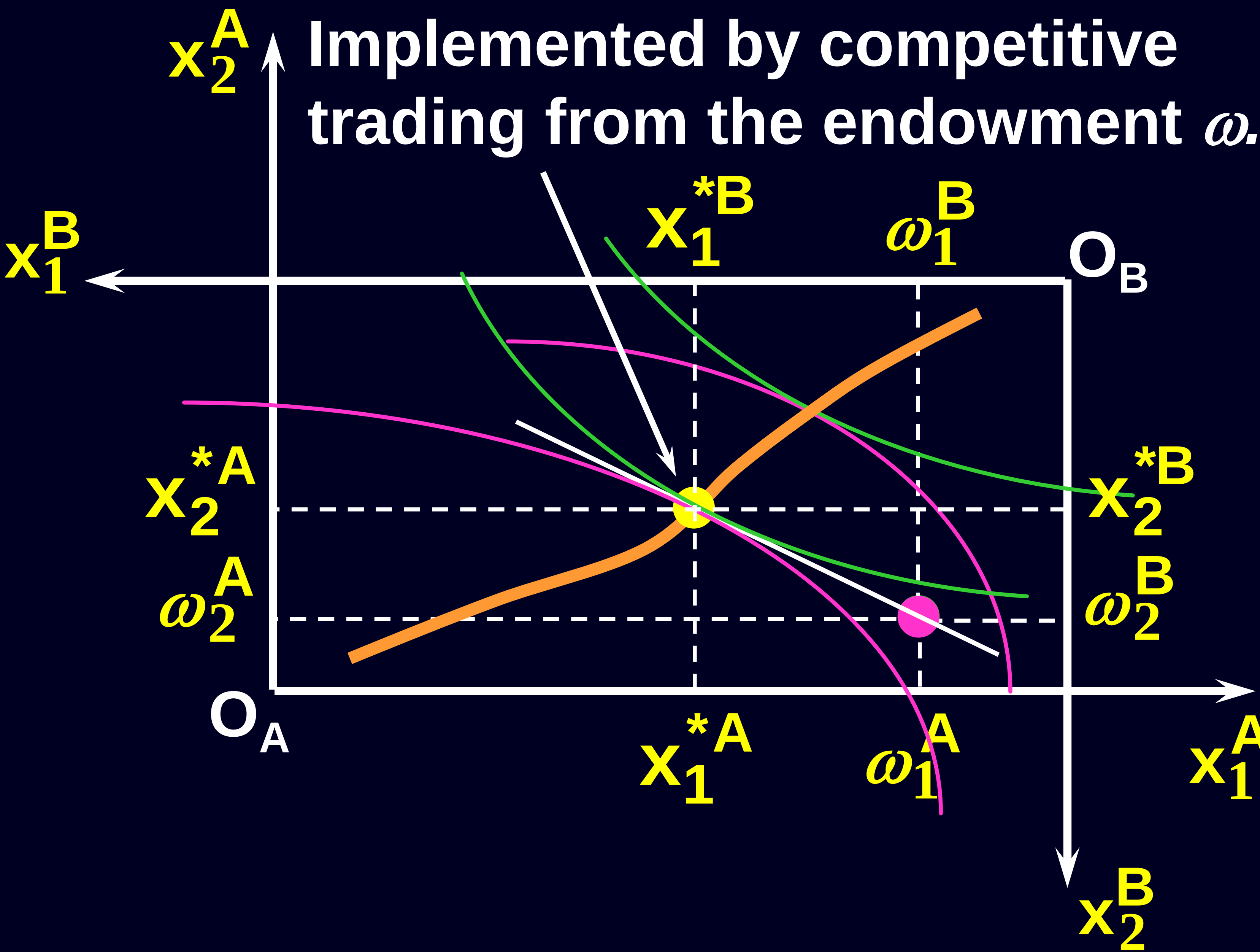
Second Fundamental Theorem



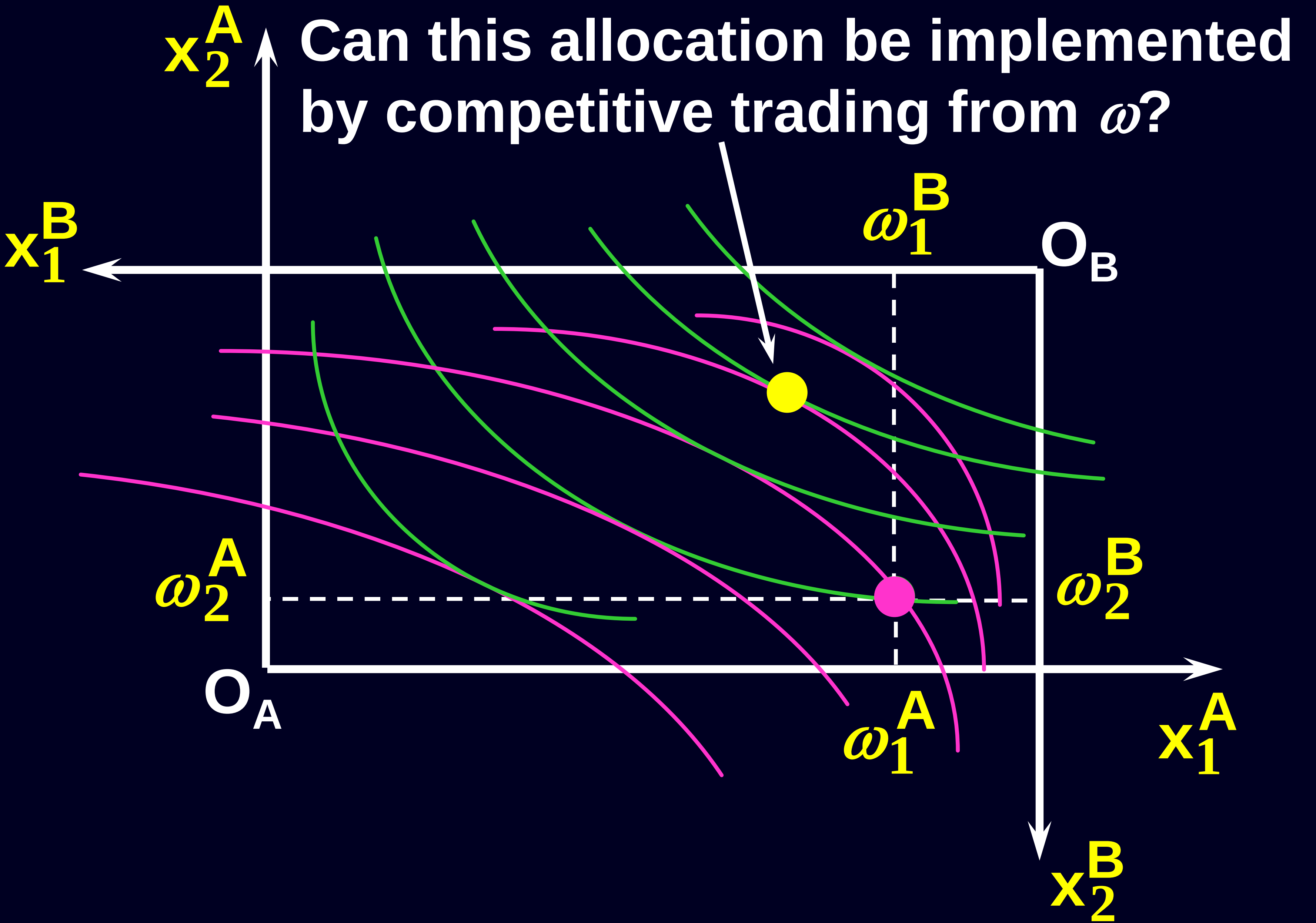
Second Fundamental Theorem



Second Fundamental Theorem

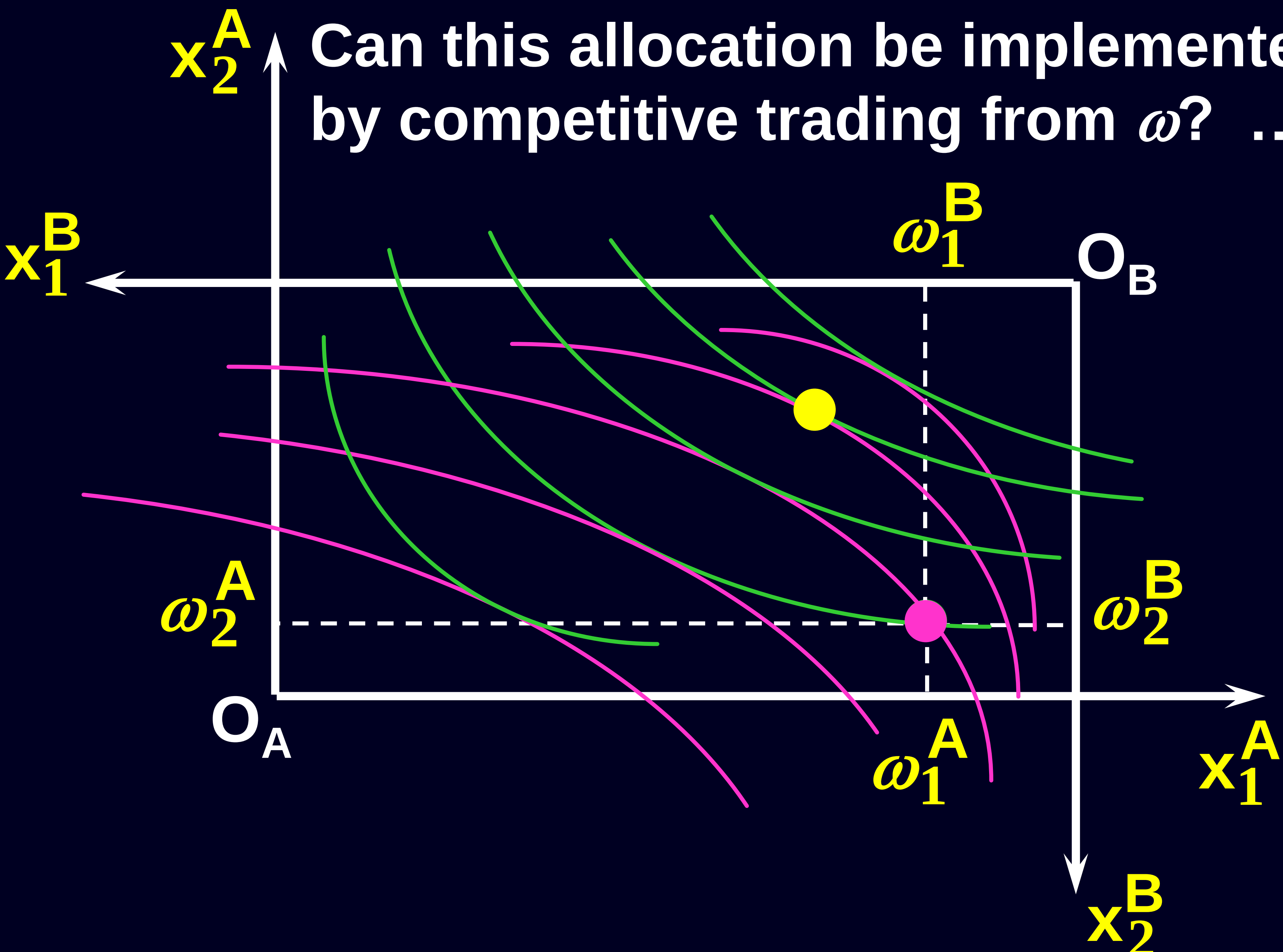


Second Fundamental Theorem



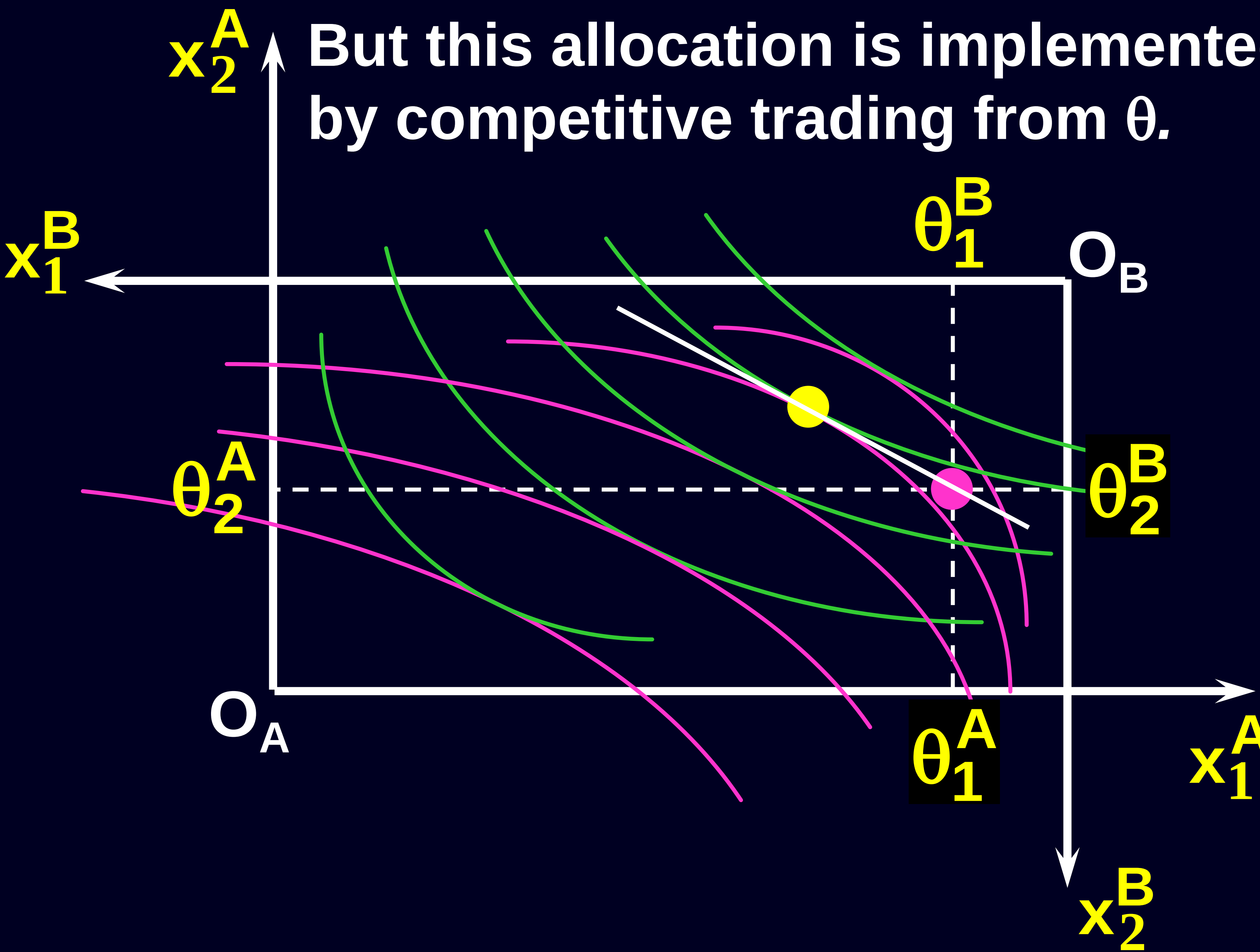
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω? ……



Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ.



Walras' Law

- **Walras' Law is an identity; i.e. a statement that is true for any positive prices (p_1,p_2), whether these are equilibrium prices or not.**

Walras' Law

- Every consumer's preferences are well-behaved so, for any positive prices (p_1,p_2), each consumer spends all of his budget.
- For consumer A:

$$p_1x_1^{*A} + p_2x_2^{*A} = p_1\omega_1^A + p_2\omega_2^A$$

For consumer B:

$$p_1x_1^{*B} + p_2x_2^{*B} = p_1\omega_1^B + p_2\omega_2^B$$

Walras' Law

$$p_1x_1^{*A} + p_2x_2^{*A} = p_1\omega_1^A + p_2\omega_2^A$$

$$p_1x_1^{*B} + p_2x_2^{*B} = p_1\omega_1^B + p_2\omega_2^B$$

Summing gives

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Rearranged,

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

That is, ...

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$
$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B)$$
$$= 0.$$

This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2 -- this is Walras' Law.

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implications of Walras' Law

So one implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of commodity 1? That is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.$$

Implications of Walras' Law

So a second implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.